International Journal of Current Engineering and Technology
E-ISSN 2277 – 4106, P-ISSN 2347 – 5161
Available at http://inpressco.com/category/ijcet

Research Article

Fixed Point Theorem for Weakly Compatible Maps in *G*-Metric Space using the Property E.A

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Received 14 February 2020, Accepted 11 April 2020, Available online 18 April 2020, **Vol.10, No.2 (March/April - 2020)**

Abstract

The concept of a $G-$ metric space was introduced by Mustafa and Sims (2006), wherein the authors discussed the topological properties of this space and proved the analog of the banach contraction principle in the context of $G-$ metric spaces. In this Paper, we use the notion of the property E.A, to prove fixed point theorems for weakly compatible maps in $G-$metric space.

Keywords: *G-Metric Space, Weak Compatibility, Property E.A, Fixed Point, Common Fixed Point Theorem.*

1. Introduction

Inspired by the fact that metric fixed-point theory has a wide application in almost all fields of quantitative sciences, many authors have directed their attention to generalize the notion of a metric space. In this respect,serveral generalized metric spaces have come through by many authors, in the last decade. Among all the generalized metric spaces, the notion of $G-$ metric space has attracted considerable attention from fixed point theorists. The concept of a $G-$ metric space was introduced by Mustafa and Sims (2006), wherein the authors discussed the topological properties of this space and proved the analog of the banach contraction principle in the context of $G-$ metric spaces. Following these results, many authors have studied and developed several common fixed-point theorems in this framework. M.Aamri and D.El Moutawakil (2002), introduced the property E.A, which is a true generalization of non$-$Compatible maps in metric spaces. Under this notion many common fixed-point theorems were studied in the literature.

In this Paper, we use the notion of the property E.A, to prove fixed point theorems for weakly compatible maps in G $-$metric space. Here, we give preliminaries and basic definitions which are helpful in the sequel. First, we introduce the concepts of a G $-$metric and G $-$metric space.

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DOI: https://doi.org/10.14741/ijcet/v.10.2.13

2. Preliminaries

Definition 1: Let$X$ be a nonempty set, and let $G: X \times X \times X \to R^+$ be a function satisfying the following axioms:

(G_1) $G(x, y, z) = 0$ if $x = y = z$,

(G_2) $0 < G(x, x, y)$, for all $x, y \in X$ with $x \neq y$,

(G_3) $G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$ with $z \neq y$,

(G_4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$(Symmetry in all three variables)

(G_5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$for all$x, y, z, a \in X$,

(Rectangle inequality)

Then the function G is called a generalized metric or more specifically a $G-$metric on X and the pair (X, G) is called a $G-$metric space.

Definition 2: Let (X, G)be a $G-$metric space, and let $\{x_n\}$ be a sequence of points in X, a point $x \in X$ is said to be the limit of the sequence $\{x_n\}$

if $\lim_{m,n\to\infty} G(x, x_n, x_m) = 0$ and one says that sequence $\{x_n\}$ is $G-$convergent to x.So, that if $x_n \to x$ or $\lim_{n\to\infty} x_n \to x$ as $n \to \infty$ in a $G-$metric space (X, G) then for each $\in > 0$, there exists $k \in N$ such that $G(x, x_n, x_m) <\in$ for all $m, n \geq k$.

Proposition 1: Let (X, G) be a $G-$metric space. Then the following are equivalent:

(1) $\{x_n\}$ is G −convergent to x,
(2) $G(x_n, x_n, x) \to 0$ as $n \to \infty$,
(3) $G(x_n, x, x) \to 0$ as $n \to \infty$,
(4) $G(x_m, x_n, x) \to 0$ as $m, n \to \infty$.

Definition 3: Let (X, G) be a G −metric space. A sequence $\{x_n\}$ is called G −cauchy if, for each $\in > 0$ there exists $k \in N$ such that $G(x_n, x_m, x_l) < \in$ for all $n, m, l \geq k$ that is if $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.

