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Research Article

Analytic Solutions of a Stochastic Banking Model with Left Truncated Inter Withdrawal Times and Upper Truncated Amount of Withdrawals

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Abstract

Stochastic Banking models (S.B.Ms) occupy an important place in modern research, dealing with cash flow analysis of a Banking System. Knowledge about the reserve level of a Banking system, play a vital role in many Fiscal policies of any economy. To have prospective and fruitful economic plans, one must have a prior knowledge about the cash reserve level available with the nation, without which the plans will be vague and ineffective. Hence in 1983 (Sarma, 1983) proposed a stochastic banking model (S.B.M) with a critical reserve level ($C \geq 0$) and obtain many results relating to the reserve level X(t) available with the system at any given time $t \geq 0$, (vide Ref .2). Later in 1991, (Sarma and Pushpangali ,1991) Proposed a S.B.M.with general linear rate of inputs and obtained explicit expressions of M/G/1/FIFO/K and G/M/1/FIFO/K S.B.Ms. Further in 1995,(Sarma and Sarma, 1995) obtained results of S.B.Ms where withdrawals or inter – withdrawals are assume to follow an Erlangian distribution. The application of this distribution to S.B.M.has more practical relevance because the service of a customer in a Bank consists of different phases like issuing of tokens, passing of the amount, making suitable entries and so on.

Thus more and more practically relevant assumptions were brought in to the model, so that the S.B.M. suggested in 1983 is more and more closer to the reality.

In this paper a practically valid and more essential assumption namely (1) Lower Truncation Of Inter – Withdrawal Times And (2) Upper Truncation Of Amounts Of Withdrawals, is incorporated into the Stochastic Banking Model in order to make the model more closer to reality and to increase the application potentiality of the model. In General a customer is not allowed to withdraw or take loan against the amount deposited by him in the form of fixed deposits, until a minimum pre - stipulated time is over. Further, he cannot withdraw the entire amount deposited by him as Loan. Only a certain percentage of amounts are sanctioned to the customer which is generally known as eligible amount for the loan and he is eligible to withdraw the amount as loan up to an maximum of that eligible amount An analytic solution of a $M_T/M^a/1/FIFO/\infty$ Stochastic Banking Model (S.B.M) is obtained, Where $\mathbf{M_T}$ represents a lower truncated Law governing the random variable of inter – withdrawal times and M^a represents a upper truncated Law governing the random variable of amount of withdrawals.

Keywords: *Lower Truncated Variable, Upper Truncated Variable, Reserve level of a Bank, Stochastic Banking Models, Critical reserve level.*

1. Introduction

Application of Stochastic storage models in banking system was first studied in 1983 by (Sarma.K.L.A.P.,1983). He introduced a random variable X(t) which represents the reserve level available with the system at time t. X(t) is assumed to be a stochastic variable because it is mainly depending upon two other independent random variables namely

(1) The Inter – Withdrawal times denoted by 'u' and (2) The amount of withdrawals denoted by 'v'. Let h(u) represents the p.d.f. of the random variable 'u' and g(v) represents the p.d.f.of the random variable 'v' . Knowledge about the reserve level available with the system at any given time play a vital role in fiscal policies. Stochastic banking models attract more and more researches to obtain solutions to many problems relating to banking system. Thus Sarma proposed a S.B.M. with the following assumptions.

1. Input fed into the system is assumed to increase linearly at a unit rate.
2. Inter – Withdrawal times (u) are assumed to follow an independent r.v. which is governed by a known probability law, whose p.d.f. is denoted by h(u).
3. Random withdrawal depletes the reserve level x (t) instantaneously.
4. Amount of withdrawals (v) are also assumed to follow an independent random variable which is governed by a known probability law whose p.d.f. is denoted by g (v).

