

Research Article

# Passive and Active Investigation of a Modified Variable Stiffness Suspension System

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Accepted 20 May 2017, Available online 22 May 2017, Vol.7, No.3 (June 2017)

## Abstract

*A theoretical active and experimental passive analysis of a modified variable stiffness model of the suspension structure is considered to advance the conventional structure. The focus idea of a structure concentrated on variable stiffness mechanisms by the additional subsystem to the structure depend on the control, pivoting arm adjusting the force between the sides, it comprises of a vertical control strut. The variation of the load transfer by turning arm has spring and damper at another side of it where the purpose of rotation is supplemented of body vehicle by subsystem as vertical support. The variation of the load transfer ratio is used by additional strut in order to control a position of a point. It's supplement of the second strut to a body of the vehicle. This investigation aims to develop the traditional suspension system in passive by modifying the structure and in active case by designing a controller using Linear Quadratic Regulation control theory. The performance of the transfer energy of vibrations for modified structure has effects on the body vehicle by control arm, so reducing energy transfer for disturbance of the road to the vehicle body at comparatively lower cost to the traditional active suspension. The experimental investigates and theoretical results shown that a better performance to improve both the ride comfort and road holding.*

**Keywords:** Variable stiffness: suspension system; Passive and active suspension; Linear Quadratic Regulation.

## 1. Introduction

The isolation of the vehicle and the passengers from the road disturbance is the primary aim of the suspension system while keeping good contact with the road. An idea suspension should be received the vehicle to reduce the acceleration of the body car, dynamic tire force and satisfy the constraint imposed on the rattle space (small deflection between the axis of body car and tire axis). The performance of the suspension structure directly effects on the stability of handling and ride comfort of vehicles. However, The traditional passive suspension has significant limitation in coordination of these two performances, unable to meet the requirement of the specification design of the vehicle. Therefore, researchers carried out investigate on non-passive suspension system **Chen Jiarui (2006)**. Moreover, can be minimized all parameters of the suspension structure simultaneously. The advantages of controlled suspension are minimized and improve the usual design trade-off is possible rather than with passive systems. Linear optimal control theory delivers a systematic method to design the active suspension controller. **H. Taghirad et al (1995)**. A fully active

suspension substitutes the damper by a hydraulically driven actuator. It is additionally not reliable as in execution debasement results at whatever point the control, unsuccessful, which might be because of either mechanical, electrical, or programming disappointments. The availability of reliable and efficient of electronic constituents, the engineers of the vehicle are established to utilize several automatic control structures to advance the behavior of the vehicle and the economy. **Ballo(2007)**.

The suspension system in passive case was considered simplest, not required external energy and cheap, but in semi-active case suspension might advance of compromise between the both, the simplicity and cheapness of passive suspension systems, but the complicated and high cost of higher-performance of the suspension system in case of fully active. Which can provide a considerable advance in car ride quality and more reliable? Consequently, The suspension systems in case semi-active are classified into two types skyhook and ground hook control strategy introduced by **Kobori et, al (1993)**. Several methods of control proposed to overawed these complications in the system. Many approaches to active control of the suspension structure such as Linear Quadratic Gaussian, non-linear and adaptive controller were established and suggested so as to

