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Research Article

Numerical Method for Optimum Performances of Fin Profiles

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Abstract

In the present work, we present a numerical method able to capture the optimum thermal performances of finned surfaces of high and low conductivity. The bidimensional temperature distribution on the longitudinal section of the fin is calculated by restoring to the finite volumes method. The heat flux dissipated by a generic profile fin is compared with the heat flux removed by the rectangular profile fin with the same length and volume. In this study it is shown that a finite volume method for quadrilaterals unstructured mesh is developed to predict the two dimensional steady-state solutions of conduction equation, in order to determine the sinusoidal parameter values which optimize the fin effectiveness. In this scheme, based on the integration around the polygonal control volume, the derivatives of conduction equation must be converted into closed line integrals using same formulation of the 'Stokes theorem'. The heat flux dissipated by generic profile fin is compared with the heat flux removed by rectangular profile with the same length and volume. The numerical method is then applied to the case of sinusoidal profiles fin that represent problems with complex geometries, which make the heat transfer fluxes as high as possible under different conditions. The optimum profile is finally shown for different sinusoidal profiles.

Keywords: *heat transfer, fin optimization, unstructured grid, Stokes theorem, sinusoidal fin.*

1. Introduction

In many engineering sectors, where high thermal fluxes must be transferred, the finned surface power removers are today an usual tool. Since finned surfaces allow evident improvements in heat transfer effectiveness, the heat exchangers field is one of the most interested in their applications.

Moreover new industrial sectors present an increasing interest in the introduction of extended surfaces for heat flux removal. In particular, the electronics industry has promoted a new interest in developing heat removers, aimed at transferring heat from electronic components to the environment, in order to reduce the working temperature and to improve the characteristics and the reliability (A. Bar-Cohen and A. D. Kraus 1990, C. W. Leung, S. D. Probert 1989, A. D. Sinder and A. D. Kraus 1987).

The optimization of heat remover longitudinal profile, in order to transfer the highest power with the smallest volume, is a problem that is not yet completely solved. Such a problem was talked for the first time in the 1920s (Y. Tsukaoto and Y. Seguchi).

The ratio of the actual heat transfer from the fin surface to that, that would transfer if the whole fin surface were at the same temperature as the base is commonly called as the fin efficiency. (D.R. Harper, W.B. Brown 1922), in connection with air-cooled aircraft engines, investigated straight fins of constant thickness, wedge-shaped straight fins and annular fins of constant thickness; equations for the fin efficiency of each type were presented and the errors involved in certain of the assumptions were evaluated.

(E.Schmidt 1926) studied the three types of fins that are: rectangular, triangular and concave parabolic from the material economy point of view. He stated that the least metal is required for given conditions if the temperature gradient is linear, and showed how the thickness of each type of fin must be varied to produce this result. The temperature gradient in conical and cylindrical spines was determined by (Focke et al 1942). In this work, Focke, like Schmidt, showed how the spine thickness must be varied in order to keep the material requirement to a minimum; he, too, found that the result is impractical and went to determine the optimum cylindrical- and conical-spine dimensions. (Avrami Melvin, J.B. Little 1942) derived equations for the temperature gradient in thick-bar fins and showed under what conditions fins might act as insulators on the basic surface. Approximate equations were also given including, as a special case, that of (D.R. Harper, W.B. Brown 1922). (K.A. Gardner 1942) derived general equations for the temperature gradient and fin efficiency in any extended surface to which a set of idealized assumptions are applicable.

(W.H. Carrier, S.W. Anderson 1944) discussed straight fins of constant thickness, annular fins of constant thickness and annular fins of constant cross-sectional area, presenting equations for fin efficiency of each. In the latter

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two cases the solutions were given in the form of infinite series.

A rather unusual application of Harper and Brown's equation was made by (K.A. Gardner 1942), in considering the ligaments between holes in heat-exchanger tube sheets as fins and thereby estimating the temperature distribution in tube sheets. (Duffin, R. J. 1959) presented a variational problem relating to cooling fins where (Maday, C.J. 1974) studied the problems of have a minimum weight of straight fin. In this work, (P. Razelos, K. Imre 1980) considered the linear variation of the thermal conductivity with temperature and assumed that the heat transfer coefficients vary according to a power law with distance from the bore. (Tsukamoto, Y., Seguchi, Y., 1984) proposed shape optimization problem for minimum volume fin. A correlation for the optimal dimensions of a constant and variable profile fins was presented in terms of a reduced heat transfer rate. Assuming that the heat transfer coefficient is a power function of the temperature difference of a straight fin of a rectangular profile and that of the ambient, (H.C. Unal 1985) obtained a closed form solution for the one-dimensional temperature distribution for different values of the exponent in the power function. An exact solution for the rate of heat transfer from a rectangular fin governed by a power law-type temperature dependence heat transfer coefficient has been obtained by (A.K. Sen, S. Trinh 1986). (Snider, A.D., Kraus, A.D. 1987) effectuated a quest for the optimum longitudinal fin profile.

The effect of fin parameters on the radiation and free convection heat transfer from a finned horizontal cylindrical heater has been studied experimentally by (R. Karaback 1992). The fins used were circular fins. The experimental setup was capable of analyzing the effect of fin diameter and spacing on heat transfer. A correlation equation for the tip temperature of uniform annular fins as a function of thermo geometric parameters and radii ratio has been obtained by (A. Campo, L. Harrison 1994).