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With the above assumptions Sarma obtained many results relating to the reserve level X(t) vide references (Sarma, 1983) and (Sarma and Venugopal,1981). A typical realization of the model with the above assumptions is given in Fig (1.1). Further (Sarma and Pushpanjali, 1991) obtained results for S.B.Ms by assuming a generalized linear rate of cash flow into the system. When the inter – withdrawal times follow an Erlangian distribution, the results relating to a banking model were obtained by (Sarma and Sarma, 1995).The concept of truncation applied in reliability and life testing problems can be used in S.B.Ms to know the reserve level X (t). In this paper a S.B.M. is considered, where inter – withdrawal times are assumed to follow a lower – truncated random variable truncated at time T because in order to draw a loan against deposits made by the customer, he has to wait a minimum pre – stipulated time T and amount of withdrawals are assumed to follow a upper – truncated random variable truncated at a point 'a' because in order to draw a loan against deposits made by the customer, he is eligible to draw up to a certain percentage of the amount deposited by him. The amount '**a**' is usually known as loan eligibility amount. A customer can withdraw any amount less than or equal to 'a'. In this paper an analytic solution is obtained for $M_T/M^a/1/FIFO/\infty$ model.

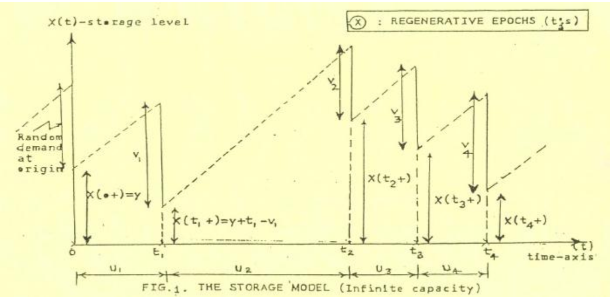

FIG.1. THE STORAGE MODEL (Infinite capacity)

2. Formulation of the model

The S.B.M. considered in this paper is based on the four assumptions explained from A_1 to A_4 in section 1. To bring the model more closer to reality two more practically valid assumptions is incorporated into the model namely.

5. Inter – withdrawal times are assumed to follow a lower – truncated random variable because in order to draw a loan against deposits made by the customer, he has to wait a minimum pre – stipulated time T.

6. Amount of withdrawals times are assumed to follow a upper – truncated random variable because in order to draw a loan against deposits made by the customer, he is eligible to draw up to a certain percentage of the amount deposited by him.

Under the assumption (5), the p.d.f. of the random variable (u) and under the assumption (6) the p.d.f. of the random variable (v) for $M_T/M^a/1/FIFO/\infty$ is assumed as :

$$h_T(u) = \frac{\lambda e^{-\lambda u}}{e^{-\lambda T}} \quad ,T \le u \le \infty \ , \ \lambda > 0 \ , T > 0 \tag{2.1}$$

$$g_a(v) = \frac{\mu e^{-\mu v}}{1 - e^{-\mu a}} \ , \ \mu > 0, \ 0 \le v \le a, \ a > 0 \tag{2.2}$$

In order to obtain solution for the reserve level X(t), we introduce two conditional p.d. functions namely M(x,y,t) and V(x,y,t) which are defined as follows.

$$M(x,y,t) \overset{def}{=} \lim_{\Delta \longrightarrow 0} P[0 < x < X(t) \le x + \Delta, X(u) > 0 \forall u \in [0,t] / X(0_-) > X(0) = X(0+) = y] / \Delta \tag{2.3}$$

and

$$V(x,y,t) \overset{def}{=} \lim_{\Delta \longrightarrow 0} P[0 < x < X(t) \le x + \Delta, X(u) > 0 \forall u \in [0,t], X(t+) < X(t) / X(0-) > X(0) = X(0+) = y] / \Delta \tag{2.4}$$