Proposition 2: If (X, G) be a G −metric space. Then the following are equivalent:

(1) The sequence $\{x_n\}$ is G −cauchy,
(2) For each $\in > 0$,there exists $k \in N$such that $G(x_n, x_m, x_m) < \in$ for all $n, m \geq k$.

Proposition 3: Let (X, G) be a G −metric space, then the function$G(x, y, z)$ is jointly continuous in all three of its variables.

Definition 4: A G −metric space (X, G) is called a symmetric G −metric space if $G(x, y, y) = G(y, x, x)$ for all $x, y \in X$.

Definition 5: A G −metric space (X, G) is said to be G −complete if every G −cauchy sequence in (X, G) is G −convergent in X.

Proposition 4: Let (X, G) be a G −metric space, then for any $x, y, z, a \in X$it follows that

(1) If $G(x, y, z) = 0$, then $x = y = z$,
(2) $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$,
(3) $G(x, y, y) \leq 2G(y, x, x)$,
(4) $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$,
(5) $G(x, y, z) \leq \frac{2}{3}\big(G(x, y, a) + G(x, a, z) + G(a, y, z)\big)$,
(6) $G(x, y, z) \leq \big(G(x, a, a) + G(y, a, a) + G(z, a, a)\big)$,

An interesting observation is that any G −metric space (X, G) induces a metric d_G on X given by $d_G(x, y) = G(x, y, y) + G(y, x, x)$ for all $x, y \in X$.

Moreover, (X, G) is G −complete if and only if (X, d_G) is complete.

It was observed that in the symmetric case ((X, G) is symmetric), many fixed point theorems on G −metric spaces are particular cases of the existing fixed point theorems in metric spaces. This allows us to readily transport many results from the metric spaces into the G −metric spaces.

On the other hand, by reasoning on the properties of the mappings, the practice of coining weaker forms of commutativity to ensure the existence of a common fixed point for self-mappings on metric spaces is still on. To read more in this direction, we refer to Di Bari and Vetro C. (2008) and the references therein. Here, for our further use, we recall only the two fundamental notions of weakly compatible mappings and property E.A, Gopal D. *et al.* (2011). Jungck G. (1976) introduced the notion of weakly compatible mappings as follows.

Definition 6: Let S and T be two self-mappings of a metric space (X, d). Then the pair (S, T) is said to be weakly compatible or coincidently commuting if they commute at their coincidence points that is if $Su = Tu$ for some $u \in X$ then $STu = TSu$.

Amari and El Moutawakil (2002) introduced a new concept of the property E.A, in metric spaces to generalize the concept of non-compatible mappings. Then, they proved some common fixed point theorems.

Definition 7: Let S and T be two self-mappings of a metric space (X, d). Then the pair (S, T) is said to satisfy the property E.A if there exists a sequence $\{x_n\}$ is in X such that $\lim_{n \to \infty} S x_n = \lim_{n \to \infty} T x_n = t$, where $t \in X$.

3. The Main Results

Theorem 3.1: Let(X, G)be a $G-$ metric space and $A, B, S, T: X \to X$ be four self-mappings such that:
$A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$;

One of the pairs (A, S) and (B, T) satisfies the property E.A;

$$G(Ax, By, By) \leq h \max \left\{ \begin{array}{c} G(Sx, Ty, Ty), G(By, Sx, Ty), \\ G(Ax, Ty, Ty), G(By, Ty, Ty), \\ \frac{1}{2}\big(G(Ax, Ty, Ty) + G(By, Sx, Sx)\big), \\ G(By, Ax, Ty), \\ \frac{1}{2}\big(G(Sx, Ty, Ty) + G(By, Ax, Ty)\big), \\ G(Ax, Sx, Sx), \\ \frac{1}{2}\big(G(Ax, Ty, Ty) + G(By, Ty, Ty)\big), \\ G(By, Sx, Sx) \end{array} \right\}$$

for all $x, y \in X$, where $h \in (0,1)$;

One of $A(X), B(X), S(X)$ and$T(X)$ is a complete subset of X;

Then the pairs (A, S) and(B, T) have a coincidence point. Further if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point in X.

