International Journal of Current Engineering and Technology
E-ISSN 2277 – 4106, P-ISSN 2347 – 5161

Available at http://inpressco.com/category/ijcet

Research Article

Thermosolutal Instability of Magneto-Hydrodynamic Flow through Porous Medium

Saroj Choudhary†*

†Department of Applied Science, JECRC, Jodhpur, Rajasthan, India

Accepted 05 March 2015, Available online 07 March 2015, **Vol.5, No.2 (April 2015)**

Abstract

Thermosolutal instability in a heterogeneous fluid layer with free boundaries has been, studied in porous medium. Employing the normal mode technique, the solution has been obtained. For the case of conducting viscous, in compressible and heterogeneous fluids, the dispersion relation has been derived and solved numerically. It has been found that magnetic field shows stabilizing influence and porosity shows destabilizing effects on the thermosolutal instability.

Keywords: *Different densities fluids, Kelvin-Helmholtz instability, magnetic field, porous medium.*

1. Introduction

The study of onset of convection in a porous medium has attracted considerable interest because of its natural occurrence and intrinsic importance in many industrial problems, in petroleum exploration, chemical and nuclear industries, in geophysics, ground water hydrology, soil sciences. The effect of a magnetic field on the stability of such a flow if of interest in geophysics, particularly in the study of Earth's core where the earth's mantle, which consists of conductivity fluid behaves like a porous medium which can become convectively unstable as a result of differential diffusion.

The understanding of the flow phenomenon in packed beds is of considerable practical importance in the interpretation of chemical reactor performance where hydrodynamic dispersion and molecular diffusion play important roles in mixing process. The investigation of thermosolutal convection is motivated by its interesting complexities as a double diffusion phenomenon. The forces in thermal convection buoyancy arise from density differences due to variations in temperatures and also from those due to variation in solute concentration. The problem of the setting up of convection currents in non-porous medium in a layer of viscous fluid was first solved by Rayleigh (Rayleigh and Lord, 1964) and Jeffreys (Jeffreys, 1926) and further elaborated by Low (Low A. R., 1929), Hales (Hales, A. L. 1936), Pellew and Southwell (Pellew *et al*, 1930) and in more details under varying assumptions was studied by Chandrasekhar (Chandrasekhar S., 1961) in his monograph. The first pioneering work concerning the buoyancy induced transport in a horizontal porous layer heated uniformly from below began with the work of Horton and Rogers (Horton *et al*, 1945). In recent past, many workers such as Prabhamani and Rudraigh (Prabhamani *et al*, 1973). Rohini (Rohini G., 1979) and Rudraian and Masuoka (Rudraiah *et al*, 1982) are used Brinkman equation as a first approximation to investigate the onset of convection in a horizontal porous layer heated from below with different physical configurations, Dandapat and Gupta (Dandapat *et al*, 1982) studied the onset of thermal convection in a layer of saturated porous medium which is subjected to random vibrations. The problem of thermosolutal instability in a horizontal layer of saturated porous medium was treated by Nield (Nield D. A., 1967) within the framework of linear perturbation theory. Sharma (Sharma R. C., 1990) studied the thermosolutal convection in compressible fluids in porous medium in the absence, separately of rotation and magnetic field. It is interesting, therefore, the study of the thermololutal instability in a heterogeneous fluid layer with free boundaries in porous medium.

1.1 Formulation of the Problem and Linearized Perturbation Equation

Here we study the thermosolutal instability in a heterogeneous fluid layer with free boundaries in porous medium. Consider a horizontal layer of fluid in porous medium of thickness *d* between two free boundaries at *Z=0* and *Z=d*. Let the fluid in pores be conducting viscous, incompressible and heterogeneous. The density of the fluid be of the form

*Corresponding author: **Dr. Saroj Choudhary**

$\rho_0 f(Z)$, where ρ_0 is the density at $Z=0$. The layer is infinite in the horizontal direction and is heated and soluted from below. The temperature gradients

$$\beta = \frac{(T_0 - T_1)}{d}$$

and

$$\beta' = \frac{(C_0 - C_1)}{d}$$

where

T_0, T_1 $(T_0 > T_1)$

and

C_0, C_1 $(C_0 > C_1)$

are the constant temperatures and concentrations of the lower and upper surfaces.