Above two conditional p.d.fs govern the reserve level at time t, the reserve level being never zero in [0,t] conditioned upon the initial reserve level y is known after a withdrawal is made at 0-. The basic difference between the M(x,y,t) and V(x,y,t) is that in the latter one an extra intermediary condition that time t is also an epoch of withdrawal is assumed i.e. at time t there should be withdrawal. In order to obtain solutions for M(x,y,t) and V(x,y,t) by using embedded regenerative process technique and using addition and multiplication theorems of probability we have the following two integral equations viz ;

$$M(x,y,t) = U(x-y)\delta(x-y-t)\int_t^\infty h_T(u)du + \int_0^t h_T(u)du \int_0^{y+u} M(x, y+u-v, t-u) g_a(v)dv \tag{2.5}$$

and

$$V(x,y,t) = U(x-y)\delta(x-y-t)h(t) + \int_0^t h_T(u)du \int_0^{y+u} M(x, y+u-v, t-u) g_a(v)dv \tag{2.6}$$

Where U(.) represents Heavy side unit step function and δ(.) is Dirac – delta function.

The reserve level x > 0 at t is the result of the two mutually exclusive and completely exhaustive cases namely (i) that there is no withdrawal during [0,t] and (ii) there is atleast one withdrawal in [0,t]. By applying addition and multiplication theorems of probability and the concept of regenerative process we obtain (2.5) and (2.6). Since the epoch of t is a point of withdrawal, h(t) in the first term on the R.H.S. of (2.6) appeared instead of an integral in the corresponding place of (2.5). Integral equations given in (2.5) and (2.6) are useful to provide solutions for M(x,y,t) and V(x,y,t) introduced in (2.3) and (2.4) respectively. These integral equations are formed using the regenerative process $\{X(t_i +)\}$imbedded in the general process {X(t)}where t_i 's are the epochs of withdrawals. The process $X(t_i +)$ is a regenerative in the sense that conditioned upon the knowledge that $X(t_i +) = y$ is known after a withdrawal is made and the process starts afresh from every such t_i.

3. Results and Discussions

Now we proceed to obtain solutions for M(x,y,t) and V(x,y,t) by using the integral equations introduced in (2.5) and (2.6) for $M_T/M^a/1/FIFO/\infty$ model. Defining the double Laplace transforms F*(s,p) of F(y,t) w.r.t.y and t as

$$F^*(s,p) = \int_0^\infty e^{-sy} dy \int_0^\infty e^{-pt} F(y,t) dt \ , \tag{3.1}$$

Re s, Re p>0

First we proceed to obtain analytic solution for M(x,y,t) in the following two theorems.

Theorem 3.1: The double Laplace transforms of M*(x,s,p) of M(x,y,t) for M_T /M^a/1/FIFO/∞ model is given by

M*(x,s,p) =

$$\left\{\frac{(s+\mu)(1-e^{-\mu a})}{[e^{-\lambda T}(1-e^{-\mu a})(\lambda+p-s)(s+\mu)-\lambda\mu(1-e^{-(s+\mu)a})-\lambda(s+\mu)e^{-sa}(1-e^{-\mu a})]}\right.$$

$$\left.[e^{-sx}-e^{-(\lambda+p)x}+e^{-s(x-T)}\frac{(1-e^{-pT})}{p}e^{-\lambda T}(\lambda+p-s)-\lambda M^*(x,\lambda+p,p)\left[\frac{\mu}{(\lambda+p+\mu)(1-e^{-\mu a})}+e^{-(\lambda+p)a}\right]]\right\} \quad (3.2)$$

Proof: Using M(x,y,t) given in (2.5) in the place of F(y,t) in equation (3.1),substituting equation (2.1) for $h_T(u)$ and equation (2.2) for $g_a(v)$ the p.d.f.s of inter withdrawal times and truncated amount of withdrawals,

M*(x,s,p)=A+B (3.3)

Where

$$A=\int_0^\infty e^{-sy}\{\int_0^\infty e^{-pt}\{U(x-y)\delta(x-y-\max(t,T))\int_{\max(t,T)}^\infty h_T(u)du\}dt\}dy \quad (3.4)$$

and

$$B=\int_0^\infty e^{-sy}\{\int_0^\infty e^{-pt}\{\int_0^t h_T(u)du\int_0^a g_a(v)M(x,y+u-v,t-u)dv\}dt\}dy \quad (3.5)$$