Proof: Suppose that the pair (B, T) satisfies the property E.A, then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$. Since $B(X) \subseteq S(X)$,there exist a sequence $\{y_n\}$ in X such that $Bx_n = Sy_n$. Hence $\lim_{n\to\infty} Sy_n = t$.
We claim that $\lim_{n\to\infty} Ay_n = t$ on contrary suppose that this is not true. Then from 3^{rd} condition, we have

$G(Ay_n, Bx_n, Bx_n) \leq$

$$h\, max \begin{Bmatrix} G(Sy_n, Tx_n, Tx_n), G(Bx_n, Sy_n, Tx_n), \\ G(Ay_n, Tx_n, Tx_n), G(Bx_n, Tx_n, Tx_n), \\ \frac{1}{2}\big(G(Ay_n, Tx_n, Tx_n) + G(Bx_n, Sy_n, Sy_n)\big), \\ G(Bx_n, Ay_n, Tx_n), \\ \frac{1}{2}\big(G(Sy_n, Tx_n, Tx_n) + G(Bx_n, Ay_n, Tx_n)\big), \\ G(Ay_n, Sy_n, Sy_n), \\ \frac{1}{2}\big(G(Ay_n, Tx_n, Tx_n) + G(Bx_n, Tx_n, Tx_n)\big), \\ G(Bx_n, Sy_n, Sy_n) \end{Bmatrix}$$

Taking the limit as $n \to \infty$, we get

$\lim_{n\to\infty} G(Ay_n, t, t)$

$$\leq h\, max \begin{Bmatrix} G(t,t,t), G(t,t,t), \\ \lim_{n\to\infty} G(Ay_n, t, t), G(t,t,t), \\ \frac{1}{2}\left(\lim_{n\to\infty} G(Ay_n, t, t) + G(t,t,t)\right), \\ \lim_{n\to\infty} G(t, Ay_n, t), \\ \frac{1}{2}\left(G(t,t,t) + \lim_{n\to\infty} G(t, Ay_n, t)\right), \\ \lim_{n\to\infty} G(Ay_n, t, t), \\ \frac{1}{2}\left(\lim_{n\to\infty} G(Ay_n, t, t) + G(t,t,t)\right), \\ G(t,t,t) \end{Bmatrix}$$

It implies that

$\lim_{n\to\infty} G(Ay_n, t, t) \leq h \lim_{n\to\infty} G(Ay_n, t, t) < G(Ay_n, t, t)$ as $h \in (0,1)$ is a contradiction. Hence $\lim_{n\to\infty} Ay_n = t$.

Now suppose that $S(X)$ is a complete subset of X, then $t = Su$ for some $u \in X$. now we will show that $Au = Su = t$.

Again from 3^{rd} condition, we have

$G(Au, Bx_n, Bx_n)$

$$\leq h\, max \begin{Bmatrix} G(Su, Tx_n, Tx_n), G(Bx_n, Su, Tx_n), \\ G(Au, Tx_n, Tx_n), G(Bx_n, Tx_n, Tx_n), \\ \frac{1}{2}\big(G(Au, Tx_n, Tx_n) + G(Bx_n, Su, Su)\big), \\ G(Bx_n, Au, Tx_n), \\ \frac{1}{2}\big(G(Su, Tx_n, Tx_n) + G(Bx_n, Au, Tx_n)\big), \\ G(Au, Su, Su), \\ \frac{1}{2}\big(G(Au, Tx_n, Tx_n) + G(Bx_n, Tx_n, Tx_n)\big), \\ G(Bx_n, Su, Su) \end{Bmatrix}$$

Taking the limit as $n \to \infty$, we get

$G(Au, t, t)$

$$\leq h\, max \begin{Bmatrix} G(t,t,t), G(t,t,t), G(Au,t,t), G(t,t,t), \\ \frac{1}{2}\big(G(Au,t,t) + G(t,t,t)\big), G(t, Au, t), \\ \frac{1}{2}\big(G(t,t,t) + G(t,Au,t)\big), G(Au,t,t), \\ \frac{1}{2}\big(G(Au,t,t) + G(t,t,t)\big), G(t,t,t) \end{Bmatrix}$$

