International Journal of Current Engineering and Technology
E-ISSN 2277 – 4106, P-ISSN 2347 – 5161

Available at http://inpressco.com/category/ijcet

Research Article

Calculation of Transitory Temperature Distribution in a Rectangular Shaft using Crank-Nicolson Scheme

[1]Muhammad Nadeem, [2]Afifa Maqsood, [3]Muhammad Moaz Safer, [1,4]Hira Fatima, [1,5]Abdul Raheem and *[6]Tazeem Fatima

[1,4,5]Department of Mathematics, Lahore Leads University, Pakistan
[2]Department of Mathematics & Statistics, Bahauddin Zakariya University, Pakistan
[3]Department of Mathematics & Statistics, University of Agriculture Faisalabad, Pakistan
*[6]Department of Mathematics, Government College University Faisalabad, Pakistan

Received 20 April 2024, Accepted 20 May 2024, Available online 24 May 2024, **Vol.14, No.3 (May/June 2024)**

Abstract

In this study, the Crank-Nicolson scheme (CNS) was employed to calculate the transient temperature distribution within a rectangular shaft. The shaft boundary was discretized into ten equal intervals, each with a grid point. The length of each interval, δx, was 0.01. Eleven grid points were utilized, with the temperatures at the first and eleventh grid points maintained at 0°C throughout the computation. The computational procedure for determining the transient temperature at the grid points was used to develop a MATLAB computer program. The computed results were then compared to the exact solutions. The accuracy of the CNS was evaluated through the use of tables and graphs.

Keywords: *Rectangular shaft, Transitory Temperature Distribution, Crank-Nicolson Scheme*

1. Introduction

The term "finite element method" was first used by from the University of California, Ray W. Clough. Together with John H. Argyris of the Stuttgart Technical University and the Imperial College in London, he is credited with developing the finite element method. One of the simplest also most established techniques for explaining differential equations is the use of finite difference approximations for derivatives.

Around 1768, L. Euler (1707–1783) already knew it in one spatial dimension and most likely expanded it to two dimensions.

Brook Taylor suggested finite differences in 1715, and In their various works from 1860 to 1939, George Bole, They were looked at by L. M. Milne-Thomson and Károly Jordan. The concept of finite differences was first introduced by Brook Taylor in 1715, and it was later developed by George Boole in 1860, L. M. Milne-Thomson in 1933, and Károly Jordan in 1939.

One of Jost Bürgi's processes involving finite differences first appeared. The finite difference method was one of the earliest methods used for the numerical solution of differential equations. It might have been used for the first time in 1768 by Euler.

*Corresponding author's ORCID ID: 0000-0002-0921-086X
DOI: https://doi.org/10.14741/ijcet/v.14.3.2

The finite difference approach is used to immediately apply the governing equations' differential form. The ideas to divide derivatives of the flow variables with a Taylor series expansion. The finite element method, which splits a structure into elements with nearby demarcated strains or stresses, has its beginnings in the engineering literature, thus, in the latter half of the 1950s, structural engineers combined well-known framework analysis with variational techniques in continuum mechanics. Performed part of the groundbreaking work, and Clough (1960) is credited with coining the phrase "finite element method."

The Crank-Nicolson method was created in 1947 in order to remedy this critical weakness in the forward and backward Euler methods [5]. It is an implicit procedure of a higher order (in time). Its accuracy in k is just first order, which is a drawback. On the other hand, extrapolation can be used to correct this, leading to a second-order system. Once more, this is a system that can be expressed in the form (3) and can therefore be resolved at each time level using conventional matrix solver methods. In order to approximate the coefficient matrix of the ordinary differential equations derived from the partial differential equation, the MLCN approach uses the Trotter Product formula of the exponential function. For many real-world applications, using excessively small time steps is unaffordable in terms of computer efficiency. A Crank-Nicholson implicit method exists and is presented here as it is. It converges on all lambda values. Value of A, which is the average of the other values of u at given points like B, C, D, and E when lambda equals one, provides the formula's simplest version. Or when k = a h squared. The amount of truncation fault presented while approximating the partial derivatives directly affects how quickly the method converges. The Crank-Nicolson technique therefore meets at rates of O (t2) and (S2). Compared to either the explicit approach or the implicit method, this method converges more quickly. One of the most used techniques for the numerical integration (cf. Integration, numerical) of diffusion problems, created by J. Crank and P. Nicolson [a1] in 1947. They thought about using an implicit finite difference method to approximate the solution of a non-linear differential system, such as the kind that appears in heat flow issues. All physical issues typically involve partial differential equations, and they have a extensive range of applications in research and engineering. It is a continuing effort to look for the best numerical techniques for solving a specific problem of partial differential equation due to the numerous scientific and technological advancements.