Let a uniform magnetic field

$$H = (0, 0, H)$$

be prevalent in the system and linearized equations are

$$\rho \frac{D\delta q}{Dt} = -\nabla \delta p + q\delta\rho - \alpha\rho_0 \delta T + \alpha' \rho_0 \delta C + \mu_e H \nabla \delta H + \rho_0 \nu \nabla^2 \delta q - \frac{\rho_0 \nu \delta q}{P} \quad (1)$$

$$\nabla . \delta q = 0 \quad (2)$$

$$\frac{\partial(\delta\rho)}{\partial t} + (\delta q . \nabla)\rho = 0 \quad (3)$$

$$\nabla . \delta H = 0 \quad (4)$$

$$\frac{\partial(\delta T)}{\partial t} - \beta(\delta q . \nabla) z = K_T \nabla^2 (\delta T) \quad (5)$$

$$\frac{\partial(\delta C)}{\partial t} - \beta'(\delta q . \nabla) z = K_s \nabla^2 (\delta C) \quad (6)$$

$$\frac{\partial(\delta H)}{\partial t} = curl(\delta q \times H) z + \eta \nabla^2 \delta H \quad (7)$$

2. Normal Mode Analysis

Analyzing the disturbances in terms of normal modes, find that the linearized perturbation equations and appropriate boundary conditions are satisfied if the dependence of physical quantities on is of the form.

$$f(z) = \exp(iK_x x + iK_y y + nt)$$

(8)

Where K_x and K_y and are horizontal wave numbers such that the wave numbers of the disturbance is

$$K = \sqrt{K_x^2 + K_y^2}$$

Also $n=n_1+n_2$ where n_1 denotes the growth rate and $n_2$the frequency of the disturbances.

Further we have written

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$\xi = \frac{\partial hy}{\partial x} - \frac{\partial hx}{\partial y}$$

Where ζ ξ and denote the z components of vortocity and current density, respectively, of puerturbation.

Using the dimension varaiables

$$D = \alpha D,\ \sigma = \frac{nd^2}{\nu},\ \rho_1 = \frac{\nu}{K_T},\ \rho_2 = \frac{\nu}{\eta},\ K = \frac{K_s}{K_T}, B = \frac{D^2}{P},$$

$$R = \frac{g\alpha\beta d^4}{K_T \nu},\ R_1 = \frac{g\alpha' \beta' d^4}{K_T \nu},\ R_2 = \frac{gd^3}{\nu^2},\ Q = \frac{\mu_e H^2 d^2}{\rho_0 \eta \nu}$$

Taking the rectilinear components from the equations (1) to (7), analysing in normal modes and eliminating some of the variable, we get following equation in view of non-dimensional quantities.

$$\begin{aligned}
&\sigma P_1 (D^2 - k^2 d^2)(D^2 - k^2 d^2 - \sigma P_1) \lfloor K (D^2 - k^2 d^2) - \sigma P_1 \rfloor \times \\
&\lfloor (D^2 - k^2 d^2 - \sigma P_2)(D^2 - k^2 d^2 - B - \sigma) - GD^2 \rfloor W \\
&-k^2 d^2 (D^2 - k^2 d^2 - \sigma P_2) \{ (D^2 - k^2 d^2 - \sigma P_2)(D^2 - k^2 d^2 - B - \sigma) - GD^2 \} \times \\
&\lfloor \{ R_2 (D^2 - k^2 d^2 - \sigma P_1) - P_2 \sigma R \} \{ K (D^2 - k^2 d^2) - \sigma P_1 \} \rfloor + P_1 \sigma R_1 (D^2 - \sigma P_1) W = 0
\end{aligned} \quad (9)$$

3. Diespersion Relation

By variational method, supposing

$$W = W_0 \sin lz, \zeta = \zeta \cos lz, \delta T = \delta T_0 \sin lz,\ \delta C = \delta C_0 \sin lz, H = H_0 \cos lz, \xi = \xi_o \cos lz$$

Where

$$l = \frac{T}{d}$$

and

$$W_0,\ \zeta_0,\ \delta T_0,\ \delta C_0, H_0,\ \xi_0$$

are constants. We get following dispersion relation

$$A_0 \sigma^5 + A_1 \sigma^4 + A_2 \sigma^3 + A_3 \sigma^2 + A_4 \sigma + A_5 = 0 \quad (10)$$