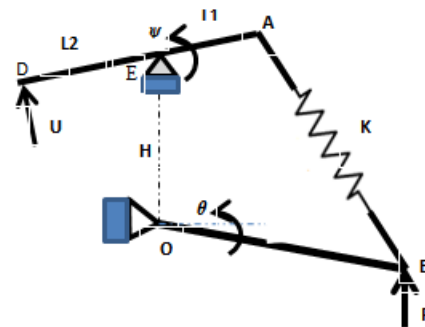
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achieve the occurring problems **M. M. M. Salem et al (2009)**. The passive suspension design was changed from a range of investigation, as recorded by the works of **Anubi et al (2013)**, offerings the design, examination, and test support of the uninvolved instance of a system suspension with variable stiffer for idea depends on an as of late composed variable stiffness component. It contains the level control and a vertical strut. **Anubi et al (2013)**, Presented the active control of the variable stiffness suspension structure for the previous idea used the idea of the nonlinear energy sink for the efficient energy of vibration in the body vehicle to control the mass. **S. Hossein Sadati, et al (2008)**. Studied eight-DOF model counting, dynamics of driver seat exposed to random disturbances of the road to study benefits of activity over the traditional passive suspension structure. An optimum control method was occupied in active suspension of the vehicle. **K.S. Hong et al (1999)**, described the design and controller system to investigate and discuss the results responses of the vehicle were achieved from a variety type of road simulation input. In conclusions, a contrast of the fuzzy active control system and (PID) control was presented utilized MATLAB simulations. **Ikbal Eski, et al (2009)**, designed by the Neural network created on the robust control system to the controller for oscillation of vehicle for the full structure of the suspension. The suggested control system was involved of a robust controller, a neural controller and a classical neural network in the automotive suspension structure. This paper presents a modified variable stiffness of the new quarter car model for suspension structure and investigation, experiment passive and theoretical active cases, The stiffness variation conception used in this investigation utilize the "reciprocal actuation to effectually of energy transfer between the traditional vertical strut and the horizontal fluctuating arm to refine the dissipation of energy in the suspension structure generally. In this study taken many types of the road generator in the passive case for the test rig (Harmonic and one cycle sine road profile) to investigate the behavior of the modified structure in these excitations, then study the active case to improve the performance of the system and Linear Quadratic Regulation control as the complementary control. Relatively, due to the number of parts moving in this model, which it can easily be combined into current old-style suspension in front and/or rear designs as an application with a double wishbone suspension system.

**Description and Mechanism of Suspension Variable Stiffness**

The model of the variable stiffness system is presented in Figure (1). The Lever arm OB of length L is stuck at an altered point O and allowed to revolve about O. The spring AB is stuck to lever arm at B and it is allowed to turn about B. The flip side A, of the spring is allowed to pivoting about E by the lever arm AD, which is joint with O at E as appeared by a double-headed arrow. The

spring AB is likewise allowed to pivot about point A. The F is the outer force is relied upon to act vertically upwards at point B without a doubt don't a loss of sweeping statement. Arm AD comprises from two component L1 and L2 and turning about E by  $\psi$ . The sign is to change the general stiffness of the structure by development L1 and L2 fluctuating latent under the effect of vertical spring-damper framework (don't appear in the figure) allude as (U) force, Consideration the system of suspension as presented in Figure (2). The schematic model and test rig are consisting of a quarter of body vehicle as wheel assembly, two dampers, two spring lower upper wishbones and road disturbance. The points O, A, B and D are presented in Fig. (1) For the mechanism of the variable stiffness of Fig. (2).



**Fig.1 Variable Stiffness Mechanism**

Vertical regulator force U used to control by rotation of arm AD with angle  $\psi$  which an opportunity to control on the mechanism for overall stiffness. The modulation of tire is considered as a linear spring for both spring rate and damping coefficient. The suppositions assumed for Fig. (2) Are brief as tails:

- 1-The vertical movement  $Z_s$  of vehicle is measured only.
- 2-The angle camber of the wheel is zero at equilibrium position
- 3- The wheel is joint to the body car by two ways: first by damper and second by the arm OC (first control arm) where  $\theta$  indicates the angular movement of the arm.
- 4- The deflection in spring, damping and tire forces are linear.
- 5- Body vehicle and wheel are expected to be particles.

**Equations of Motion**

The quarter model of automobile for three degrees of freedom Fig. (2) Is employed in the suspension system. This model can capture bounce angle of wishbone ( $\theta$ ), angle of control arm ( $\psi$ ) and the vertical vehicle movement  $z_s$ . Therefor the general coordinates of the structure are presented as: **Anubi et al (2013)**,

$$q = [z_s \quad \theta \quad \varphi]^T \tag{1}$$

The kinetic, potential and damping energies are obtained from the structure as

$$K.E = \frac{1}{2}m_s\dot{z}_s^2 + \frac{1}{2}m_t(\dot{z}_s + L_c\dot{\theta})^2 + \frac{1}{2}I_1\dot{\theta}^2 + \frac{1}{2}I_2\dot{\varphi}^2 \tag{2-a}$$