i) Crank Nicolson scheme is a finite difference approach that is a second order method in time and one of the numerical strategies for solving a partial differential equation.
ii) For the diffusion problem and numerous other kinds of partial differential equation, the Crank Nicolson scheme is conditionally stable.
iii) It is a strong numerical approach that can offer a solution with a higher order of precision. Starting with the difference between the solutions at two times, it approximates the first order time derivative time.
iv) It was developed by John Crank and Phyllis Nicolson in the middle of the 20th century, who used it to numerically solve the heat equation for the first time. The crank Nicolson approach provides a smaller truncation error for both the time and the spatial dimensions when compared to other numerical algorithms.

[5] It is frequently utilized to analysis transitory issues in engineering and scientific situations. In financial mathematics, the diffusion equation can be used to simulate a variety of phenomena. The Crank Nicolson scheme is also used in these contexts.

[6]At each level of time, it just necessitates the resolution of a fairly straightforward set of linear equations.

[7] The most popular technique for resolving parabolic partial differential equations is the Crank Nicolson approach. John Crank and Phyllis Nicholson created the Crank Nicolson Method for resolving heat equations in the mid of the 20th century.

A technique for quantitatively analysis heat conduction partial differential equations was investigated. This combines the stability of an implicit method with the accuracy of a second-order approach in both space and time. There are problems with first-order temporal truncation in both the explicit (ahead Euler) and implicit (reverse Euler) methods. This means that to produce an accurate result, brief time growths must be used. Owing of its stability, the explicit technique als0 has extra 1imitations. Excessively tiny time increments are prohibitive in many practical applications due to worries about computer efficiency.

It is an implicit method of higher order (in time). The Crank-Nicolson method only needs the solution of a simple system of linear equations and is numerically stable at any time level.

The two-dimensional heat equation was solved using the Crank-Nicholson method, with the geometric domain and initial conditions taken from a gray scale image. The heat distribution profile was simulated for different points of the domain for 4 minutes from the beginning and displays those distributions in the figures above, where the colour pixels indicate the temperature.

The heat distribution profile was simulated for different points of the domain for 4 minutes from the beginning and displays those distributions in the figures above. As a result, it is suggested that digital picture processing could be a useful tool for other tasks as well as numerical modelling of the heat equation. Although a color image might improve results and applications, it should be emphasised that our study was based on a grey image for the sake of simplicity, time efficiency, and space savings. The Crank-Nicolson method, a finite difference technique used in numerical analysis, can be used to numerically solve the heat equation as well as other partial differential equations. It is a second-order temporal approach. It is numerically stable, has an implicit Runge-Kutta technique that may be articulated, and is implicit in time. In the middle of the 20th century, John Crank and Phyllis Nicolson created the technique.

Diffusion equations and numerous other equations are proved to be unconditionally stable using the Crank-Nicolson method. If the ratio of the time step t times the thermal diffusivity to the square of the space step, x 2, is significant (usually more than 1/2 per Von Neumann stability analysis), the approximation solutions could still contain (decaying) spurious oscillations. Due to its stability and immunity to oscillations, the less accurate reverse Euler method is typically used where large time increments or high spatial resolution are necessary.