In this study, Campo and Harrison considered constant heat transfer coefficient along the fin. The optimum dimensions of circular fins of trapezoidal profile with variable thermal conductivity and heat transfer coefficients have been obtained by P. Razelos, K. Imre. Performance and optimum dimensions of different cooling fins with a temperature dependent heat transfer coefficient have been presented by (K. Laor, H. Kalman 1996). In this work, Laor and Kalman considered the heat transfer coefficient as a power function of temperature and used exponent values in the power function that represent different heat transfer mechanisms such as free convection, fully developed boiling and radiation. The optimum dimensions of circular fins with variable profile and temperature dependent thermal conductivity have been obtained by (S.M. Zubair et al 1996). With the help of symbolic computational mathematics, (A. Campo, R.E. Stuffle 1996) presented a simple and compact form correlation that facilitates a rapid determination of fin efficiency and tip temperature in terms of fin controlling parameters for annular fins of constant thickness. (Giampierto Fabbri 1997) considered polynomial profile heat removers and he propose a genetic algorithm in order to determine the polynomial parameter values. Nevertheless, for many situations, an ultimate solution has not yet been found the problem of optimizing the profiles of the fins.

(Lien-Tsaiyu, Cha'o-Kuang Chen 1999) presented the transient temperature response of a convective–radiative rectangular profile annular fin under a step temperature change occurring in its base. They have assumed constant heat transfer coefficient along the fin and used a hybrid method of Taylor transformation and finite difference approximation. The temperature distribution was implemented by employing natural cubic spline fitting. (Esmail M. A. Mokeimer 2002) studied the performance of annular fins of different profiles subjects to locally variable heat transfer coefficient. (Florin Bobaru and Srinivas Rachakouda 2004) presented a numerical approach able to determine the dependence of optimal shapes profiles of thermal fins on the conductivity parameters. (R. Karvenin and T. Karvenin 2010) presented a method for finding the plate fin for maximizing total heat transfer when cooled by forced or natural convection.

In this work, we consider sinusoidal profile heat removers and we propose a numerical method in order to determine the temperature distribution that can find the optimum geometries for maximum heat transfer.

2. Mathematical model and assumptions

The mathematical analysis, in the above cited articles, for the heat transfer from fins, was based on some or all of the following assumptions:

1. Steady heat flow.
2. The fin material is homogeneous and isotropic.
3. There are no heat sources in the fin itself.
4. The heat flow to or from the fin surface at any point is directly proportional to the temperature difference between the surface at that point and the surrounding fluid.
5. The thermal conductivity of the fin is constant.
6. The heat transfer coefficient is the same over all the fin surface.
7. The temperature of the surrounding fluid is constant.
8. The temperature of the base of the fin is uniform.
9. The fin thickness is so small compared to its length and width that temperature gradient normal to the surface may be neglected.

In the orthogonal coordinate system we will refer to a heat remover with longitudinal section symmetrical with respect to the *x* axis and with a rectangular profile, as shown in Fig. 1, then with the proposed model that described by the sinusoidal function *y(x)*, as shown in Fig. 2. The fin width and length L, is immersed in a fluid with a constant bulk temperature T_F. Moreover, the fin base temperature T_0 is known.

In order to calculate the heat flux removed by such a fin it is necessary to determine the temperature distribution in the longitudinal section (plane *xy*). This distribution must satisfy the Laplace's equation:

$$\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=0 \tag{1}$$

With the boundary conditions:

$$T(0,y)=T_0 \tag{2}$$

$$\left[\frac{\partial T}{\partial x}\right]_{L,y} = -\frac{h_f}{k}\left[T(L,y) - T_F\right] \quad (3)$$

$$\left[\frac{\partial T}{\partial x}\right]_{x,0} = 0 \quad (4)$$

$$\left[\frac{\partial T}{\partial x}\right]_{x,f(x)} - \left[\frac{df}{dx}\frac{\partial T}{\partial x}\right]_{x,f(x)} = -\frac{h\sqrt{\left(\frac{df}{dx}\right)^2 x}}{k}\left[T(x,f(x)) - T_F\right. \quad (5)$$

h and h_f being the convective heat transfer coefficients for the longitudinal fin surface and for the final surface and for the final transversal one, respectively, k being the thermal conductivity of the fin. Due to the complexity of the problem it is convenient to determine the temperature distribution numerically using for example our method.

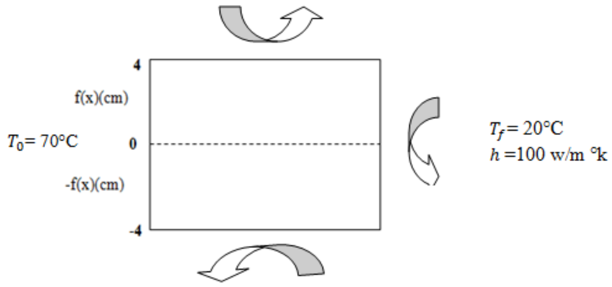


Fig.1 Longitudinal section of a symmetrical profile for an rectangular profile

3. Effectiveness of the fin

The fin performance can be evaluated on the basis of the compared effectiveness, i.e. the ratio between the heat flux (Q_d) dissipated by the heat remover with a generic profile and the heat flux (Q_r) removed by a fin of the same volume and length and with rectangular profile:

$$E = \frac{Q_r}{Q_r} \quad (6)$$

Let us then consider a rectangular fin of width $2\bar{f}$,$\bar{f}$ being the average value of *f(x):*

$$\bar{f} = \frac{1}{L}\int_0^L f(x)dx \quad (7)$$

The temperature distribution on the longitudinal section of such a fin must satisfy equation (1), the boundary conditions (2)-(4) and the following:

$$\left[\frac{\partial T}{\partial x}\right]_{x,f(x)} = -\frac{h}{k}\left[T(x,f(x)) - T_F\right. \quad (8)$$