$$P.E = \frac{1}{2}k_t(z_s + L_c\theta - r)^2 + \frac{1}{2}k_s(L_B\theta - L_1\varphi)^2 + \frac{1}{2}k_uL_2^2\varphi^2 \tag{2-b}$$

$$D.E = \frac{1}{2}C_s(L_c\dot{\theta} - L_1\dot{\varphi})^2 + \frac{1}{2}C_t(\dot{z}_s + L_c\dot{\theta} - \dot{r})^2 + \frac{1}{2}I_1\dot{\theta}^2 + \frac{1}{2}C_uL_2^2\dot{\varphi}^2 \tag{2-c}$$

### State Space Analysis And Linearization

The states variable are introduced as,  $[x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T = [z_s \ z'_s \ \theta \ \theta' \ \psi \ \psi']^T$  the equation of motion written in the state equation as follows: **W.J. Palm (2007)**

$$\begin{aligned} \dot{x}_1 &= x_2, \quad \dot{x}_2 = F_1(x_1, x_2, x_3, x_4, x_5, x_6, z_r) \\ \dot{x}_3 &= x_4, \quad \dot{x}_4 = F_2(x_1, x_2, x_3, x_4, x_5, x_6, z_r) \\ \dot{x}_5 &= x_6, \quad \dot{x}_6 = F_3(x_3, x_4, x_5, x_6) \end{aligned} \tag{3}$$

$$F_1 = \frac{-1}{Dy} \{ (Dk_t + k_tL_c)z_s + (k_tL_cD + k_tL_c^2 + k_sL_B^2)\theta + (C_tD + C_tL_c)\dot{z}_s + (C_tL_cD + C_tL_c^2 + C_sL_B^2)\dot{\theta} - c_sL_1L_B\varphi' - k_sL_1L_B\varphi - (k_tD + k_tL_c)z_r - (C_tD + C_tL_c)\dot{z}_r \} \tag{3-a}$$

$$F_2 = \frac{-1}{D\theta} \{ (D'C_t + C_tL_c)\dot{z}_s + (c_tL_cD' + C_tL_c^2 + C_sL_B^2)\dot{\theta} + (k_tD' + k_tL_c)z_s + (k_tL_cD' + k_tL_c^2 + k_sL_B^2)\theta - c_sL_1L_B\varphi' - k_sL_1L_B\varphi - (k_tD' + k_tL_c)z_r - (C_tD' + C_tL_c)\dot{z}_r \} \tag{3-b}$$

$$F_3 = \frac{-1}{I_2} \{ k_sL_1L_B\theta + C_sL_1L_B\dot{\theta} - (k_sL_1^2 + k_uL_2^2)\varphi - (C_sL_1^2 + C_uL_2^2)\dot{\varphi} \} \tag{3-c}$$

Where,

$$\begin{aligned} Dy &= m_tL_c - (m_s + m_t)(m_tL_c^2 + I_1)/m_tL_c \\ D\theta &= -\frac{m_tL_c^2 + I_1}{m_s + m_t} + (m_tL_c^2 + I_1), \\ D &= \frac{m_tL_c^2 + I_1}{m_tL_c} \quad D' = \frac{-m_tL_c}{m_s + m_t} \end{aligned}$$

The new model of the system is linearized because of the small angle approximation and the fact that its equilibrium is zero. The linearization is needed model forms that do not contain the sine or cosine function and whose equilibrium is nonzero.

$$\begin{aligned} \dot{x} &= Ax + B_aF_a + Bz_r, \\ y &= Cx + D_aF_a + Dz_r \end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial x_3} & \frac{\partial F_1}{\partial x_4} & \frac{\partial F_1}{\partial x_5} & \frac{\partial F_1}{\partial x_6} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial x_3} & \frac{\partial F_2}{\partial x_4} & \frac{\partial F_2}{\partial x_5} & \frac{\partial F_2}{\partial x_6} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{\partial F_3}{\partial x_3} & \frac{\partial F_3}{\partial x_4} & \frac{\partial F_3}{\partial x_5} & \frac{\partial F_3}{\partial x_6} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ a21 & a22 & a23 & a24 & a25 & a26 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ a41 & a42 & a43 & a44 & a45 & a46 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & a63 & a64 & a65 & a66 \end{bmatrix} \tag{4}$$