Where

$$A_0 = P_1^2 x^2$$

$$A_1 = 2P_1^2 x (Bx + \pi^2 x^2 + Q) + P_1 (1 + K) \pi^2 x^3$$

$$\begin{aligned}
A_2 = {} & P(2K+1)\pi^2 x \{ Bx + \pi^2 x^2 + Q \} - \frac{k^2}{l^2} x P_1 (R - R_1) + P_1^2 \{ Bx + \pi^2 x^2 + Q \}^2 \\
& - P_1^2 \pi R_5 \frac{k^2}{l^2} x + K \pi^4 x^4
\end{aligned}$$

$$A_3 = P_1(1+K)\pi^2 x\left\{Bx+\pi^2x^2+Q\right\}^2 - \frac{k^2}{l^2}\pi^2 x(KR-R_1) - \frac{k^2}{l^2}P_1(R-R_1)\times$$
$$\left\{B\left((1+y^2)\right)+\pi^2\left(1+y^2\right)^2+Q\right\} - \pi^3\frac{k^2}{l^2}x^2P_1(1+K)R_5$$
$$-\pi\left\{P_1^2\frac{k^2}{l^2}R_5 - 2K\pi^3x^3\right\}\left\{Bx+\pi^2x^2+Q\right\}$$

$$A_4 = K\pi^4x^2\left\{Bx+\pi^2x^2+Q\right\}^2 - K\pi^5\frac{k^2}{l^2}\pi^3R_5 - \pi^3\frac{k^2}{l^2}P_1x(1+K)\left\{Bx+\pi^2x^2+Q\right\}R_5$$
$$-\pi^2\frac{k^2}{l^2}x(KR-R')\left\{Bx+\pi^2x^2+Q\right\}$$

$$A_5 = -K\pi^5\frac{k^2}{l^2}x^2\left\{Bx+\pi^2x^2+Q\right\}R_5$$

$$y=\frac{k}{l}1+y^2=x$$

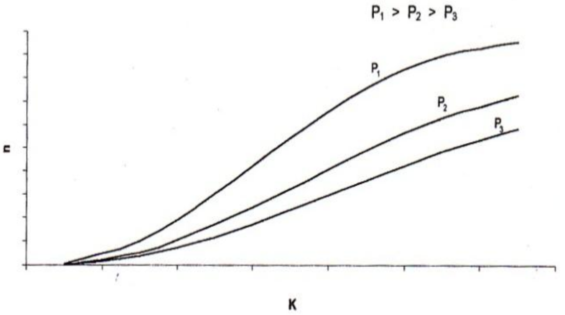

Fig 1: Variation of the growth rate with respect to wave number

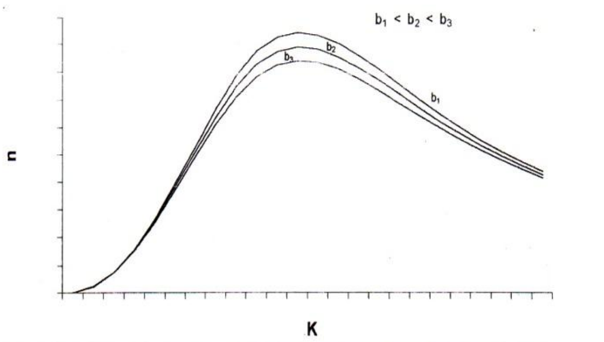

Fig 2: Variation of the growth rate with respect to wave number

4. Numerical Analysis and Conclusion

It has been carried out positive real part of smallest root of the dispersion relation for different values of physical parameters and represented the stabilizing / destabilizing influence by plotting the graph in wave number against growth rate of disturbance keeping other parameters fixed. It is found that magnetic field shows stabilizing (Fig. 1) and porosity shows dest. (Fig. 2) effects on the thermosolutal instability.

References

Rayleigh, Lord (1964), On convective currents in a horizontal layer of fluid when the higher temperature is on the underside, *Phil. Mag. 32*, 529-46.

Jeffreys H., (1926), The Stability of layer of fluid heated below, *Phil. Mag.*, 2 833-44.

Low, A. R. (1929), On the criterion for stability of a layer of viscous fluid heated from blow, *Proc. Roy. Soc. (London) A*, 125, 180-95.

Hales, A. L. (1936), Mon. Not. R. Astr. Soc. Geophys. Suppl., 3, 372-80.

Pellew, A and Southwell, R. V. (1930), On maintained convective motion in a fluid heated from below, Proc. Roy. Soc. (London) A, 176, 312-43.