Since both longitudinal and final transversal surfaces are plane we can assume h equal toh_f. By integrating equation (1) with the above boundary conditions the following solutions is obtained (H. S. Carslaw and J. C. Jaeger 1947) :

$$T(x,y) = T_F + \frac{2h(T_0 - T_F)}{k}\sum_{n=1}^{\infty}\left[\frac{1}{\left(\alpha_n^2 + \frac{h^2}{k^2}\right)\bar{f} + \frac{h}{k}}\frac{\cos(\alpha_n y)}{\cos(\alpha_n \bar{f})} \times \frac{\alpha_n \cosh[(\alpha_n(L-x)] + \frac{h}{k}\sinh[\alpha_n(L-x)]}{\alpha_n \cosh(\alpha_n L) + \frac{h}{k}\sinh(\alpha_n L)}\right] \quad (9)$$

being α_n the solution of the equation:

$$\alpha \tan(\alpha\bar{f}) = \frac{h}{k} \quad (10)$$

The heat flux dissipated for unit of length is then:

$$Q_r = 4h(T_0 - T_F)\sum_{n=1}^{\infty}\left[\frac{tan(\alpha_n\bar{f})}{\left(\alpha_n^2 + \frac{h^2}{k^2}\right)\bar{f} + \frac{h}{k}}\frac{\alpha_n \sinh(\alpha_n L) + \frac{h}{k}\cosh(\alpha_n L)}{\alpha_n \cosh(\alpha_n L) + \frac{h}{k}\sinh(\alpha_n L)}\right] \quad (11)$$

We can calculate the heat flux dissipated by the remover for unit of width in the following way:

$$Q_d = 2\left(\sum_i g_{oi}(T_0 - T_i) + g_{h0}(T_0 - T_F)\right) \quad (12)$$

g_{oi}being the thermal conductance between the fin base and the *ith* element, where it is zero for all the elements which are not adjacent to the fin base. While g_{h0} being the thermal conductance between the fin base and the coolant fluid.

4. Numerical procedure

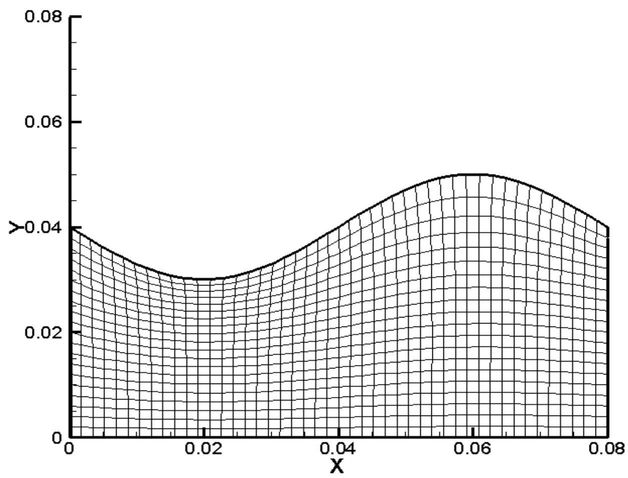
We now propose the numerical method which is able to determine the values of the fin profile describing parameters which allow the highest compared effectiveness. We will consider heat removers for which the profile function $f(x)$ has a sinusoidal form and we have 2 cases:

$$f_1(x) = w - a_0 \sin\left(\frac{2\pi x}{\lambda_0}\right) \quad (13)$$

$$f_2(x) = w + a_0 \sin\left(\frac{2\pi x}{\lambda_0}\right) \quad (14)$$

a_0, λ_0 being the amplitude and length of the sinusoid respectively.

At the beginning we have to fix the fin profile on one undulation to compare the performance of the two geometries and changing the amplitudes from 0.01to 0.035 cm. in this way the fin profile which have best performance is then selected and reproduced in all the study.



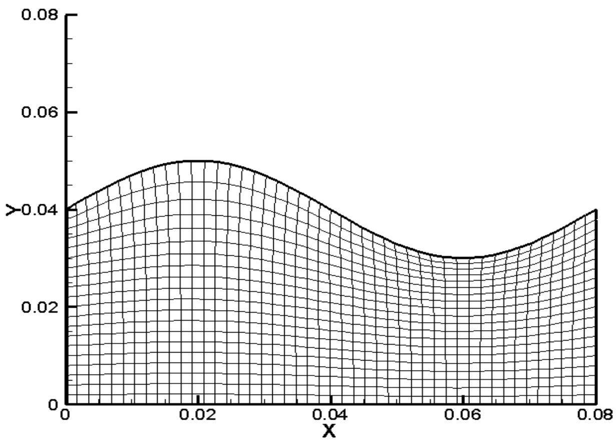


Fig. 2 Longitudinal section of the two cases $f_1(x)$ and $f_2(x)$.

The Laplace's equation is integrated in space using a finite volume method that is developed for an unstructured grid made up of quadrilaterals (G.K. Despotis, and S. Tsangaris 1995, G. K. Despotis, and S. Tsangaris 1996, S. Boivin, F. Cayré, and J. M. Hérard 2000, N. Piskounov 1987).

For the integration around finite volume, the derivations of the flow equation must be converted into closed line integrals using same formulation of the Stokes theorem, which is described by the following equation:

$$\oint_E \vec{T}\,.\overrightarrow{dr} = \iint_s rot\,\vec{T}\,.\vec{n}dS \tag{15}$$

Where $\overrightarrow{dr}$ is the elementary arc, dS is the elemntary surface and is the normal vector to this surface. The computational domain is discretized on a quadrilateral unstructured grid where each node is the centre of polygonal cell constituted of four elements; all computed variables are stored at the centers of the polygonal as:

Approximation of the first derives

The convective terms are calculated at the node P (fig.2). The nodal finite volume descritization scheme is used for the discretization of the convective terms that appear in the governing equation. The first differences are calculated as:

$$\left(\frac{\partial T}{\partial x}\right)_c = \frac{1}{A_C}\int_{Sc} T.dy = \frac{1}{A_C}\sum_{i=1}^{NC}\frac{T_{i+1}+T_i}{2}(y_{i+1}-y_i) \tag{16}$$

$$\left(\frac{\partial T}{\partial y}\right)_c = \frac{1}{A_C}\int_{Sc} T.dx = \frac{1}{A_C}\sum_{i=1}^{NC}\frac{T_{i+1}+T_i}{2}(x_{i+1}-x_i) \tag{17}$$

Where $A_C$is the area of the polygonal control volume (1,2,3,…NE), T the temperature and x,$y$are the coordinate of the polygonal vertices, and I refers to the vertices number of external polygonal control volume.

Approximation of the second derives

This terms must be calculated at the node P and this achieved by computing the second order derivatives at the same point. The required second differences may be computed as:

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_c = \left[\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)\right]_C = \frac{1}{A_{CI}}\int_{SCI} T.dy = \frac{1}{A_{CI}}\sum_{i=1}^{NC}\left(\frac{\partial T}{\partial x}\right)_E (y_{i+1}-y_i) \tag{18}$$

$$\left(\frac{\partial^2 T}{\partial y^2}\right)_c = \left[\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)\right]_C = \frac{1}{A_{CI}}\int_{SCI} T.dx = \frac{1}{A_{CI}}\sum_{i=1}^{NC}\left(\frac{\partial T}{\partial y}\right)_E (x_{i+1}-x_i) \tag{19}$$

$A_{CI}$is the area of polygonal control volume (2,4,…NE) (fig.2) and I refer to the vertices number of internal polygonal control volume. Where, the first differences at the middle of the edge are defined as:

$$\left(\frac{\partial T}{\partial x}\right)_E = \frac{1}{A_E}\int_{SE} T.dy = \frac{1}{A_E}\sum_{i=1}^{4}\frac{T_{i+1}+T_i}{2}(y_{i+1}-y_i) \tag{20}$$

$$\left(\frac{\partial T}{\partial y}\right)_E = -\frac{1}{A_E}\int_{SE} T.dx = -\frac{1}{A_E}\sum_{i=1}^{4}\frac{T_{i+1}+T_i}{2}(x_{i+1}-x_i) \tag{21}$$

$A_E$is the area of the quadrilateral control volume ((1),(2),(3),(4))(Fig.2.) and the four vertices of quadrilateral control volume.

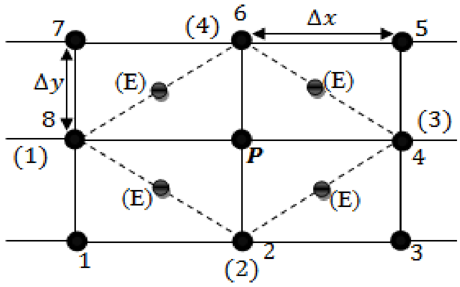


Fig. 3 The computational control volume structure

5. Results

In this study, the numerical method has been utilized in order to optimize the sinusoidal profiles, and we select two types of materials: for the thermal fin high conductivity materials (aluminum of 200/m.°k) and for the low conductivity (titanium of 20/m.°k).

Examination of errors and accuracy

The numerical errors are calculated to show how the errors are improved by refining meshes of (60,120,240 and 480 nodes) and the order of accuracy is achieved. The maximum ($\boldsymbol{E_{max}}$) and root – min – square (RMS) errors were calculated to compare with the analytical solution of the rectangular fin for the aluminum metal. These errors are defined by:

$$E_{max} = \left|(T_{analytic-}T_{numeric}\right| \tag{22}$$

$$RMS = \sqrt{\frac{\sum_{i=1}^{n}(T_{analytic-}T_{numeric})^2}{n}} \tag{23}$$

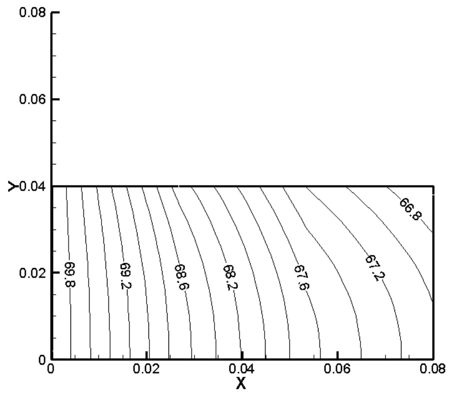
To examine the accuracy quantitatively, the maximum and RMS errors depending on grid sizes (nodal number) are presented in Fig 5. the results are obtained when the

thermal conductivity diffusion is equal to 200 w/m °k and the coefficient of heat transfer is equal to 100w/m² °k .
Therefore, the error decreases at the same rate of root grid size. The solid lines in (fig.3.) shows how the root grid size decrease as the grid number increase (grid size decreases). The maximum and RMS errors decrease at the same rate this verifies that the order accuracy of the method is achieved. The numerical solution and the analytical solution for the aforesaid cases were almost typical. Such a comparison was a validation for our numerical method.

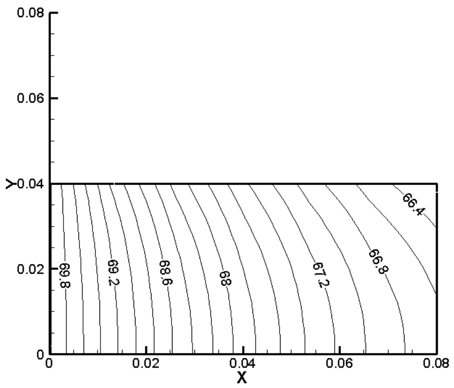
A FORTRAN program has been used to solve the heat transfer governing equations the rectangular fin profiles and show the local temperature along the fin surface.

The program is used to solve the equations of Stokes's theorem for all cases under study to get the temperature distribution along the fin (Fig 4).

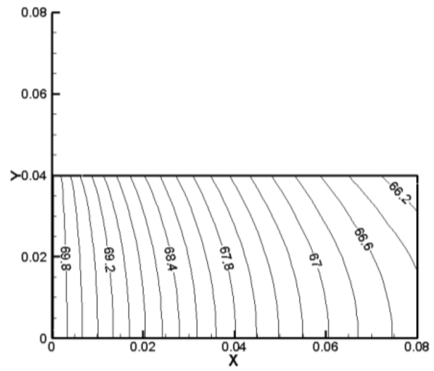
This temperature distribution is then used to calculate the local heat transfer rate along the fin.



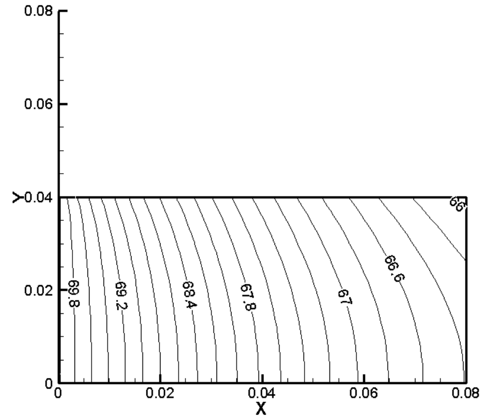
60 nodes



120 nodes



240 nodes



480 nodes

Fig. 4 Temperature distribution in different grid size

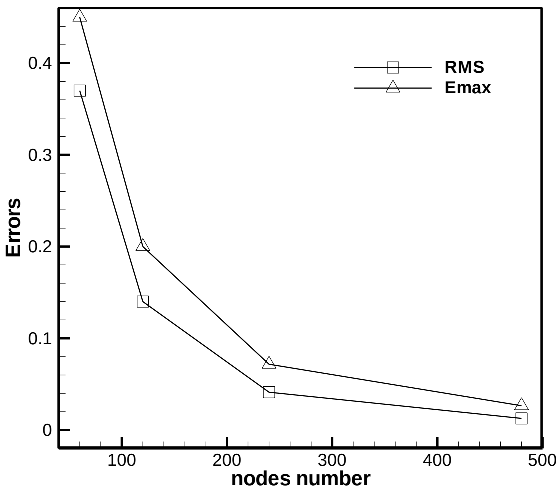


Fig. 5 The maximum and RMS error according to the grid size

The difference between the fin effectiveness that is obtained numerically via 480 nodes with respect that is obtained analytically was negligible. So, that nodes number has been adopted throughout the work.

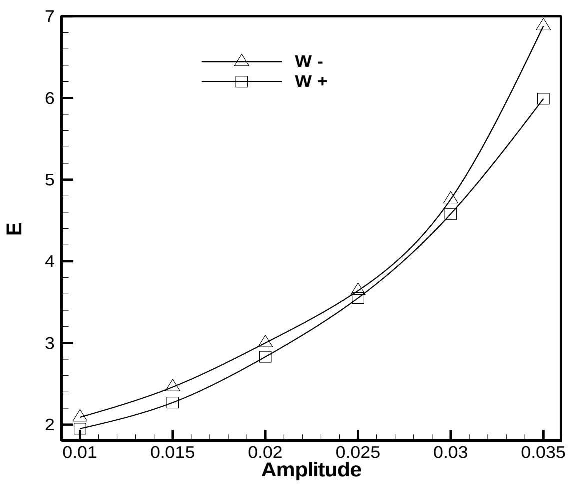


Fig. 6 The compare effectiveness

The compare effectiveness of the two cases that are given by the equations (13,14) , in fact, always grows with the number of undulation, but the oscillation that sublimate than down gives best performances than the opposite case Fig 6.

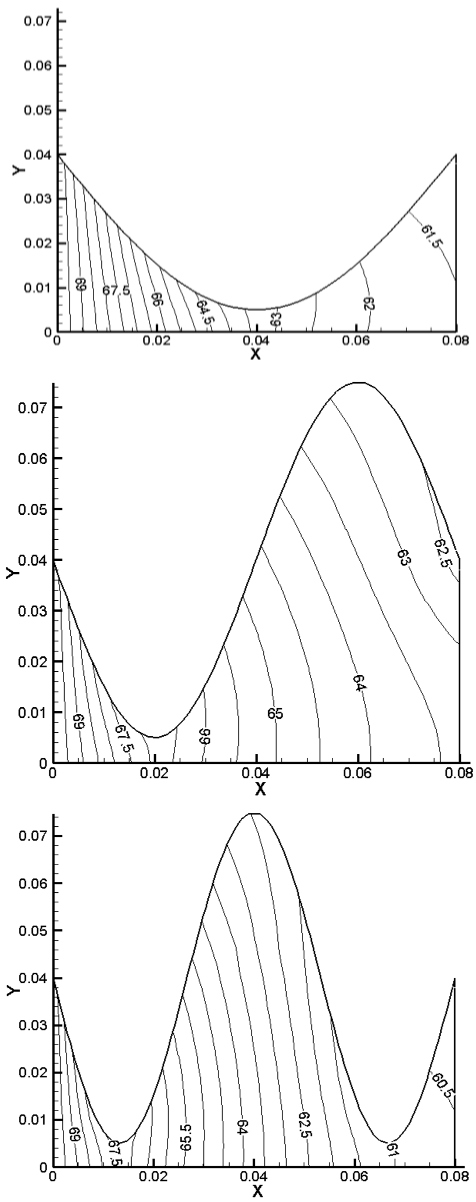
The proposed numerical method has been utilized in order to optimize the sinusoidal profiles of aluminum (k=200 w/m.°k) and the titanium (k=20 w/m.°k) fins with coefficient of heat transfer equal to 100 w/m².°k.