After some simplifications by considering two cases for t<T and t>T we have

$$A=\frac{e^{-sx}-e^{-(\lambda+p)x}}{e^{-\lambda T}(\lambda+p-s)}+\frac{e^{-s(x-T)}}{p}[1-e^{-pT}] \quad (3.6)$$

Similarly by substituting equations (2.1) and (2.2) in equation (3.5) and after some simplifications

We have

$$B=\frac{\lambda}{e^{-\lambda T}(1-e^{-\mu a})(\lambda+p-s)}\left[\frac{M^*(x,s,p)\left[\mu(1-e^{-(s+\mu)a})+(1-e^{-\mu a})(s+\mu)e^{-sa}\right]}{(s+\mu)}-\frac{M^*(x,\lambda+p,p)\left[\mu(1-e^{-(\lambda+p+\mu)a})+(\lambda+p+\mu)(1-e^{-\mu a})e^{-(\lambda+p)a}\right]}{(\lambda+p+\mu)}\right] \quad (3.7)$$

Adding equations (3.6) and (3.7) and after some simplications we obtain equation (3.2).

Theorem 3.2: The time dependent solution of M(x,y,t) is well determined in terms of M*(x,s,p) using equation (3.2).

Proof: We first observe the R.H.S of equation (3.2) which involves one unknown constant namely M*(x,λ+p,p) obtaining which M*(x,s,p) is completely determined. Further we notice that M*(x,s,p) is analytic in s in the right half plane i.e. s > 0 we then observe that the denominator in equation (3.2)

i.e.

$$[e^{-\lambda T}(1-e^{-\mu a})(\lambda+p-s)(s+\mu)-\lambda\mu(1-e^{-(s+\mu)a})-\lambda e^{-sa}(1-e^{-\mu a})(s+\mu)]$$

is a quadratic in s and has two roots of which one root is with positive real part. Let this real part be denoted by δ. Since M*(x,s,p) is analytic in s for Re.s > 0 the numerator of equation (3.2) also should vanish at this δ. Thus we obtain a non – homogenous linear equation involving M*(x,λ+p,p), solving which the unknown constant can be determined and hence M*(x,s,p) is completely known. Inverting M*(x,s,p) with respect to s and p successively M(x,y,t) is completely determined.

Now we proceed to obtain time dependent solution for V(x,y,t) in the following in the following two theorems.

Theorem 3.3: The double Laplace transforms of V*(x,s,p) of V(x,y,t) for M_T/M^a/1/FIFO/∞ model is given by

V*(x,s,p) =

$$\left\{\frac{\lambda(s+\mu)(1-e^{-\mu a})}{[e^{-\lambda T}(1-e^{-\mu a})(\lambda+p-s)(s+\mu)-\lambda\mu(1-e^{-(s+\mu)a})-\lambda e^{-sa}(1-e^{-\mu a})(s+\mu)]}\right.$$

$$\left.[e^{-sx}-e^{-(\lambda+p)x}-V^*(x,\lambda+p,p)\left[\frac{\mu(1-e^{-(\lambda+p+\mu)a})}{(\lambda+p+\mu)(1-e^{-\mu a})}+e^{-(\lambda+p)a}\right]]\right\} \quad (3.8)$$

Proof: Using V(x,y,t) given in equation (2.6) in the place of F(y,t) in equation (3.1) substituting equation (2.1) for $h_T(u)$ and equation (2.2) for $g_a(v)$ the p.d.f.s of inter withdrawal times and truncated amount of withdrawals, using $h(t)=\frac{\lambda e^{-\lambda t}}{e^{-\lambda T}}$ (3.9)

we have

V*(x,s,p)=C+D (3.10)