Chandrasekhar, S. (1961), Hydrodynamics and Hydromagnetic Stability, *Oxford University Press*, London.

Horton, C. W. and Rogers, F. T., Jr. (1945), Convective current in porous medium, *J. App. Phys.*, 16, 367-370.

Prabhamani, R. P. and Rudraiah, N. (1973), Stability of hydromagnetic thermonvective flow through porpus medium, Trans. ASME, J. Appl. Mech, 40, 879.

Gasser, R. D. and Kazimi (1976), M. S., Trans. ASME, *J. Heat Transfer*, 98, 49.

Rohini, G. (1979), PhD. Thesis, Bangalore University.

Rudraiah, N. and Mausoka, T. (1982), Asymptotic analysis of natural convection through horizontal porous layer, *Int. J. Engng.*, 20(1), 22-39.

Dandapat, B. S. and Gupta, A. S. (1982), Thermal instability in a porous medium with random vibrations, Acta Mechanica, 43, 37-47.

Nield, D. A. (1967), J. Fluid Mech., 20, 545-58.

Sharma, R. C. (1990), Thermsolutal convecton in compressible fluids in porous medium, *Jour. Math. Phy. Sci.*, 24(4), 265-81.

Watson, C., Zweibel, E.G., Heitsch, F. and Churchwell, E. (2004), Kelvin-Helmholtz Instability in a Weakly Ionized Medium, *Astrophys. J.*, 608, 274-281.

Allah, M.H.O. (2002), Rayleigh-Taylor Instability with Surface Tension, Porous Media, Rigid Planes and Exponential Densities, *Indian J. Pure and Applied Maths*, 33, 1391-1404.

Benjamin, T.B. and Bridges, T.J. (1997), Reappraisal of the Kelvin-Helmholtz Problem Part 1, Hamiltonian Structure, J. Fluid Mech., 333, 301-325.

Bhatia, P.K. and Mathur, R.P. (2006), Stability of Viscous Rotating Gravitating Streams in a Magnetic Field, *Z. Naturforsch.*, 61, 258-262

Bhatia, P.K. and Sharma, A. (2003), Kelvin-Helmholtz Instability of Two Viscous Superposed Conducting Fluids, *Proc. Natl. Acad. Sci.*, 73, 497-520.

Khan, A. and Bhatia, P.K. (2001), Stability of Two Superposed Viscoelastic Fluids in a Horizontal Magnetic Field, *Ind. J. Pure and Appl. Maths*, 32, 99-108.

Khan, A. and Bhatia, P.K (2003), Stability of a Finitely Conducting Compressible Fluid through Porous Medium, Ganita Sandesh, 17, 35-42.

Kumar, P. and Lal, R. (2005), Stability of Two Superposed Viscoelastic Fluids, Thermal Sciences, 9, 87-95.

Kumar, P., Lal, R. and Singh, G.J. (2006), MHD Instability of Rotating Superposed Walters B' Viscoelastic Fluids through a Porous Medium, J. Porous Media, 9, 463-468.

Kumar, P., Lal, R. and Singh, M. (2007), Hydrodynamic and Hydromagnetic Stability of Two Stratified Rivlin-Ericksen Elastico-Viscous Superposed Fluids, *Int. J. Appl. Mech. and Eng.*, 12, 645-853.

Gupta, U. and Aggarwal, P. (2011), Thermal instability of compressible Walters' (Model B') fluid in the presence of hall currents and suspended particles, *Thermal Science*, vol. 15(2), pp. 487-500.

Chander Bhan Mehta, Susheel Kumar and Sanjeev Gangta (2013), Instability of Stratified Walters Fluid in Porous Medium in the Presence of Suspended Particles and Magnetic Field, International Journal of Physical and Mathematical Sciences, Vol. 4, No.1.

Rana, G.C., and Kumar, S. (2010), Thermal instability of Rivlin - Ericksen Elastoviscous Rotating Fluid Permeating with suspended particles under variable gravity field in porous medium, *Studia Geotechnica et Mechanica*, Vol. XXXII, No.4.

Rana, G.C., Banyal, A.S., and Chand, J. (2012), Rayleigh¬ Taylor instability of two stratified Rivlin Ericksen superposed Fluids Permeated with suspended particles in porous medium, *Int. Journal of Applied Sciences and Engineering Research*, Vol.1, No.2.