The method was utilized by choosing the number of undulation which describes the fin profile equal to a value from 1 to 3 undulations. We imposed the half-high of the reproduced fin samples and varying the amplitude from 0.01 cm to 0.035 cm.

The temperature distributions of the sinusoidal profiles that gives best performance while a_0= 0.035 are reported in Fig 7.

➢ *Temperature Distribution*

1. Conductivity k=200 w/m.°k



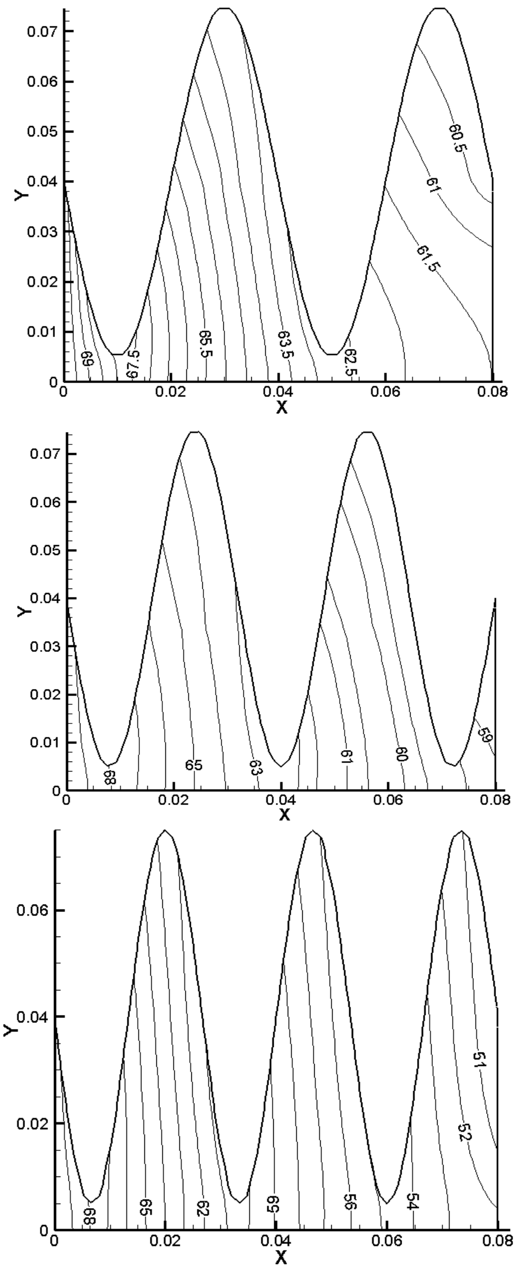
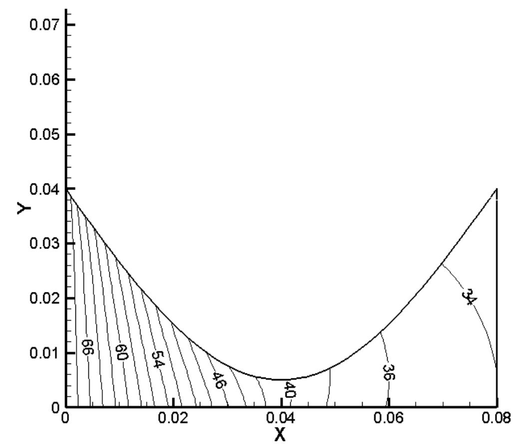
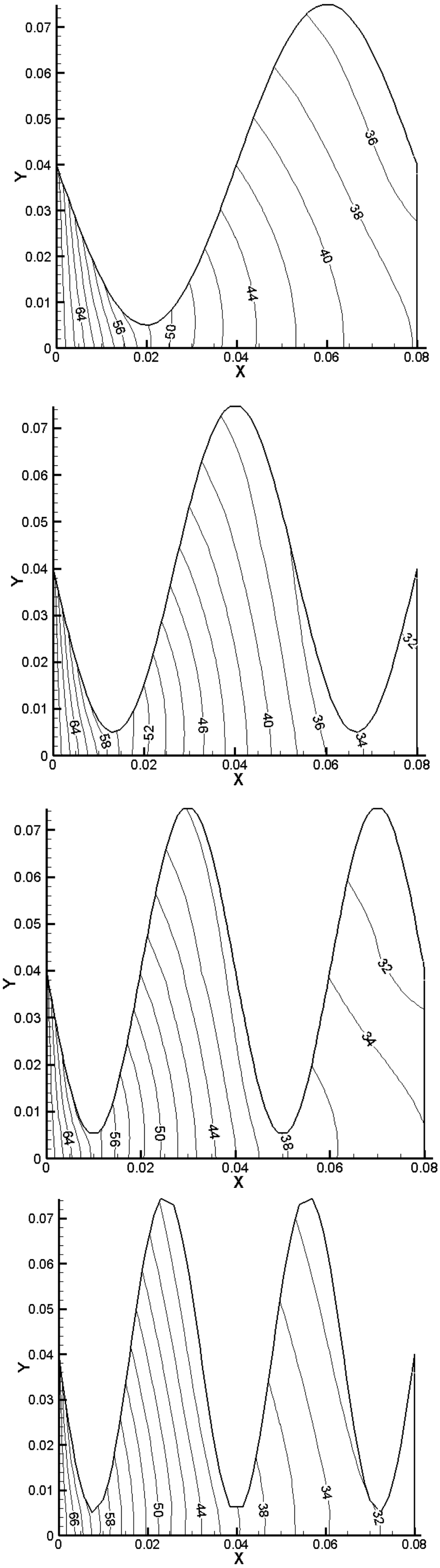


Fig. 7 Temperature distribution in different undulations with a_0= 0.035 cm and k =200 w/m.°k.

2. Conductivity k=20 w/m.°k





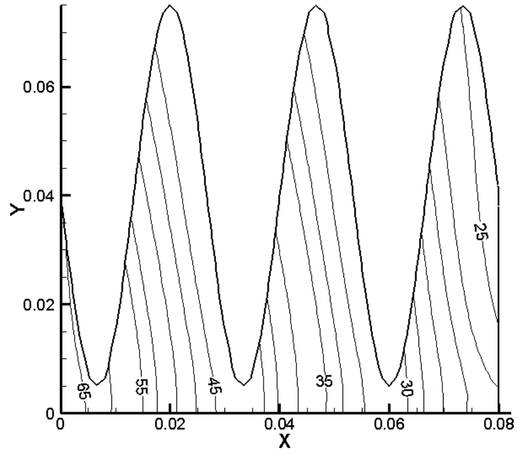


Fig. 8 Temperature distribution in different undulations with a_0= 0.035 cm and k =200 w/m.°k.

In order to better understand the compromise between the requirement of extending the heat transfer surface as much as possible and that of making the longitudinal thermal conduction easier. The heat flux on the longitudinal section of the fins with an optimum performance on the third undulation in sinusoidal profiles as shown in Fig 9 and Fig 10.

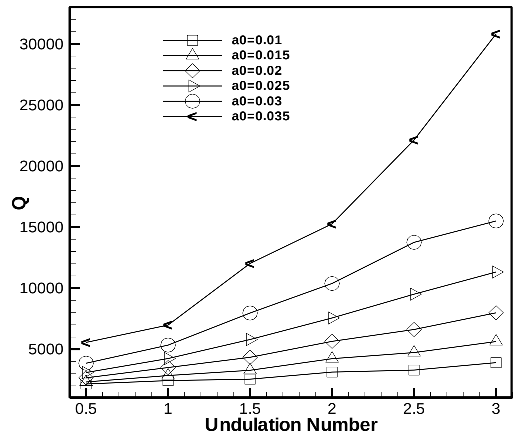


Fig. 9 The heat flux in different sinusoidal profiles with k =200 w/m.°k and h =100 w/m².°k

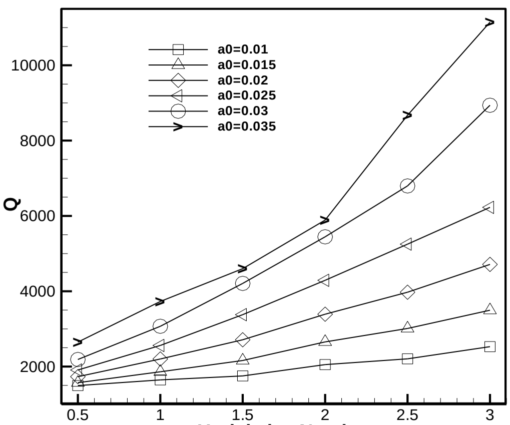


Fig. 10 The heat flux in different sinusoidal profiles with k =20 w/m.°k and h =100 w/m².°k

In Fig 9 and Fig 10 the highest values obtained for the compared heat dissipaters are shown vs the order of the sinusoidal profile both materials in different amplitudes. In both cases an increasing trend is evident.

Thermal cooling fins are normally made from high conductivity materials to increase their effectiveness, but in high temperature conditions, if cooling fins are required, a highly conductive material might not be usable due to low melting temperature point. The alternative is to use a material that has a high melting point and, in general, these materials have lower conductivity parameters.

The unit conductivity material presents a noticeable difference between the temperature at the base of the fin and the temperature at the tip of the fin.

Conclusion

The numerical method seems able to solve the problem of optimizing the longitudinal profile of fin, in order to improve its performances compared with those of a rectangular longitudinal section fin.

The optimization examples shown in the article demonstrate that it is possible to noticeably increase the compared heat dissipaters of fin by introducing undulations in its profile.

A more correct solution for the problem of optimizing heat removers will be obtained with our numerical method proposed under different conditions from the deformation of the geometries and by varying the conductivity parameters. So, it will be interesting to take the changes in the type of materials that which represents the thermal conductivity by the variation of the profile into account. In the case of the third order of sinusoidal profile, for example, a very narrow channel is created in the base of the fin.

And for the efficiency of the method of the discretization error can be reduced by way of the mesh grid is refined and the order of convergence is defined by the mesh refinement that these errors may improve.

In the numerical method optimization examples presented we did not consider undulation number of fin profiles greater than the third, that it is not yet too difficult to build for the producer.

Finally, it must be noticed that in the numerical solution for the optimum performance of fin profile examples presented a constant temperature has been assigned to the base of the fin for each value of y coordinate. In practical applications the base temperature of the fin cannot always assumed to be constant. In many problems, in fact, the entity assigned is the heat flux, which is to be removed from a wall surface with the help of fins. In such a condition, if the fins are thin and largely spaced, noticeable temperature variations occur on the surface to be cooled and, in particular, at the fin base. Neglecting this variation can result in errors of more than 20% in calculating the heat flux removed by the fin (E. M. Sparrow and L. Lee 1975, N. V. Suryanarayana 1977). In order to obtain a more correct solution for the optimizing the profile of fins under the above quoted conditions, it is then convenient to utilize our numerical method proposed which reproduced also portion of the wall with heat assigned to the side opposite to the fins.

References

A. Bar-Cohen and A. D. Kraus (1990), Advances in Thermal Modeling of Electronic Components and System, ASME Press Series. Vol. 2, pp. 41-107, New York.

C. W. Leung, S. D. Probert (1989), Steady-state heat transfer from vertical fins protruding upwards from horizontal bases, Int. J. Energy, 62, 94-101.

A. D. Sinder and A. D. Kraus (1987), the quest for the optimum longitudinal fin profile, Heat Transfer Engng 8(2), 19-25.

Y. Tsukaoto and Y. Seguchi, Shape optimization problem for minimum volume fin, heat transfer _ Japon Res 13, 1-19.

D.R. Harper, W.B. Brown (1922), Mathematical Equations for Heat Conduction in the Fins of Air-Cooled Engines, National Advisory Committee for Aeronautics, Report no. 158

E. Schmidt, Die Warmeubertrgung durch Rippen, Zeit (1926). V.D.I. 70 885–889, see pp. 947–951

R. Focke, Die Nadel als Kuhelemente, Forsch. Geb. Ingenieurw (1942). 13,34–42.

Avrami Melvin, J.B. Little, Diffusion of heat through a rectangular bar and the cooling and insulating effect of fins, I. The steady state, J. Appl. Phys. 225– 264.

K.A. Gardner (1942), Heat exchanger tube sheet temperature, Refiner Nat. Gasoline Manuf. 71–77.

W.H. Carrier, S.W. Anderson (1944), The resistance to heat flow through finned tubing, Heat. Piping air cond. 10 , 304–320.

Duffin, R. J. (1959), A variationnal problem relating to cooling fins. J. Math. Mech. 8, 47-56.

Maday, C.J.(1974), The minimum weight one-dimensional straight fin. ASME J. Eng. Ind. 96, 161-165.

P. Razelos, K. Imre (1980), The optimum dimensions of circular fins with variable thermal parameters, Trans. ASME, J. Heat Transfer 102 , 420–425.

Tsukamoto, Y., Seguchi, Y. (1984), Shape optimization problem for minimum volume fin. Heat Transfer Japanese Research 13 (1). 1-19.

H.C. Unal (1985), Determination of the temperature distribution in an extended surface with a non-uniform heat transfer coefficient, Int. J. Heat Mass Transfer 28 (12) 2270– 2284.

A.K. Sen, S. Trinh (1986), An exact solution for the rate of heat transfer from rectangular fin governed by a power law-type temperature dependence, Trans. ASME, J. Heat Transfer 108 , 457–459.

Snider, A.D., Kraus, A.D. (1987), The quest for the optimum longitudinal fin profile. Heat Trans. Engrg. 8 (2), 19-25.

R. Karaback (1992), The effect of fin parameter on the radiation and free convection from a finned horizontal cylindrical heater, Energy Convers. Mgmt. 33 (11) 997–1005.

A. Campo, L. Harrison (1994), Prediction of safe tip temperature in uniform annular fins for the design of thermal exchange equipment via sympolic mathematics, Int. Commun. Heat Mass Transfer 21 (4) 531–538.

K. Laor, H. Kalman (1996), Performance and optimum dimensions of different cooling fins with a temperature dependent heat transfer coefficient, Int. J. Heat Mass Transfer 39 (9) 1993-2003.

S.M. Zubair, A.Z. Al-garni, J.S. Nizami (1996), The optimal dimensions of circular fins with variable profile and temperature-dependent thermal conductivity, Int. J. Heat Mass Transfer 39 (16) 3431–3439.

A. Campo, R.E. Stuffle (1996), Symbolic mathematics for calculation of thermal efficiencies and tip temperatures in annular fins of uniform thickness, Int. J. Heat Mass Transfer 40 (2) 490–492.

Giampierto Fabbri (1997)., A genetic algorithm for fin profile optimization, Int. J. Heat Mass Transfer. Vol. 40, No. 9, pp. 2165.

Lien-Tsaiyu, Cha'o-Kuang Chen (1999), Application of the taylor transformation to the transient temperature response of an annular fin, Heat Transfer Eng. 20 (1) 78–87.

Esmail M. A. Mokeimer (2002), Performance of annular fins with different profiles subject to variable heat transfer coefficient. Int. J. Heat Mass Transfer. 45,3631-3642.

Florin Babaru, Srinivas Rachakonda (1999), Optimal shape profiles for cooling fins of high and low conductivity, Int. J. Heat and Fluid Flow 20 ,634-641.

R. Karvenin and T. Karvenin (2010) , Optimum geometry of fixed volume plate fin for maximizing heat transfer. Int. J. Heat Mass Transfer. 53 ,5380-5358.

H. S. Carslaw and J. C. Jaeger (1947), Conduction of Heat in Solids, Chap. 5, Section 63. Clarendon Press, Oxford.

G.K. Despotis, and S. Tsangaris (1995), Fractional step method for the solution of incompressible Navier-Stockes equations on unstructured triangular meshes, Int. J. Num. Meth. Fluids, vol. 20, pp. 1273–1288.

G. K. Despotis, and S. Tsangaris (1996), A finite volume method for the Solution of the extruDate swell problem on unstructured triangular meshes, Int. J. Num. Meth. Heat Fluid Flow, vol. 6, pp. 65–83.

S. Boivin, F. Cayré, and J. M. Hérard (2000), A Finite volume method to solve the Navier-Stokes equations for incompressible flows on unstructured meshes Int. J. Therm. Sci., vol.39 , pp. 806–825.

N. Piskounov (1987), Calcul intégral et différentielle tome 2, union Soviétique 11ime Edition.

E. M. Sparrow and L. Lee (1975), Effects of fin base temperature depression in multifin array, J. Heat Transfer 97(8), 463-465.

N. V. Suryanarayana (1977), Two-dimensional effects on heat transfer rates from an array of straight fins, J. Heat transfer 99(2), 129-132.