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Research Article

Spacecraft Tumbling Chaotically Under Gravity and Aerodynamic Torque

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Abstract

In this paper, the half width of the chaotic separatrix has been estimated by Chirikov's criterion. Through surface of section method, it has been observed that the aerodynamic torque parameter and the mass distribution parameter play an important role in changing the regular motion into chaotic one.

***Keywords**: Chaos, Poincare section, Solar System, Aerodynamic Torque*

1. Introduction

Astronomers have uncovered certain kinds of instabilities that occur throughout the solar system, in the motions of Saturn's moon Hyperion, in gaps in the asteroid belt between Mars and Jupiter and in the orbits of the system's planets themselves. Inarrea, M. and Lanchares, V. (2000) described the chaotic behavior of a dual – spin spacecraft with time – dependent moments of inertia in free motion. Ciraolo and Pettini (2002) discussed the quantitative description of Hamiltonian chaos, based on Riemannian geometry of Newtonian dynamics. Dvorak, Rudolf (2003) showed that the chaotic motion has been found in the motion of the planets and appears to be present on even a larger level in extra-solar planetary system. Winter, and Murray, D. (1997) reviewed analytical models for studying the dynamical behavior of objects near interior, mean motion resonances in the context of the planar, circular, restricted three-body problem. Bachelard and Chandre (2006) discussed a method to reduce or enhance chaos in Hamiltonian flows with two degrees of freedom. In this paper, it has been discussed that the aerodynamic torque plays a significant role in changing the motion of revolution into liberation or infinite period separatrix.

2. Equation of Motion

Let us consider a rigid satellite S moving around the earth E in a Circular orbit such that the orbital plane coincides with the equatorial plane of the earth (Fig 1). The body is assumed to be tri-axial body with principal moments of inertia A < B < C at its centre of mass. Here C is the moment of inertia about the spin axis which is perpendicular to the orbital plane. These principal axes are taken as our co-ordinate axes x, y, z; the z axis being perpendicular to the orbital plane. Let r be the instantaneous radius vector of the centre of mass of the satellite, θ the angle that the long axis of the satellite makes within a fixed line EF lying in the orbital plane and δ/2 the angle between the radius r and the long axis.

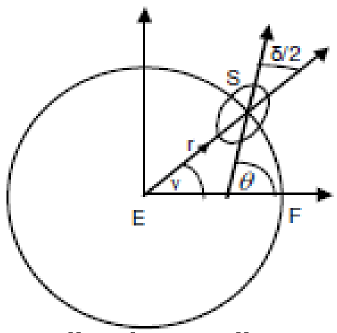

Fig. 1 Satellite planar oscillation in circular orbit

The equation of motion for the non-linear motion of a satellite under the influence of aerodynamic torque in a circular orbit is obtained as

$$\frac{d^2\theta}{dt^2} = \frac{-\mu}{2r^3}\left[n^2 \sin\delta - \varepsilon\left(\alpha^2 + b\alpha + d\right)\sin\alpha\right] \quad (1)$$

3. Estimation of Resonance Width

Taking $n^2 = w_0{}^2 = \frac{3(B-A)}{C}$, $\delta = 2(\theta - \alpha)$ and using Fourier like Poisson-Series, Equation (1) represents the equation of motion of disturbed pendulum given by

$$\frac{d^2 x_p}{dt^2} + f'(x_p) = m_p g'(x_p, t) \quad (2)$$

where $v_p = \theta - pt$, $x_p = 2v_p$, $k_{1p}{}^2 = w_0{}^2 H(p,e)$,
$m_p = -\varepsilon$, $g'(x_p,t) = \sin t^3 + b\sin t^2 + d\sin t$

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$f'(x_p) = k_{1p}{}^2 \sin x_p$

For the unperturbed part of Equation (2)

$$\left(\frac{dx_p}{dt}\right)^2 = c_{1p} + 2k_{1p}{}^2 \cos x_p$$

where c_{1p} is constant of integration.

The motion to be real if $c_{1p} + 2k_{1p}{}^2 \geq 0$. There are three Categories of motion depending upon $c_{1p} > 2k_{1p}{}^2$, $c_{1p} < 2k_{1p}{}^2$ and $c_{1p} = 2k_{1p}{}^2$

3.1 CATEGORY-I: $c_{1p} > 2k_{1p}{}^2$

In case of perturbed pendulum by making use of the theory of variation of parameters Brown and Shook (1964)[2], since $m_p, k_{1p}{}^2$ are small quantities, so rejecting second or higher order terms $\frac{dc_{1p}}{dt} \cong 0$ so, c_{1p} is a constant up to second order of approximation and $\frac{d^2 l_p}{dt^2} \cong -\frac{2m_p c_{1p} d}{n_p} \sin l_p$. If $l_p = x_p$, then

$$\left(\frac{dx_p}{dt}\right)^2 = c_{2p} + 2k_{2p}{}^2 \cos x_p$$

where c_{2p} is constant of integration and

$$k_{2p}{}^2 = \frac{2m_p c_{1p} d\,\varepsilon}{n_p}$$

Again it gives three types of motion, Type I is that in which $\frac{dx_p}{dt}$ is never zero, Type II is that in which $\frac{dx_p}{dt}$ =0, at 0 or π,

<u>For Type-I</u>, the solution is

$$x_p = N_p t + \varepsilon_{2p} + \frac{k_{2p}{}^2}{N_p{}^2} \sin(N_p t + \varepsilon_{2p}) + ...$$

where $\frac{1}{N_p} = \frac{1}{2\pi} \int_0^{2\pi} \frac{dx_p}{(c_{2p} + 2k_{2p}{}^2 \cos x_p)^{1/2}}$, which is the case of revolution.

<u>For the Type II,</u> the solution is

$x_p = \lambda_p \sin(p't + \lambda_0)$

where $p' = \omega_0 \sqrt{\frac{2m_p d\,\varepsilon H(p,e)}{n_p{}^3}}$, λ_p and λ_0 being arbitrary constants. This is the case of libration.

<u>TYPE III,</u> occurs when

$$c_{2p} = 2k_{2p}{}^2 = 2\omega_0{}^2 \frac{2m_p d\,\varepsilon H(p,e)}{n_p{}^3}$$

The solution is $x_p + \pi = 4\tan^{-1}\exp(k_{2p}t + \alpha_0)$, where α_0 is an arbitrary constant and the other having a particular value.

When $t \to \pm\infty$, $x_p \to \pm\pi$, at both places, $\left(\frac{dx_p}{dt}\right) = 0$ and all higher derivatives of x_p approach to zero. This is the case of infinite period separatrix which is asymptotic forward and backward in time to the unstable equilibrium.

3.2 CATEGORY-II: $c_{1p} < 2k_{1p}{}^2$

In case of perturbed equation again, using the theory of variation of parameters, we get

$$\frac{dc_{1p}}{dt} \cong \frac{m_p}{k_{1p}} \cos l_p \left(\sin t^3 + b \sin t^2 + d \sin t\right),$$

$$\frac{dl_p}{dt} \cong k_{1p} - \frac{m_p}{k_{1p} c_{1p}} \sin l_p \left(\sin t^3 + b \sin t^2 + d \sin t\right),$$

When n_1 is even,

$$\frac{d^2 l_p}{dt^2} \cong \frac{-m_p d}{k_{1p} c_{1p}} \sin l_p, \quad \frac{d^2 l_p}{dt^2} + k_{3p}{}^2 \sin l_p = 0, \quad k_{3p}{}^2 = \frac{m_p d}{k_{1p} c_{1p}} > 0$$

which is again the equation of pendulum. As in previous case this equation gives us revolution, libration and infinite period separatrix motion. On the other hand, if n_1 is odd.

$$\frac{d^2 l_p}{dt^2} = -\frac{m_p d}{k_{1p} c_{1p}} \sin l_p = k_{3p}{}^2 \sin l_p$$

When l_p is small, the solution of above equation is given by $l_p = e^{k_{3p}t} + e^{-k_{3p}t}$

3.2 CATEGORY-III: $c_{1p} = 2k_{1p}{}^2$.

The unperturbed solution is

$x_p + \pi = 4\tan^{-1} e^{k_{1p}t} + \alpha_0$,

where α_0 is an arbitrary constant and the other having a specific value. This is the case of infinite period separatrix which is asymptotic forward and backward in time to the unstable equilibrium. Near the infinite period separatrix broadened by the high frequency term into narrow chaotic band (Chirikov (1979)[4]), for small n, the half width of the chaotic separatrix is given by

$$\omega_p = \frac{I_p - I_p^s}{I_p^s} = 4\pi\varepsilon_1 \lambda^3 e^{-\pi\lambda/2},$$

where ε_1 is the ratio of the coefficient of the nearest perturbing high–frequency term to the coefficient of the perturbed term, and λ= $\frac{\Omega}{\omega}$ is the ratio of the frequency difference between the resonant term and the nearest non resonant term (Ω) to the frequency of small–amplitude liberations (ω).

4. The Spin Orbit Phase Space

It is known that most of the Hamiltonian systems give regular and irregular trajectories. Henon and Heiles (1964) have shown that the phase space is divided into two regions in which trajectories behave chaotically or quasi-periodically. In this paper, the spin orbit problem is 2π-periodic in dimensionless time and surface of section have been drawn by looking at the trajectories stroboscopically with period 2π. The section has been drawn with *dq/dv* versus ν at every periapse passage. Since the orientation denoted by q is equivalent to the orientation denoted by π + q, therefore, the interval restricted from θ to π. It may be observed that the chaotic separatrix surrounds each of the resonance states and each of these chaotic zones is separated from others by non-resonant quasi-periodic rotation trajectories.

All the possibilities are shown from Figure 2 to Figure 5 for various values of ε, n, b and d. Figure 2 is plotted for different values of n when ε, b and d are fixed. Figure 3 is drawn for different values of ε when n, b and d are fixed. Figure 4 is drawn for different values of b for fixed values of ε, n and d. Figure 5 is drawn for different values of d for fixed values of ε, n and b. It has been observed that as ε and n changes, the regular curves disintegrate and this disintegration increases with the increase in aerodynamic torque parameter ε and n.

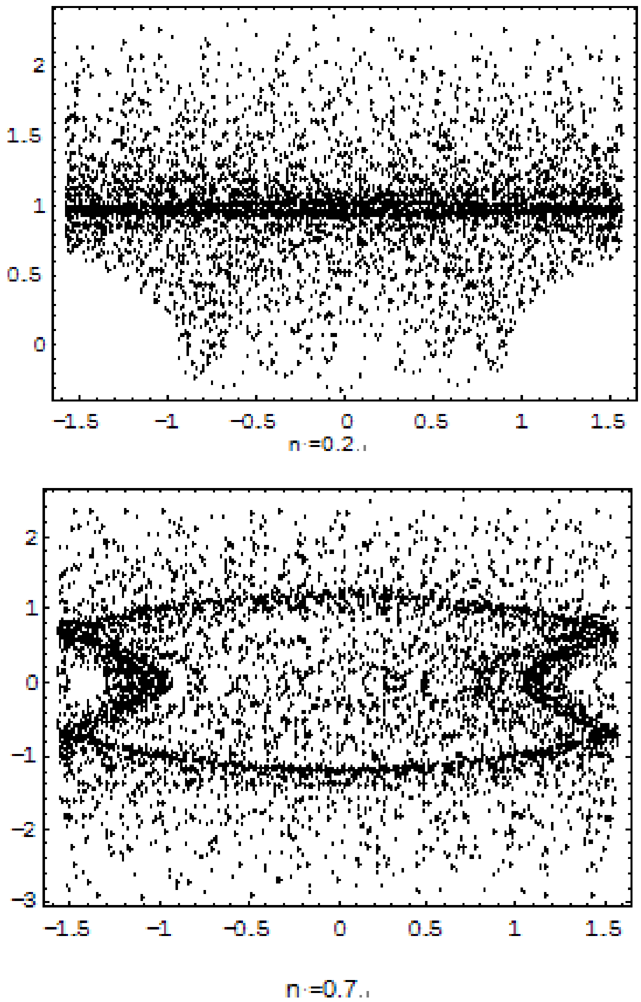


Fig. 2 ε = 0.000001, b=0.9, d=0.4, n=0.2, 0.7

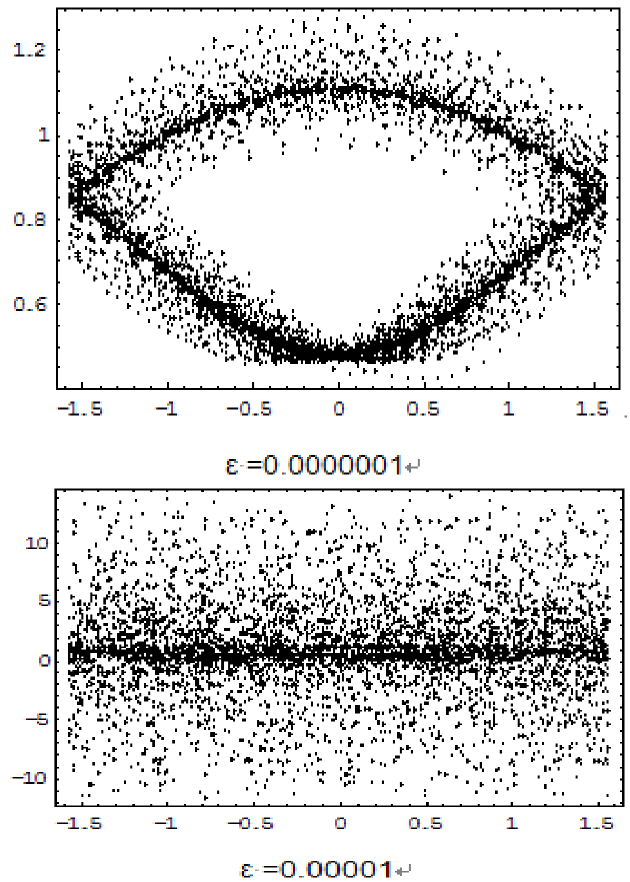


Fig. 3 ε = 0.0000001, ε = 0.00001, b = 0.5, d = 0.7, n = 0.5

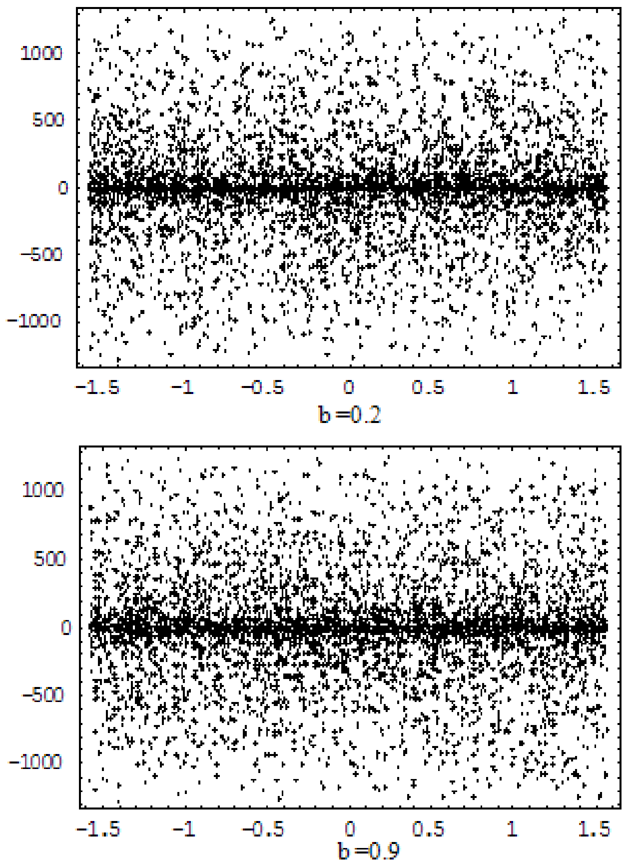
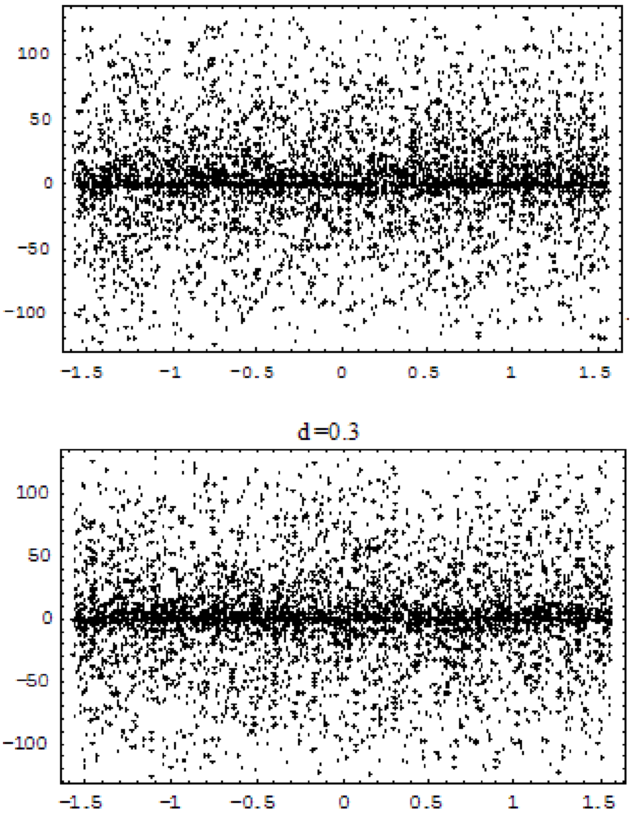


Fig. 4 ε = 0.001, b = 0.2, b=0.9, d = 0.5, n = 0.3



d=0.3

d=0.8

Fig. 5 ε = 0.0001, b = 0.3, d=0.3, d = 0.8, n = 0.6

5. Conclusion

It has been observed that the aerodynamic torque plays a very significant role in changing the motion of revolution into libration or infinite period separatrix. The half width of the chaotic seperatrices estimated by the Chrikov's criterion is not affected by the aerodynamic torque. In the spin–orbit phase space the regular curves start disintegrating due to aerodynamic torque and the irregular mass distribution of the satellite and this disintegration increases with the increase in ε and n.

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