Mathematical Formulation of problem

3.1 Derivation

The governing equation for solving the problem under discuss can be derived as follows;

By the law of conservation of energy

$$E_{in}-E_{out}= E_{stored} \quad (3.1)$$

$$f x - f x + \delta x = \rho c A \delta x \frac{\partial T}{\partial t} \quad (3.2)$$

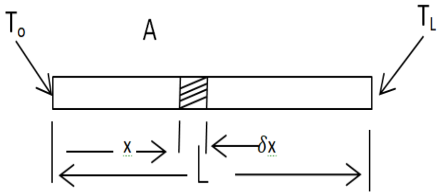

Figure 1. Transitory heat conduction in a rectangular shaft where f_x and $f_{x+\delta x}$ are the heat fluxes entering into and leaving from the cross sectional area A of the element of shaft respectively as shown in figure 1. Also, L is the length of shaft along x axis, ρ is the density and C is specific heat. The varation of temperature depends on x and t. By the Fourier's law, the amount of heat flux depending on the temperature gradient is given as follows;

$$f x = -\lambda A \frac{\partial T}{\partial t} \quad (3.3)$$

Where ƛ is thermal conductivity coefficient
Using equation (3.3) and equation (3.2) and then applying the expansion of Taylor' series to the heat flux term $f_{(x+\delta x)}$ we have

$$-\lambda A \frac{\partial T}{\partial t} - \left[-\lambda A \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda A \frac{\partial T}{\partial x}\right)\delta x - \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\lambda A \frac{\partial T}{\partial x}\right)\delta x^2 - \cdots\cdots\right] = \rho c A \delta x \frac{\partial T}{\partial t}$$

$$-\lambda A \frac{\partial T}{\partial t} + \lambda A \frac{\partial T}{\partial t} + \frac{\partial}{\partial x}\left(\lambda A \frac{\partial T}{\partial x}\right)\delta x + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\lambda A \frac{\partial T}{\partial x}\right)\delta x^2 + \cdots\cdots = \rho c A \delta x \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x}\left(\lambda A \frac{\partial T}{\partial x}\right)\delta x + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\lambda A \frac{\partial T}{\partial x}\right)\delta x^2 + \cdots\cdots = \rho c A \delta x \frac{\partial T}{\partial t}$$

$$\left[\frac{\partial}{\partial x}\left(\lambda A \frac{\partial T}{\partial x}\right) + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\lambda A \frac{\partial T}{\partial x}\right)\delta x^{\square} + \cdots\cdots\right]\delta x = \rho c A \delta x \frac{\partial T}{\partial t}$$

Divide both side of above equation by δ_x, we find

$$\frac{\partial}{\partial x}\left(\lambda A \frac{\partial T}{\partial x}\right) + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\lambda A \frac{\partial T}{\partial x}\right)\delta x^{\square} + \cdots\cdots = \rho c A \frac{\partial T}{\partial t}$$

Talking limit as $\delta x \to 0$, we obtain

$$\lim_{\delta x \to 0}\left[\frac{\partial}{\partial x}\left(\lambda A \frac{\partial T}{\partial x}\right) + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\lambda A \frac{\partial T}{\partial x}\right)\delta x^{\square} + \cdots\cdots\right] = \lim_{\delta x \to 0} \rho c A \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x}\left(\lambda A \frac{\partial T}{\partial x}\right) + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\lambda A \frac{\partial T}{\partial x}\right)(0) + \cdots\cdots = \rho c A \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x}\left(\lambda A \frac{\partial T}{\partial x}\right) = \rho c A \frac{\partial T}{\partial t} \quad (3.4)$$

If the thermal conductivity ƛ is constant, then equation (3.4) becomes

$$\frac{\partial}{\partial x}\left(\lambda A \frac{\partial T}{\partial x}\right) = \rho c A \frac{\partial T}{\partial t} \quad (3.5)$$

Where A is the cross sectional area of the rectangular shaft.

Dividing both sides of equation (3.5) by $\rho c A$, we get

$$\frac{\lambda}{\rho c}\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad (3.6)$$

Which is the parabolic equation and Crank Nicolson Scheme (CNS) will be applied to (3.5) for calculating the transitory temperature distribution in a rectangular shaft. The length of this shaft is taken as unity and it is made from a material such that

$$\frac{\lambda}{\rho c} = 1$$

With the initial temperature along the shaft is $\sin(\pi x)$. The problem statement can be summarized as under

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \qquad 0 \le x \le 1 \quad (3.7)$$