$$\begin{aligned} a21 &= \frac{-(Dk_t + k_tL_c)}{Dy}, \quad a22 = \frac{-(DC_t + C_tL_c)}{Dy}, \\ a23 &= \frac{-(DL_c k_t + k_tL_c^2 + k_sL_B^2)}{Dy} \\ a24 &= \frac{-(DL_c C_t + C_tL_c^2 + C_sL_B^2)}{Dy}, \quad a25 = \frac{k_sL_1L_B}{Dy} \\ a26 &= \frac{C_sL_1L_B}{Dy}, \quad a41 = \frac{-(D'k_t + k_tL_c)}{D\theta} \\ a42 &= \frac{-(D'C_t + C_tL_c)}{D\theta}, \quad a4 = \frac{-(D'L_c k_t + k_tL_c^2 + k_sL_B^2)}{D\theta} \\ a4 &= \frac{-(D'L_c C_t + C_tL_c^2 + C_sL_B^2)}{D\theta}, \quad a45 = \frac{k_sL_1L_B}{D\theta} \\ a46 &= \frac{C_sL_1L_B}{D\theta}, \quad a63 = \frac{k_sL_1L_B}{D\theta}, \quad a64 = \frac{C_sL_1L_B}{D\theta} \\ a65 &= \frac{-(k_sL_1^2 + k_uL_2^2)}{I_2}, \quad a66 = \frac{-(C_sL_1^2 + C_uL_2^2)}{I_2} \end{aligned}$$

$$B = \begin{bmatrix} 0 & \frac{\partial F_1}{\partial z_r} & 0 & \frac{\partial F_2}{\partial z_r} & 0 & \frac{\partial F_3}{\partial z_r} \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{k_tD + k_tL_c}{Dy} & 0 & \frac{k_tD' + k_tL_c}{D\theta} & 0 & 0 \end{bmatrix}^T \tag{5}$$

$$B_a = \begin{bmatrix} 0 & \frac{\partial F_1}{\partial F_a} & 0 & \frac{\partial F_2}{\partial F_a} & 0 & \frac{\partial F_3}{\partial F_a} \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{L_2}{I_2} & 0 \end{bmatrix}^T \tag{6}$$

D and C matrices depended on the output values from the system and the  $Z_r(t)$  presents the road movement indication as a surface of the road profile and the speed of the automobile. The term  $L_c$  and  $L_B$  are the length of pint O to tire and to point B respectively.

### LQR Controller Design

The method of the optimal control with performance index has the ability to accomplish optimal closed loop for linear output or input feedback. In a Linear Quadratic Regulator approach, the strategy of control is found the control vector  $u(t)$  such that an assumed performance index as **S. Hossein Sadati (2008)**.

$$J_{LQR} = \int H(x, u)dt \tag{7}$$

Where  $H(x, u)$  is a quadratic function or Hermitian function, which is important linear controlling law.

$$u(t) = -H_{LQR}x(t) \tag{7-a}$$

Where  $H_{LQR}$  is a  $LQR$  matrix of gains. In  $LQR$  controller, the index of quadratic performance can be written as:

$$J_{LQR} = \int_0^{\infty} (x^T Q x + u^T R u) dt \tag{8}$$

Consider the feedback regulator of the state variable as:

$$u = Kx \tag{8-a}$$

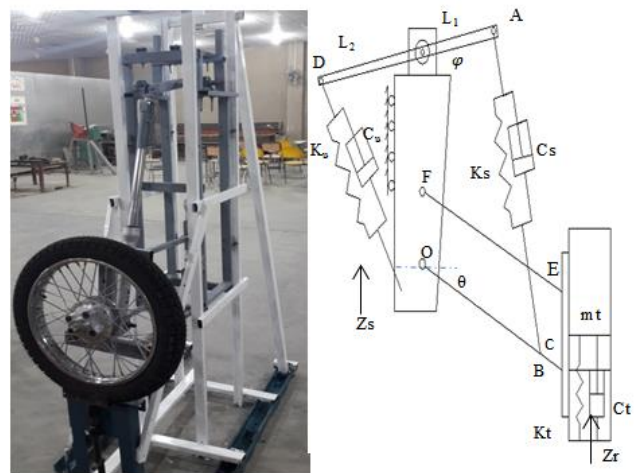
Where:  $K$  is the matrix of gains for state feedback.

It represents the controller input limitation as well as the performance characteristic requirement. The optimal controller of assumed structure is exposed as a controller strategy to reduce the index of performance. The system equation is  $\dot{x} = Ax + Bu$ , Where the  $u(t) = K_{LQR} x$ , and  $u$  it is referred to the Force Actuator. While,  $Q$  and  $R$ , Are weighting matrices positive definite.

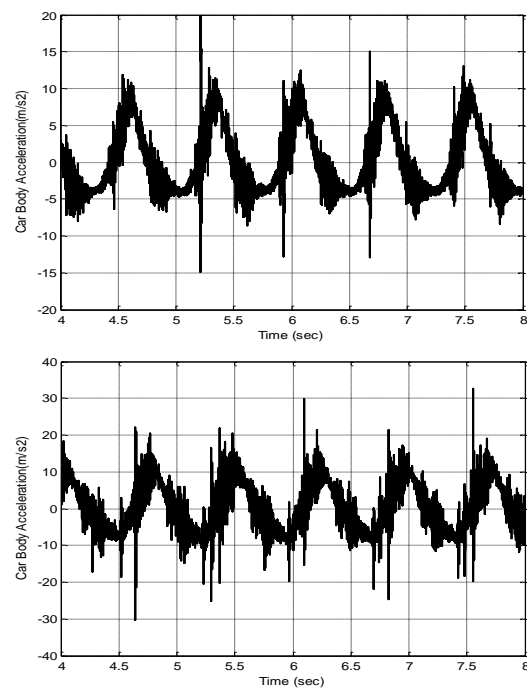
### Experiment (Test Rig And Results)

The experimental system is exposed in Figure (2). It is a test rig of quarter car scaled down to (1:10) ratio associated with a passenger car average in **Anubi et al (2013)**,. A body of a quarter vehicle is passable to move up-and-down along a fixed structure. The approximate structure was made of steel framing and then loaded with a weight 18.5 kg added to weight quarter car model to get on total weight 31.5kg. The struts are 2014 Super MTR front suspensions. The road producer is a modest mechanism (slider-crank) activated by motor EED geared down to (17.95:1) ratio, Three accelerometers are involved, first in the frame of a quarter car, second on the wheel hub, and third on the generator of the road. Data gaining was done by the LABVIEW data NI, USB-6234. The results obtained from this part explain the advantage of the modified structure in this field by comparing with the traditional suspension system, that has gotten by making the test rig, operate according to passive traditional suspension system, that mean the rig operate with one strut, while the proposed suspension can be gotten by make two struts work instantaneously. Two examinations were carried out, harmonic sinusoidal and one cycle sinusoidal test. For these tests, the road producer is actuated by a continuous torque of the AC motor, where the system up and down movement with 10cm high. The procedure of experimental repeated many times to validate the reputability of the test. The figures (3, and 4) shown that the value of acceleration of body vehicle for the using models to get an implementation of the proposed suspension system in simple way to exceed the complication in the actual suspension system such as weight and additional of the specification of the system and to take the idea of the suspension structure only. Figure (3) displays the vertical acceleration of the vehicle, it refers to the ride comfort, Where the lower