Where

$$C=\int_0^\infty e^{-sy}\{\int_0^\infty e^{-pt}\{U(x-y)\delta(x-y-t)\frac{\lambda e^{-\lambda t}}{e^{-\lambda T}}\}dt\}dy \quad (3.11)$$

and

$$D=\int_0^\infty e^{-sy}\{\int_0^\infty e^{-pt}\{\int_0^t \frac{\lambda e^{-\lambda u}}{e^{-\lambda T}}du\int_0^a \frac{\mu e^{-\mu v}}{1-e^{-\mu a}}V(x,y+u-v,t-u)dv\}dt\}dy \quad (3.12)$$

After some simplifications of equation (3.11) we have

$$C=\frac{\lambda[e^{-sx}-e^{-(\lambda+p)x}]}{(\lambda+p-s)} \quad (3.13)$$

Similarly after some simplifications of equation (3.12) We have

$$D=\frac{\lambda}{e^{-\lambda T}(1-e^{-\mu a})(\lambda+p-s)}\left[\frac{M^*(x,s,p)\left[\mu(1-e^{-(s+\mu)a})+(1-e^{-\mu a})(s+\mu)e^{-sa}\right]}{(s+\mu)}-\frac{M^*(x,\lambda+p,p)\left[\mu(1-e^{-(\lambda+p+\mu)a})+(\lambda+p+\mu)(1-e^{-\mu a})e^{-(\lambda+p)a}\right]}{(\lambda+p+\mu)}\right] \quad (3.14)$$

Adding equations (3.13) and (3.14) and after some simplifications we obtain equation (3.8).

Hence the proof.

Theorem 3.2: The time dependent solution of V(x,y,t) is well determined in terms of

V*(x,s,p) using equation (3.8).

Proof: We first observe the R.H.S of equation (3.8) which involves one unknown constant namely V*(x,λ+p,p) obtaining which V*(x,s,p) is completely determined. Further we notice that V*(x,s,p) is analytic in s in the right half plane i.e. s > 0 we then observe that the denominator in equation(3.8)

i.e.

$$[e^{-\lambda T}(1-e^{-\mu a})(\lambda+p-s)(s+\mu)-\lambda\mu(1-e^{-(s+\mu)a})-\lambda e^{-sa}(1-e^{-\mu a})(s+\mu)]$$

is a quadratic in s and has two roots of which one root is with positive real part. Let this real part be denoted by δ. Since V*(x,s,p) is analytic in s for Re.s > 0 the numerator of equation (3.8) also should vanish at this δ. Thus we obtain a non – homogenous linear equation involving V*(x,λ+p,p), solving which the unknown constant can be determined and hence V*(x,s,p) is completely known. Inverting V*(x,s,p) with respect to s and p successively V(x,y,t) is completely determined.
Hence the proof.

It is important to note that the basic difference between M(x,y,t) and V(x,y,t) is that in the latter one, t is the epoch of withdrawal and hence the difference came in A and C terms given in equations (3.6) and (3.13). Further an extra constant λ which is the average of the law governing inter-withdrawal times came as a multiplicative constant in V*(x,s,p) when compared with M*(x,s,p) which is natural because it is a point of withdrawal in V(x,y,t).

Thus in this paper we obtained analytic solutions of M(x,y,t) and V(x,y,t) for $M_T/M^a/1/FIFO/\infty$ model. In these results, if we substitute T = 0 and a = ∞ (i.e. if there is no lower truncation of inter-withdrawal times and upper truncation of amount of withdrawals) we obtain the results obtained by (Sarma, 1983).

Thus in this paper an attempt is made to bring the S.B.M. introduced by (Sarma, 1983) closer to the reality by introducing lower truncated inter- withdrawal times and upper – truncated amount of withdrawals obtained analytic solution of M(x,y,t) and V(x,y,t) for $M_T/M^a/1/FIFO/\infty$ model. Some more practically valid assumptions like

1. The capacity of the reserve level can be considered as finite i.e.k<∞.
2. Explicit results for M(x,y,t),V(x,y,t) introduced in this Paper ($M_T/M^a/1/FIFO/\infty$, M_T /M^a/1/FIFO/k models).

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