With initial and boundary conditions;

$$T(x,0) = \sin(\pi x) \quad (3.8)$$

And

$$\left.\begin{aligned} T(o,t) &= 0 \\ T(1,t) &= 0 \end{aligned}\right] \quad (3.9)$$

In order to find the general solution of equation (3.7), the Laplace transform of both sides of this equation is taken with respect to t,

$$L\left\{\frac{\partial^2 T}{\partial x^2}\right\} = L\left\{\left[\frac{\partial T}{\partial t}\right\}\right.$$

$$\frac{d^2 \bar{T}}{dx^2}(x, s) = s\bar{T}(x,s) - T(x,0)$$

$$\frac{d^2 \bar{T}}{dx^2}(x,s) - s\bar{T}(x,s) = -T(x,0)$$

Using the initial condition

$$\frac{d^2 \bar{T}}{dx^2}(x,s) - s\bar{T}(x,s) = -\sin(\pi x) \quad (3.10)$$

Which is a second order linear ordinary differential equation

The general solution of (3.10) is found as follows.
The equation (3.10) can be written as

$$\left(\frac{d^2}{dx^2} - s\right)\bar{T}(x,s) = -\sin(\pi x)$$

Or

$$(D^2 - s)\,\bar{T}(x,s) = -\sin(\pi x)$$

The A.E is

$$m^2 - s = 0$$

Or

$$m^2 = s$$
$$m = \pm\sqrt{s}$$

The complementary function is

$$\bar{T}_C(x,s) = A(s)e^{\sqrt{s}x} + B(s)e^{-\sqrt{s}x} \quad (3.11)$$

Where A(s) and B(s) are arbitrary constants
Using the boundary conditions
T (0, t) = 0, T (1, 0) = 0

Talking Laplace transform of both sides
L {T (0, t)} = 0
And
L {T (1, t) } = 0
i.e $\bar{T}(1,s) = 0$
and
$\bar{T}(1,s) = 0$

A(s) + B(s) = 0 (i)

$A(s)e^{\sqrt{s}x} + B(s)e^{-\sqrt{s}x} = 0$ (ii)

From equations (i) and (ii), we get

A(s)=0 and B(s)=0

$\bar{T}_c$(x,s)=0

The particular integral is

$\bar{T}_P(x,s) = \frac{\sin(\pi x)}{s+\pi^2}$

$$\bar{T}(x,s) = \bar{T}_C(x,s) + \bar{T}_P(x,s) \quad (3.12)$$

$$\bar{T}(x,s) = 0 + \frac{\sin(\pi x)}{s^2+\pi^2}$$

$$\bar{T}(x,s) = +\frac{\sin(\pi x)}{s^2+\pi^2}$$

Talking the inverse Laplace transform of both sides of equation (3.12), we have

$L^{-1}\{\bar{T}_p(x,s)\} = L^{-1}\{\frac{\sin(\pi x)}{s+\pi^2}\}$

Or

$T(x,t) = L^{-1}\{\frac{1}{s+\pi^2}\}\ \sin(\pi x)$

$T(x,t) = e^{-\pi^2 t}\sin(\pi x)$

The rectangular shaft is divided into ten equal intervals and length of each subinterval is taken

$\delta x = 0.1$

Equal subintervals each of length$\delta x = 0.1$. There are eleven grid points for which the temperature at grid point numbers. One and eleven are maintained at zero degree throughout the computational procedure.

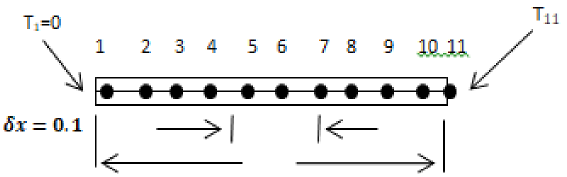

Figure 2 shows a rectangular shaft with ten equal intervals and grid point numbers.

The Crank –Nicolson scheme is used to find a temperature distribution in a rectangular shaft. This scheme starts from approximating the first order time derivative term in the form of difference of the solutions at time steps n and n+1 as

$$\frac{\partial T}{\partial t} = \frac{T_i^{n+1} - T_i^n}{\delta t} \quad (3.13)$$

The scheme also approximates the second order spatial derivative term in the form as under.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{2}\left(\frac{T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1}}{(\delta x)^2} + \frac{T_{i+1}^{n} - 2T_i^{n} + T_{i-1}^{n}}{(\delta x)^2}\right) \quad (3.14)$$

The parabolic partial differential equation representing the transitory heat conduction in a rectangular shaft is given by equation (3.6) that is

$$\frac{\lambda}{\rho c}\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

Putting the equation (3.13) and (3.14) in above equation we get

$$\frac{\lambda}{\rho c}\frac{1}{2}\left(\frac{T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1}}{(\delta x)^2} + \frac{T_{i+1}^{n} - 2T_i^{n} + T_{i-1}^{n}}{(\delta x)^2}\right) = \frac{T_i^{n+1} - T_i^n}{\delta t} \quad (3.15)$$

$$\frac{\lambda}{2\,\rho c}\frac{(T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1} + T_{i+1}^{n} - 2T_i^{n} + T_{i-1}^{n})}{(\delta x)^2} = \frac{T_i^{n+1} - T_i^n}{\delta t}$$

$$\frac{\lambda\,\delta t}{2\,(\delta x)^2 \rho c}(T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1} + T_{i+1}^{n} - 2T_i^{n} + T_{i-1}^{n}) = T_i^{n+1} - T_i^n \quad (3.16)\ a$$

Where $\propto = \frac{\lambda\,\delta t}{(\partial x)^2 \rho c}$ is the parameter

Then from equation (3.15), we have

$$\frac{\propto}{2}(T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1} + T_{i+1}^{n} - 2T_i^{n} + T_{i-1}^{n}) = T_i^{n+1} - T_i^n$$

$$\propto T_{i+1}^{n+1} - 2\propto T_i^{n+1} + \propto T_{i-1}^{n+1} + \propto T_{i+1}^{n} - 2\propto T_i^{n} + \propto T_{i-1}^{n} = 2T_i^{n+1} - 2T_i^n$$

Or

$$\propto T_{i+1}^{n+1} - 2\propto (T_i^{n+1}) + \propto T_{i-1}^{n+1} - T_i^{n+1} = -T_i^n - \propto T_{i+1}^{n+1} + 2\propto T_i^{n} - \propto T_{i-1}^{n}$$

Or

$$-\propto T_{i+1}^{n+1} + (2 + 2\propto)T_i^{n+1} - \propto T_{i-1}^{n+1} + \alpha T_i^{n+1} = \propto T_{i+1}^{n} + (2 - 2\propto)T_i^{n} + \propto T_{i-1}^{n}$$

Or

$$-\propto T_{i+1}^{n+1} + 2(1+\propto)T_i^{n+1} - \propto T_{i+1}^{n+1} = \propto T_{\square}^{n} + 2(1-\propto)T_i^{n} + \propto T_{i+1}^{n} \quad (3.16)$$

The left hand side of the equation (3.16)contains the unknown temperatures at the grid points i-1, i and i+1 at the new time step n+1, whereas the right hand side consist of the known temperatures at the time step n. the schematic computational diagram corresponding to the figure 2

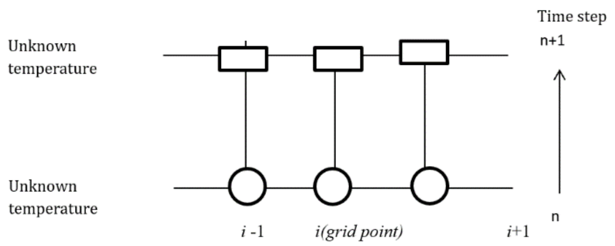

Figure 1. Schematic computational diagram of Crank Nicolson scheme. The equation (3.16) is to all grid point for which the temperature is unknown. The future application leads to a tridiagonal system of equations which can be solved for the temperature solutions.

$\frac{\lambda}{\rho c} = 1,\ \partial x = 0.1,\ \partial T = 0.02$

So $\alpha = \frac{\lambda \delta t}{\rho c(\delta x)^2} = \frac{\lambda}{\rho c} = \frac{\delta t}{(\delta x)^2} = 1\,.\left(\frac{0.02}{(0.1)^2}\right) = 2$

In the case under discussion

$$-2T_{i-1}^{n+1} + 6T_i^{n+1} - 2T_{i+1}^{n+1} = 2T_{i-1}^{n} - 2T_i^{n} + 2T_{i+1}^{n} \quad (3.17)$$

$i = 2\,,\quad T_i = 0$

$$-2T_1^{n+1} + 6T_2^{n+1} - 2T_3^{n+1} = 2T_1^{n} - 2T_2^{n} + 2T_3^{n}$$

$$6T_2^{n+1} - 2T_3^{n+1} = -2T_2^n + 2T_3^n$$

$i = 3$

$$-2T_2^{n+1} + 6T_3^{n+1} - 2T_4^{n+1} = 2T_2^n - 2T_3^n + 2T_4^n$$

$$i = 4$$

$$-2T_3^{n+1} + 6T_4^{n+1} - 2T_5^{n+1} = 2T_3^n - 2T_4^n + 2T_5^n$$

$$i = 5$$

$$-2T_4^{n+1} + 6T_5^{n+1} - 2T_6^{n+1} = 2T_4^n - 2T_5^n + 2T_6^n$$

$$i = 5$$

$$-2T_4^{n+1} + 6T_5^{n+1} - 2T_6^{n+1} = 2T_4^n - 2T_5^n + 2T_6^n$$

$$i = 6$$

$$-2T_5^{n+1} + 6T_6^{n+1} - 2T_7^{n+1} = 2T_5^n - 2T_6^n + 2T_7^n$$

$$i = 7$$

$$-2T_5^{n+1} + 6T_6^{n+1} - 2T_7^{n+1} = 2T_5^n - 2T_6^n + 2T_7^n$$

$$i = 8$$

$$-2T_7^{n+1} + 6T_8^{n+1} - 2T_9^{n+1} = 2T_7^n - 2T_8^n + 2T_9^n$$

$$i = 9$$

$$-2T_8^{n+1} + 6T_9^{n+1} - 2T_{10}^{n+1} = 2T_8^n - 2T_9^n + 2T_{10}^n$$

$$i = 10$$

$$-2T_9^{n+1} + 6T_{10}^{n+1} - 2T_{11}^{n+1} = 2T_9^n - 2T_{10}^n + 2T_{11}^n$$

$$\begin{bmatrix} 6 & -2 & \square & \square & \square \\ -2 & 6 & -2 & \square & \square \\ \square & -2 & 6 & -2 & \square \\ \square & \ddots & \ddots & \ddots & \square \\ \square & \square & -2 & 6 & -2 \\ \square & \square & \square & -2 & 6 \end{bmatrix} \begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ \vdots \\ T_9 \\ T_{10} \end{bmatrix}^{n+1} =$$

$$\begin{bmatrix} -2 & -2 & \square & \square & \square \\ 2 & -2 & 2 & \square & \square \\ \square & 2 & -2 & 2 & \square \\ \square & \ddots & \ddots & \ddots & \square \\ \square & \square & 2 & -2 & 2 \\ \square & \square & \square & 2 & -2 \end{bmatrix} \begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ \vdots \\ T_9 \\ T_{10} \\ \square \\ \square \\ sssss \end{bmatrix}^{n} \qquad (3.1)$$

Numerical Results and Discussions

The numerical results and discussions are given. The tables of results and the graphical representation of exact and computed values for different time intervals are under:

Table 1

At $\delta t = 0.02$

Sr. No	Computed Values	Exact Values	Difference
1	0.0000	0.0000	0.0000
2	0.2539	0.2537	0.0002
3	0.4830	0.4826	0.0004
4	0.6648	0.6642	0.0006
5	0.7815	0.7811	0.0004
6	0.8217	0.8212	0.0005

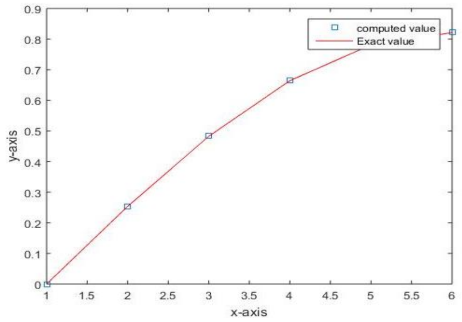

Figure 4.1: Graphical representation of exact and computing values at $\delta t = 0.02$

Table 2

At $\delta t = 0.06$

Sr. No	Computed Values	Exact Values	Difference
1	0.0000	0.0000	0.0000
2	0.1714	0.1711	0.0003
3	0.3261	0.3253	0.0008
4	0.4488	0.4477	0.0011
5	0.5276	0.5263	0.0013
6	0.5548	0.5533	0.0015

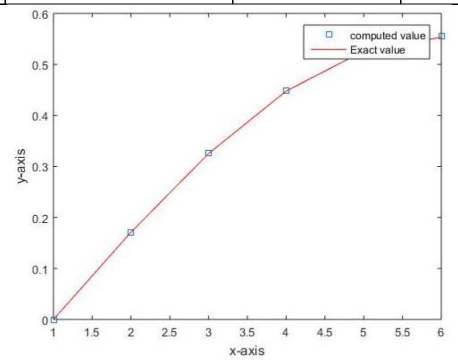

Figure 4.2: Graphical representation of exact and computing values at $\delta t = 0.06$

Table 3

At $\delta t = 0.10$

Sr. No	Computed Value	Exact Value	Difference
1	0.0000	0.0000	0.0000
2	0.1157	0.1153	0.0004
3	0.2202	0.2196	0.0006
4	0.3030	0.3018	0.0012
5	0.3562	0.3547	0.0015
6	0.3746	0.3730	0.0016

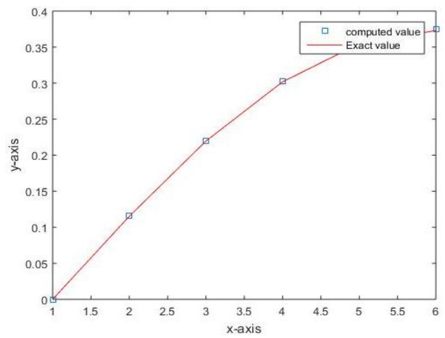

Figure 4.3: Graphical representation of exact and computing values at $\delta t = 0.10$

Table 4
At $\delta t = 0.14$

Sr. No	Computed Value	Exact Value	Difference
1	0.0000	0.0000	0.0000
2	0.0781	0.0776	0.0005
3	0.1486	0.1479	0.0007
4	0.2046	0.2034	0.0012
5	0.2405	0.2391	0.0014
6	0.2529	0.2516	0.0013

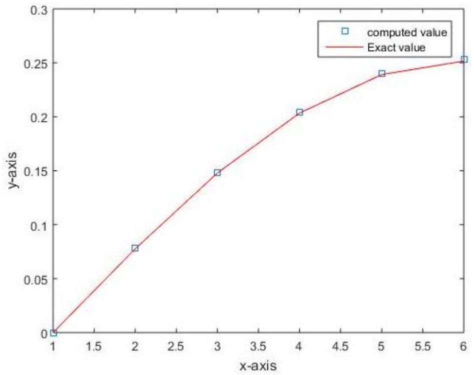

Figure 4.4: Graphical representation of exact and computing values at $\delta t = 0.14$

Table 5
At $\delta t = 0.18$

Sr. No	Computed Value	Exact Value	Difference
1	0.0000	0.0000	0.0000
2	0.0528	0.0523	0.0005
3	0.1004	0.0995	0.0009
4	0.1381	0.1372	0.0009
5	0.1624	0.1611	0.0013
6	0.1707	0.1696	0.0011

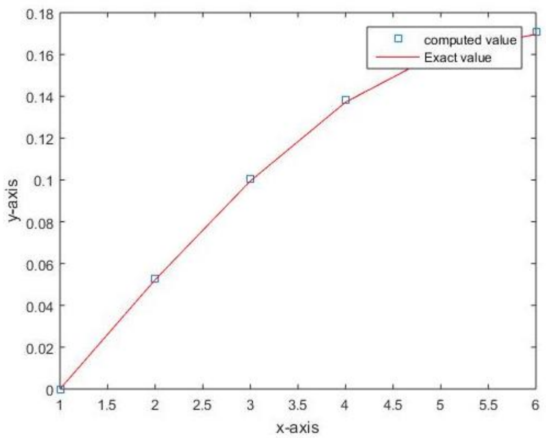

Figure 4.5: Graphical representation of exact and computing values at $\delta t = 0.18$

Discussions

In tables 1, 2, 3, 4, and 5, the differences between exact and computed values are of minor nature and the graphical representations of exact and computed values for different time intervals show that the computed results and the exact results are approximately the same. It is found from tables and graphs that exact and computed results are in good agreement.

Conclusions

In this thesis Crank Nicolson Scheme (CNC) is used for finding the transitory temperature distribution in a rectangular shaft. For this purpose the surface of the shaft is discretized into ten equal intervals with the grid point numbers .The transitory temperature for grid point numbers 2 through 10 are calculated .The computational procedure for determining the transitory temperatures at the grid point has been used for developing a MATLAB computer program .The computed grid point temperatures at different times are compared to the exact solutions It is found that the computed results are in good agreement with the exact results for the problem under consideration .Thus it is established that this scheme is efficient , accurate and time saving for this problem . Moreover, it is very useful to calculate temperature distribution on the surfaces of planets and stars.

References

[1] Necthee fernandee and rakhee badkankar, overview of a Crank Nicolson method to solve parabolic partial differential differential equation ,international journal of science and technology research volume 7 , Issice 12 December 2016

[2] Crank, J., & Nicolson, P. (1947, January). A practical method for numerical evaluation of solutions of partial differential equations of the heat-conduction type. In Mathematical proceedings of the Cambridge philosophical society (Vol. 43, No. 1, pp. 50-67). Cambridge University Press.

[3] Thomas, J. W. (2013). Numerical partial differential equations: finite difference methods (Vol. 22). Springer Science & Business Media.

[4] Smith, R. C. (2005). Smart material systems: model development. Society for Industrial and Applied Mathematics.

[5] Amoah-Mensah, J., Boateng, F. O., & Bonsu, K. (2016). Numerical solution to parabolic PDE using implicit finite difference approach. *Math. Theory Model*, *6*(8), 74-84.

[6] Wilmott, P., Howison, S., & Dewynne, J. (1995). The mathematics of financial derivatives: a student introduction. Cambridge university press.

[7] S.Sathyapriya et al. A study on Crank Nicolson method for solving parabolic partial differntial equations, International Journal for Research Trends and Innovation vol. 6

[8] Duffy, D. J. (2013). Finite difference methods in financial engineering: a partial differential equation approach. John Wiley & Sons.

[9] Sivapalan, M., Bates, B. C., & Larsen, J. E. (1997). A generalized, non-linear, diffusion wave equation: theoretical development and application. Journal of Hydrology, *192*(1-4), 1-16..

[10] Duffy, D. J. (2004). A critique of the crank nicolson scheme strengths and weaknesses for financial instrument pricing. *The Best of Wilmott*, 333.

[11] J.M Sanz – Serna ,M.p. Calvo 1994: Numerical Hamiltanions Problems, Cahpman and Hall.

[12] Smith, G. D., Smith, G. D., & Smith, G. D. S. (1985). Numerical solution of partial differential equations: finite difference methods. Oxford university press..

[13] Verwer, J. G., & Sanz-Serna, J. M. (1984). Convergence of method of lines approximations to partial differential equations. CWI. Department of Numerical Mathematics [NM].

[14] Akrivis, G. D. (1992). Finite difference discretization of the Kuramoto-Sivashinsky equation. Numerische Mathematik, *63*, 1-11.

[15] Chung, S. K., & Ha, S. N. (1994). Finite element Galerkin solutions for the Rosenau equation. Applicable Analysis, *54*(1-2), 39-56.

[16] Kuria, I. M., & Raad, P. E. (1995). An implicit multidomain spectral collocation method for stiff highly non-linear fluid dynamics problems. Computer methods in applied mechanics and engineering, 120(1-2), 163-182.

[17] Fairweather, G., & López-Marcos, J. C. (1996). Galerkin methods for a semilinear parabolic problem with nonlocal boundary conditions. Advances in Computational mathematics, 6(1), 243-262.