It implies that

$$\lim_{n\to\infty} G(Au, t, t) \leq h \lim_{n\to\infty} G(Au, t, t) < G(Au, t, t)$$

Hence $Au = Su$ (1)

Therefore u is a coincidence point of the pair (A, S), the weak compatibility of A and S implies that $ASu = SAu$ and hence $AAu = ASu = SAu = SSu$.
Since $A(X) \subseteq T(X)$, there exists $v \in X$
such that $Au = Tv$ (2)

Again from 3^{rd} condition, we have

$G(Au, Bv, Bv)$

$$\leq h\, max \begin{Bmatrix} G(Su, Tv, Tv), G(Bv, Su, Tv), \\ G(Au, Tv, Tv), G(Bv, Tv, Tv), \\ \frac{1}{2}\big(G(Au, Tv, Tv) + G(Bv, Su, Su)\big), \\ G(Bv, Au, Tv), \\ \frac{1}{2}\big(G(Su, Tv, Tv) + G(Bv, Au, Tv)\big), \\ G(Au, Su, Su), \\ \frac{1}{2}\big(G(Au, Tv, Tv) + G(Bv, Tv, Tv)\big), \\ G(Bv, Su, Su) \end{Bmatrix}$$

From (1) and (2) we have

$G(Tv, Bv, Bv)$

$$\leq h\, max \begin{Bmatrix} G(Au, Au, Au), G(Bv, Tv, Tv), \\ G(Tv, Tv, Tv), G(Bv, Tv, Tv), \\ \frac{1}{2}\big(G(Tv, Tv, Tv) + G(Bv, Tv, Tv)\big), \\ G(Bv, Tv, Tv), \\ \frac{1}{2}\big(G(Tv, Tv, Tv) + G(Bv, Tv, Tv)\big), \\ G(Au, Au, Au), \\ \frac{1}{2}\big(G(Tv, Tv, Tv) + G(Bv, Tv, Tv)\big), \\ G(Bv, Tv, Tv) \end{Bmatrix}$$

It implies that $Tv = Bv$

Thus $Au = Su = Tv = Bv = t$ (3)

Now we take $x = t$ and $y = v$ then from 3^{rd} condition, we have

$G(At, Bv, Bv)$

$$\leq h\, max \begin{Bmatrix} G(St, Tv, Tv), G(Bv, St, Tv), \\ G(At, Tv, Tv), G(Bv, Tv, Tv), \\ \frac{1}{2}\big(G(At, Tv, Tv) + G(Bv, St, St)\big), \\ G(Bv, At, Tv), \\ \frac{1}{2}\big(G(St, Tv, Tv) + G(Bv, At, Tv)\big), \\ G(At, St, St), \\ \frac{1}{2}\big(G(At, Tv, Tv) + G(Bv, Tv, Tv)\big), \\ G(Bv, St, St) \end{Bmatrix}$$

On using (3), we obtain $At = t$.

Since the pair (A, S) is weakly compatible, therefore $ASu = SAu = At = St$.
Hence we have $At = St = t$.
The same result can be obtained for the pair (B, T).

For Uniqueness: Suppose that there exist another fixed point z of A, B, S and T such that $x = t$ and $y = z$. Then by condition 3^{rd}, we have

$$G(At, Bz, Bz) \le h\, max \left\{ \begin{array}{c} G(St, Tz, Tz), G(Bz, St, Tz), \\ G(At, Tz, Tz), G(Bz, Tz, Tz), \\ \frac{1}{2}\big(G(At, Tz, Tz) + G(Bz, St, St)\big), \\ G(Bz, At, Tz), \\ \frac{1}{2}\big(G(St, Tz, Tz) + G(Bz, At, Tz)\big), \\ G(At, St, St), \\ \frac{1}{2}\big(G(At, Tz, Tz) + G(Bz, