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Research Article

Minimum Weight Optimization of a Gear Train by using Genetic Algorithm

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Abstract

Gear design optimization consists of several objectives such as centre distance, weight, fatigue strength, tooth deflection depending on the requirements. Among these objectives the weight of a gear train is one of the best considerable designs, since many power transmission applications require low weight. The advanced optimization technique, Genetic Algorithm (GA) is used to find the optimal combination of design parameters for minimum weight of a gear train. The results of the proposed algorithm of various new design variable values along with additional constraints are compared with all required parameters. From the results it is observed that Genetic Algorithm gives better solutions for gear design.

Keywords: *Optimal weight Design, Gear train, Genetic Algorithm.*

Nomenclature

a Centre distance (mm)
b Face width (mm)
b_i Constraint quantities
b_w Thickness of web (mm)
d_1, d_2 Diameter of pinion, gear shaft (mm)
D_i Inside diameter of rim (mm)
d_0 Outside diameter of boss (mm)
d_p Drilled hole diameter (mm)
D_r Dedendum circle diameter (mm)
F(x) Objective function
F_p Wear load (N)
F_s Induced bending load (Lewis formula)(N)
$g_i(x)$ Constraints
K_v Velocity factor
K_w Load factor
l Length of boss (mm)
l_w Thickness of rim (mm)
m Module (mm)
n Number of drilled holes
N_1, N_2 Speed of pinion, gear shaft (rpm)
P Power to be transmitted (kw)
v Pitch line velocity (m/s)
x Design variable vector
y Lewis tooth form factor
Z_1, Z_2 Number of teeth on pinion, gear
ρ Density of gear material (mg/m3)
σ Gear material strength (mpa)
τ Shaft shear strength (mpa)
Φ Pressure angle (^{0})

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1. Introduction

The design of many engineering systems can be a complex process. Assumptions must be made to develop realistic models that can be subjected to mathematical analysis by the available methods, and the models must be verified by experiments. Many possibilities and factors must be considered during problem formulation. Economic considerations play an important role in designing cost-effective systems. To complete the design of an engineering system, designers from different fields of engineering usually must cooperate. For example, the design of a high-rise building involves designers from architectural, structural, mechanical, electrical, and environmental engineering as well as construction management experts. Design of a passenger car requires cooperation among structural, mechanical, automotive, electrical, chemical, hydraulics design, and human factors engineers. Thus, in an interdisciplinary environment considerable interaction is needed among various design teams to complete the project. For most applications the entire design project must be broken down into several subproblems, which are then treated somewhat independently. Each of the subproblems can be posed as a problem of optimum design.

2. Literature Review

In the past various researchers had used Genetic Algorithm (GA) for different application. A brief review is carried out here by studying several research papers.

The Genetic Algorithms (GA's) were developed by Prof. John Holland and his students at the University of Michigan during the 1960s and 1970s.

There have been a number of studies attempting to optimize gears with the aid of computers. Yallamti Murali Mohan and T. Seshaiah prepared a computer program in 'C' language for optimization of spur gear by Genetic Algorithm (GA) technique [Yallamti Murali Mohan *et al*.]. This involves about the optimization of spur gear set for its centre distance, weight and tooth deflections are taken as main objective functions and the decision variable such as module, face width and number of teeth on pinion, which are subjected to constrained bending stress of gear train and contact stresses as a nonlinear constrained problem. This problem was solved by non- traditional optimization technique called Genetic Algorithm. Anjali Gupta presented that the use of Genetic Algorithm for the optimization problem of the spur gear set to reduce the weight by considering the centre distance has only one objective function with bending stress limit of gear and surface stress as a nonlinear constraints. And the design problem was optimized b y comparing the problem with traditional method [Anjali Gupta].

Tong and Walton described an interactive program to design internal gear pairs. The program was having large built-in databases for tooth cutters and materials. A complete design was provided including all the necessary information for manufacture [B.S. Tong *et al*.]. Savsani *et al*. presented two advanced optimization algorithms known as particle swarm optimization (PSO) and simulated annealing (SA) to find the optimal combination of design parameters for minimum weight of a spur gear train. Because many high-performance power transmission applications are requires low weight. They considered two cases of optimal design of a spur gear train and the objective considered in both the cases minimize total weight [Savsani *et al*.].

Yokota *et al*. formulated an optimal weight design problem of a gear for a constrained bending strength of gear, torsional strength of shafts, and each gear dimension as a nonlinear integer programming problem and solved the same using an improved genetic algorithm (GA). However, certain constraints were not satisfied and the obtained solution was not optimum [Yokota *et al*.].

However, GA provides a near optimal solution for a complex problem having large number of variables and constraints. So, this project is intended to Genetic Algorithm (GA) for solving the optimal weight design problem.

In the present work, an effort is made to verify if any improvement in the solution is possible by employing Genetic Algorithm (GA) to the same optimization model formulated by Yokota [Yokota *et al*.]. The gear design problem presented by Yokota [Yokota *et al*.] contains five design variables and five constraints. However, in the present work, the design problem is modified with five design variables and eight constraints. Modified design is optimized using Genetic Algorithm (GA) to have the confidence for the global optimum.

3. Genetic Algorithm

Genetic algorithms are search algorithms based on mechanics of the natural selection and the natural genetics (Holland, 1975). Genetic algorithm exploits the idea of the survival of the fittest and the interbreeding population to create a novel and innovative search strategy. A population of the strings representing solution to the specified problem is maintained by genetic algorithm, which then iteratively creates the new population from the old by ranking the strings and interbreeding the fittest to create the new strings, which are closer to the optimum solution to a specified problem.

The three basic operators in the genetic algorithm i.e. reproduction, crossover and mutation are discussed below:

Reproduction

The reproduction operator allows individual strings to be copied for possible inclusion in the next generation. The chance that a string will be copied is based on the strings fitness value. The different types of reproduction operators are Proportional selection, tournament selection, truncation selection, linear ranking selection and exponential ranking selection. The selection of the particular scheme depends on the problem domain being explored.

Crossover

Crossover refers to the blending of chromosomes from the parents to produce new chromosomes for the offspring. The genetic algorithm selects two strings at a random from the mating pool. It is then decided whether to crossover using a parameter called crossover probability. If the crossover takes place then a random slicing point is chosen in the string. The sliced regions are then mixed to create two new strings.

Mutation

Although crossover can generate a staggering amount of different strings, there may not be enough variety of stings to ensure that the entire problem is covered. This may lead to converging on strings that are not quite close to the optimum it seeks. To overcome this problem a mutation operator is introduced into a genetic algorithm. For each string element in each string in the mating pool the algorithm checks to see if it should perform mutation and if it should the string element is flipped (in case of binary strings). The mutation thus helps to prevent the population from stagnating and maintains the diversity throughout the iterations. However, the mutation probability should be kept very low as a high mutation rate will destroy fit strings and degenerate the algorithm into a random walk.

The performance of the genetic algorithm thus mainly depends on population size, number of generations, crossover rate and mutation rate. Genetic algorithms are preferred when near optimal improved conditions instead of exact optimum solution are cost effective and acceptable for implementation by the manufacturer. It is a derivative free approach for near optimal points search direction, and may be applied to continuous or discrete response function. Although GA has advantages over the traditional techniques, it has following limitations:

i) In GA, all offspring are accepted and their parent strings are abandoned at the end of every generation regardless of their fitness values. This gives rise to a risk that a good parent string may be replaced with its deteriorated child string. Thus the improvement on the average performance of child population over parent population cannot be always guaranteed.
ii) In GA, only good parent strings are given chance to produce offspring without any consideration of the possibilities of generating better offspring by others.
iii) They are not efficient when convergence speed is taken into consideration.
iv)Genetic algorithm does not guarantee optimal solution.

4. Optimum Design Problem Formulation Process of a Gear Train

The formulation of an optimum design problem involves translating a descriptive statement of it into a well-defined mathematical statement. The tasks to be performed in each of the foregoing steps are described to develop a mathematical formulation for the design optimization problem [Jasbir *et al.*].

It is also important to understand the solution process for optimization of a design problem. The optimization methods are iterative where the solution process is started by selecting a trial design or a set of trial designs. The trial designs are analyzed and evaluated, and a new trial design is generated. This iterative process is continued until an optimum solution is reached.

4.1. Step 1: Project/Problem Description

The purpose of this project is to design a gear train, as shown in Figure 3.3, to optimize the weight of gear train with respect to their design requirements. The gear trains are widely used in high-performance power transmission applications like automotive industries, machine tools, aerospace industries etc., because of its wide applications it is important to optimize the design of a gear train.

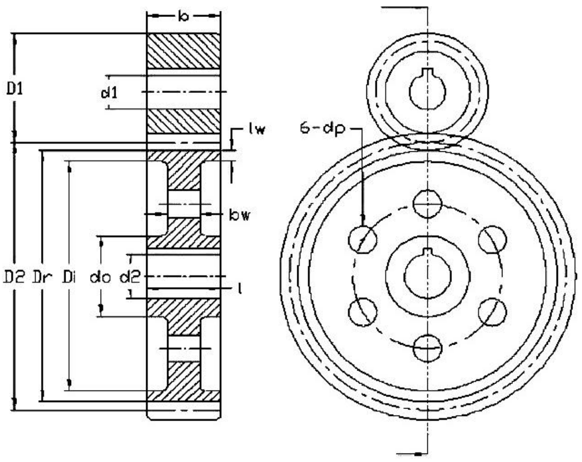


Figure 1: Basic single stage spur gear geometry

4.2. Step 2: Data and Information Collection

To formulate this design optimization problem [8], the following data is necessary.

$F_s = \pi K_v K_w \sigma m y$, $F_p = \frac{2 K_v K_w D_1 b Z_2}{(Z_1+Z_2)}$, $D_r = m(a Z_1 - 2.5)$, $l_w = 2.5 * m$, $D_i = D_r - 2l_w$, $b_w = 3.5 * m$, $d_o = d_2+25$, $d_p = 0.25(D_i - d_o)$, $D_1 = m Z_1$, $D_2 = a m Z_1$, $N_2 = \frac{N_1}{a}$, $Z_2 = \frac{Z_1 D_2}{D_1}$, $V = \frac{\pi D_1 N_1}{60000}$, $b_1 = \frac{1000 P}{V}$, $b_3 = \frac{4.97 x 10^6 x P}{N_1 \tau}$, $b_4 = \frac{4.97 x 10^6 x P}{N_2 \tau}$,
And the input values are, a = 4; ρ = 8; P = 7.5; n = 6; σ = 294.3; y = 0.102; b_2 = 0.193; τ = 19.62; K_w = 0.8; K_v = 0.389.

4.3. Step 3: Definition of Design Variables

For optimal weight design problem, the following five design variables are defined as b = face width in mm, d_1 = diameter of pinion in mm, d_2 = diameter of gear in mm, Z_1 = number of teeth on pinion and m = module in mm.

4.4. Step 4: Optimization Criterion

The optimal gear design problem defined by Yokota [Yokota *et al.*] consists of a non-linear objective function involving five design variables The design objective is to minimize the weight of the gear train. Therefore, the optimization criterion, or objective function is given as

$$\text{Weight} = F(x) = \left(\frac{\pi\rho}{4000}\right)\left[bm^2Z_1^2(1+a^2) - (D_i^2 - d_o^2)(l - b_w) - nd_p^2b_w - (d_1^2 + d_2^2)b\right]$$

Here x = Design vector = (b, d_1, d_2, Z_1, m)

4.5. Step 5: Formulation of Constraints

CASE1: The optimal gear design problem defined by Yokota [Yokota *et al.*] consists of five non-linear constraints involving five design variables. The design variables with their ranges and the constraints are given below.
For,

Bending strength of tooth, $G_1(x) = F_s \geq b_1$

Surface durability, $G_2(x) = \frac{F_s}{F_p} \geq b_2$

Torsional strength of shaft for pinion,

$$G_3(x) = d_1^3 \geq b_3$$

Torsional strength of shaft for gear,

$G_4(x) = d_2^3 \geq b_4$

Centre distance, $G_5(x) = \frac{(1+a)mZ_1}{2} \leq b_5$

And, the constraints on the design variables are imposed as,

$20 \leq b \geq 32$

$10 \leq d_1 \geq 30$

$30 \leq d_2 \geq 40$

$18 \leq Z_1 \geq 25$

$2.75 \leq m \geq 4$

CASE2: The above design problem is defined by Yokota [Yokota *et al.*] which are modified with same design variables and different bound limits along with eight constraints in which three additional constraints are added [R.C. Juvinall *et al.*] for previous constraints are given below:

For,

Bending strength of tooth, $G_1(x) = F_s \geq b_1$

Surface durability, $G_2(x) = \frac{F_s}{F_p} \geq b_2$

The check of interference,

$$G_3(x) = \frac{\sin^2\phi\, D_1(2D_2+D_1)}{(4\ m)} - D_2 - 1 \geq 0$$

Ensure uniform load distribution, $G_4(x) = \frac{b}{m} \geq 8$

and $G_5(x) = \frac{b}{m} \leq 16$

Torsional strength of shaft for pinion,

$G_6(x) = d_1^3 \geq b_3$

Torsional strength of shaft for gear,

$G_7(x) = d_2^3 \geq b_4$

Centre distance, $G_8(x) = \frac{(1+a)mZ_1}{2} \leq b_5$

Finally, the constraints on the design variables are imposed as,

$20 \leq b \geq 35$

$15 \leq d_1 \geq 32$

$30 \leq d_2 \geq 50$

$18 \leq Z_1 \geq 25$

$1 \leq m \geq 4$

Where the value of $\Phi = 25^0$.

In this project, the original design problem formulated by Yokota [Yokota *et al.*] is considered as case1 and the modified design as case2.

5. Weight Optimization of a Gear Train by Genetic Algorithm Solver in Mat Lab:

To use Global Optimization Toolbox software [Matlab User's Guide; Amos Gilat *et al.*], need to

1. Define the objective function in the MATLAB language, as a function file or anonymous function.
2. Define the constraint(s) as a separate file or anonymous function.

CASE 1: By Considering the Yokota [Yokota et al.] Design Ranges:

1) In the Problem setup and Results enter @min_weight as fitness function. The @ character indicates that this is a function handle of the file min_weight.m. And number of variables, lower bounds, upper bounds and constraint function as @gear-constraint. And also specify the population size and crossover fraction in the options pane.
2) To run the genetic algorithm, click the Start button. The tool displays the results of the optimization in the Run solver and view results pane.
3) The following message appears in the box below the Start button

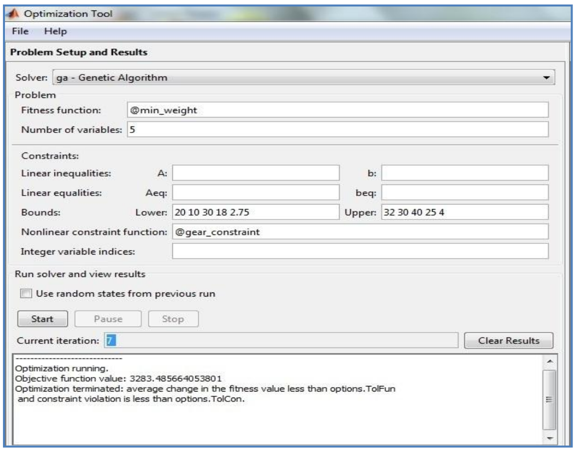


Figure 2: Value of Objective function

The message tells that

⇒ The search for a constrained optimum ended because the derivative of the objective function is nearly 0 in directions allowed by the constraint.

⇒ The constraint is satisfied to the requisite accuracy.

1. The minimizer x appears under Final point.

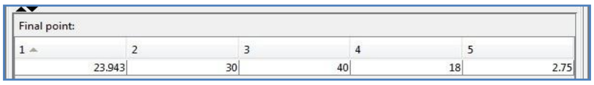


Figure 3: Optimum values of design variables.

CASE 2: For Expanded Ranges of Design Variables

1. To run the Genetic algorithm passes the following details on the Problem setup and Results pane.

Fitness function = @min_weight.

Number of Variables = 5

Lower bounds = [20 15 30 18 1]

Upper bounds = [35 32 50 25 4]

Constraint function = @new-constraint.

And also mention the population size, crossover fraction and display level on Options pane.

2. Now click on the start button to run the program.

3. After completion of the iterations the solutions will displayed on result window along with the optimum values of design variables appears under Final point pane.

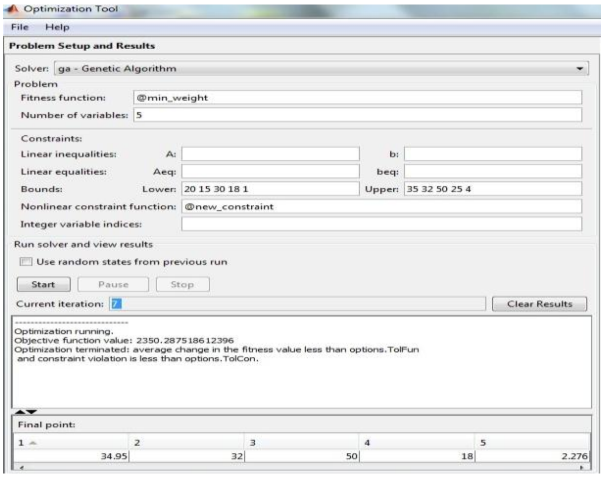


Figure 4: Results & Optimum values of design variables

The genetic algorithm solver handled the nonlinear objective function and nonlinear inequality constraints with different bound limits of design variables.

5. Results and Discussions

Considering the problem as optimization of gear train with design variables such as module, face width, diameters of gear & pinion and number of teeth on pinion, minimizing the weight of gear train is taken as an objective function and are subjected to constraints such as bending strength of tooth, torsional strength of shafts for pinion & gear and centre distance. From this data, the problem is solved in non-traditional method (Genetic algorithm) with additional constraints for the check of interference and uniform load distribution. Genetic algorithms solutions are solved by using software MATLAB 2014a. The following formulated results are obtained from the 'ga' solver.

By solving the constrained optimization problem using Genetic Algorithm the following results are obtained.

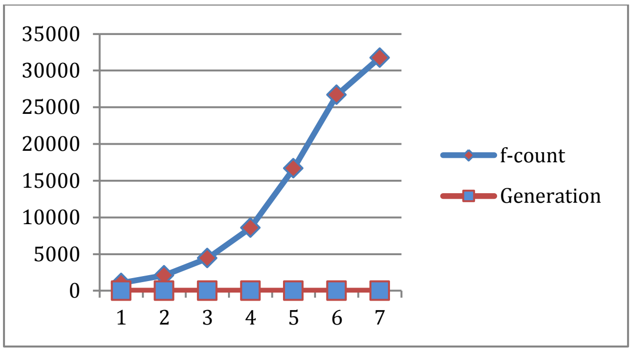
CASE1: Output values for Yokota [Yokota et al.] considered ranges of parameters

Table 1: Genetic Algorithm results for Yokota [Yokota *et al.*] ranges of problem

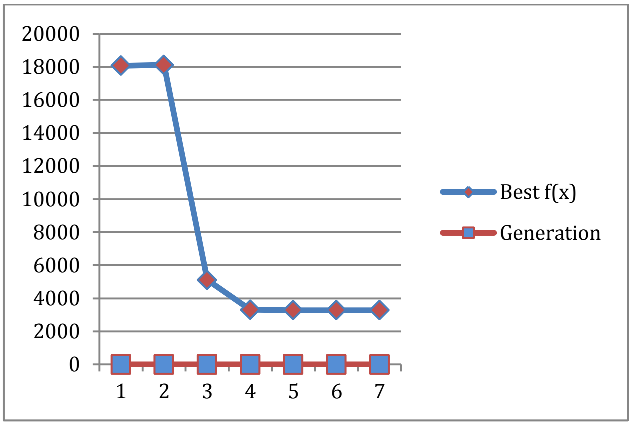
Generation	f-count	Best f(x)
1	1060	18060
2	2100	18108.1
3	4440	5107.53
4	8560	3309.25
5	16660	3283.76
6	26680	3283.49
7	31760	3283.49

The above table presents the output values generated with function evaluations and optimum function value by making the iterations.

The below graph will shows that the maximum iterations generated at the function evaluation of 31760.



Graph 1: Graph between function count and No. of iterations



Graph 2: Graph between Optimum objective value and No. of iterations

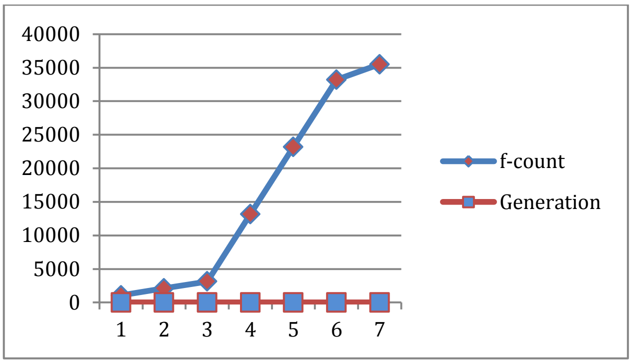
From the above graph the optimum function value lies at 7[th] iteration is 3283.49.

CASE2: Output for Expanded ranges of parameters (New constraint)

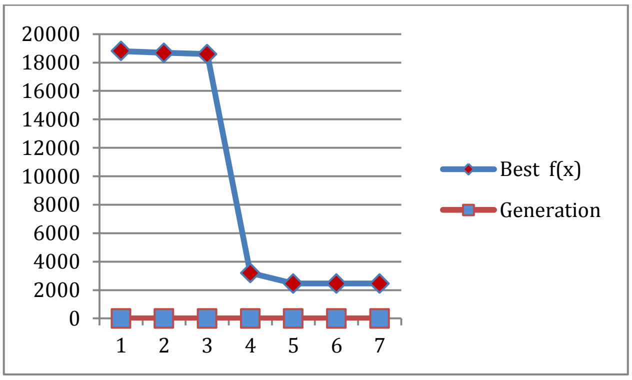
Similarly for expanded ranges of design variables i.e., with additional constraints and new ranges of design values for decision variables can be represented by the following table and graphs.

Table 2: Genetic Algorithm results for expanded ranges of problem

Generation	f-count	Best f(x)
1	1060	18890
2	2100	18707.1
3	3140	18686.6
4	13160	3077.2
5	23180	2361.67
6	33200	2350.36
7	35480	2350.29



Graph 3: Graph between function count and No. of iterations



Graph 4: Graph between Optimum objective value and No. of iterations

The above two graphs mention that the optimum function value presents at 7th iteration is 2350.29, which is obtained when the function count is at 35480. Finally by comparing the above two cases the optimum values of design variables can be formulated as in below table3.

Table 3: Comparison of optimization results from case1 and case2

S.NO	Design Variables	GA^a	GA^b
1	Weight (g)	3283.49	2350.29
2	b (mm)	23.943	34.95
3	d_1 (mm)	30	32
4	d_2 (mm)	40	50
5	Z_1	18	18
6	m (mm)	2.75	2.276
7	Function count	31760	35480

GA^a – Results for Yokota [Yokota *et al.*] considered ranges of design variables presents in case1.
GA^b - Results for Expanded ranges of design variables presents in case2.

Conclusion and Future Scope

From the results shown in table 3, it is observed that modified gear design has shown an additional weight reduction of 28.42% compared to the result reported by Yokota [Yokota *et al.*] using Genetic Algorithm. Application of Genetic Algorithm with expanded variable ranges has shown the corresponding weight reduction is better than the result obtained by Yokota [Yokota *et al.*]. Finally it is concluded that the gear parameters and weight obtained from the new design of gear train by Genetic Algorithm gives more optimal than the result reported by Yokota [Yokota *et al.*] design approach.

The problem can be extended the design variable such as hardness number, which plays a crucial role for the surface fatigue strength. And other recently developed evolutionary algorithm PSO can also be tried to solve this problem. Similar approach can be followed in case of other applications, such as minimization of mass of spring, minimization of weight of pulley system and minimization of total weight of column.

References

Yallamti Murali Mohan, T.Seshaiah (2012), Spur Gear Optimization by Using Genetic Algorithm, Vol. 2, Issue 1.

Anjali Gupta (2014), Design Optimization of the Spur Gear Set, Vol. 3 Issue 9.

B.S. Tong, D.A. Walton (1987), A computer design aid for internal spur and helical gears, International Journal of Machine Tools and Manufacture 27 (4), 479–489.

V. Savsani, R.V. Rao, D.P. Vakharia (2010), Optimal weight design of a gear train using particle swarm optimization and simulated annealing algorithms, Mechanism and Machine Theory 45 531–541

T. Yokota, T. Taguchi, M. Gen (1998), A solution method for optimal weight design problem of the gear using genetic algorithms, Computers & Industrial Engineering 35 523–526.

Jasbir S. Arora (2012), Introduction to optimum design, Elsevier Inc. Amsterdam.

R.C. Juvinall, K.M. Marshek (2000), Fundamentals of Machine Components Design, Wiley, New York.

Matlab (2004–2014), Global Optimization Toolbox User's Guide, The MathWorks, Inc,.

Amos Gilat, Matlab (2011), An Introduction with Applications, John Wiley & Sons, Inc. New York.