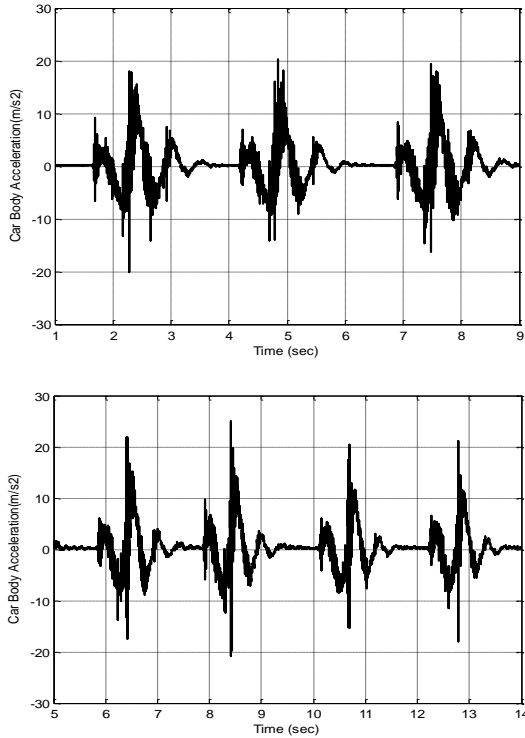
value of the acceleration (better the ride comfort) compared with vertical acceleration of traditional system. As appreciated in this figure, the modified structure is best ride friendly suspension for harmonic testing by redaction the acceleration of the structure nearly 40% for passive case suspension. The second experimental test of the proposed suspension is one cycle test, it depends on control the motor by hand (on-off) to get on one cycle as result excitation of test rig to measure the body vehicle, wheel and road generator acceleration. Figure (4) display vertical acceleration of the body car in case modified and a traditional suspension system for this test, it is also seen that the acceleration reduction about 30% of the modified structure as compared with the acceleration of the traditional suspension system.



**Fig.2** Left- The Test Rig of Proposed Model. Right- Schematic Model of Proposed Quarter Car



**Fig.3** The acceleration of body car for Harmonic Excitation A- New model, B- Traditional Model



**Fig.4** The acceleration of body car for One cycle harmonic excitation A- New model, B- Traditional Model

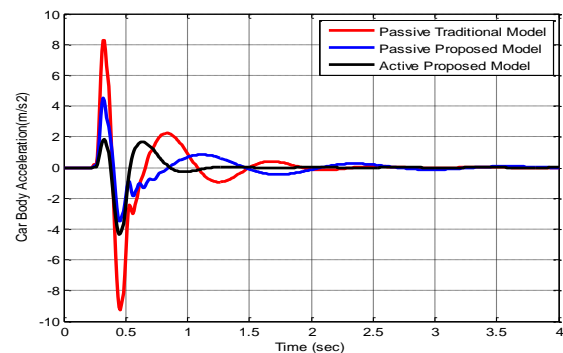
**Results and Discussions of Simulation for Passive and Active Proposed Suspension System**

To research the comportment of full-scale quarter vehicle structure as responses relating suspension deflection which were hard to implement experimentally and excitation. That is hard to implement experimentally, accurate imitations completed utilizing MATLAB, where the system was modeled. The properties of using horizontal and additional strut and stiffness of tire are given as “Renault M’egane Coup’e” model. These values are given in Table [1] and added to these values the dynamic parameters of the suspension structure to complete the suspension and tire deflection responses are associated with the traditional suspension system and the modified model. For traditional suspension system fixed the second strut to get one strut work only, but for the new model allow for two struts work in harmony to control the motion of the arm between the two struts. The information on this model to investigate the compartment of the modified suspension structure given in model **Anubi et al (2013)**. In a simulation of time-domain for traveling vehicle with a horizontal steady speed of 40 kmph is exposed to the bump of height 10 cm. This part of an investigation described the effect active control system to reduce the acceleration value of the vehicle to provide ride comfort for passengers and in order to exhibit benefiting of a modified system by comparing between active suspension and passive in the case proposed and traditional suspension system.

**Table 1** The Value Of Dynamic And Kinematic Parameters

Parameters	Value	Parameters	Value
$m_s$	315 kg	$I_a$	0.1 kg m <sup>2</sup>
$m_t$	37.5 kg	$k_t$	210 kN/m
$m_a$	3 kg	$k_s$	29 kN/m
$I$	0.0018 kg m <sup>2</sup>	$K_u$	29 kN/m
$C_s$	1500N.s/m	$L_B$	0.375m
$C_u$	1500N.s/m	$L_D$	0.4m
$L_2$	0.2m	$L_1$	0.2m

The advantage of the active vibration system reduces the effects of transmitted a force to the structure and reduced the output displacement caused by an input displacement. The program MATLAB is used to estimate the active control response to the proposed structure by Linear Quadratic Regulation method for three important parameters have a greater effect on the performance of the structure for suspension, Acceleration of the body vehicle, Suspension deflection and Tire deflection. Figure (5) indicates the acceleration of the body vehicle for active proposed suspension structure and comparison with passive proposed and traditional system, where the active system shows that car body acceleration for active is less than the proposed acceleration about 40% reduction, while reduced about 70% with respect to traditional suspension system tending to better ride comfort. From figure (6) can see that, the suspension deflection for active suspension system has reduced about 30% from traditional system and it has been stable for the proposed system in short time to provide with acceleration good ride comfort. The handling of the vehicle is presented by the tire deflection as a figure (7) reducing both tire and suspension deflection to the minimum possible value provide better handling. The comfortable of the passengers is provided by the controller on the acceleration of vehicle body in an active suspension system to reduce acceleration to the minimum possible value after passing road profile. **K.S. Hong et al (1999)**



**Fig.5** Car Body Acceleration of Active Control System

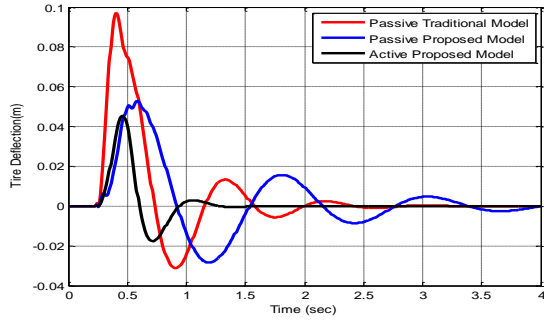


Fig.6 Suspension Deflection of Active Control System

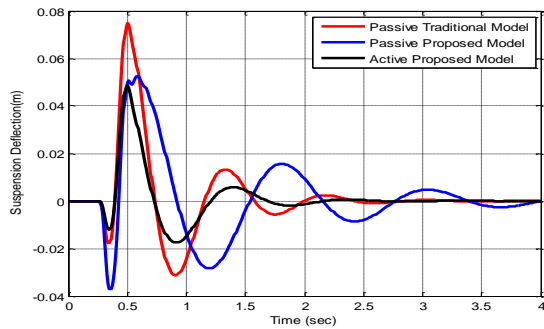


Fig.7 Tire Deflection of Active Control System

**Conclusions**

The fundamental idea of this investigation improve the performance suspension systems of a vehicle by modifying structure replaced the traditional system to development the ride comfort and road handling. While, the past investigations developed any three procedures passive, semi-active, or active suspension or by an improved new method in the control system for semi and active suspension the new variable stiffness system was presented. This investigation comprised of a configuration, study, simulation, and experimentation of a modified suspension with variable stiffness structure. The outline depended on the indication of the mechanism for variable stiffness. A theoretical modulation was presented in order to an exhibition the better performance of vehicle for ride comfort and road handling. The experimental results were obtained and corresponding with the simulation by the Matlab program to explain the difference between them and to verify outcomes, although the difference between experimental and simulation results but it has good agreement. The study considered the simulations for the active vibration control for suspension system used linear, quadratic regulation (LQR) to indicate that the resultant of suspension for variable stiffness structure has considerably improved performance than the old-style constant stiffness complement.

**Table 2** List of Symbols

Symbol	Description	Unit
H	Height of the pivot bar	m
$I_1, I_2$	Moment of inertia of the minor wishbone and controller arm	Kg m
$k_1, k_2$	Spring constants	N/m
$K_s, K_t, K_u$	Stiffness of suspension and tire spring and control strut spring	N/m
$C_s, C_t, C_u$	Damping coefficient of suspension and tire and control strut damping	N sec/m
$L_1, L_2$	Distances of the vertical springs ( $k_1$ and $k_2$ ) from the midpoint of the lever	m
$m_s, m_t, m_a$	Sprung, unsprung and control masses.	kg
$z_s$	Vertical movement of the sprung mass.	m
$z_r$	Vertical movement of the road.	m
F	An external force.	N

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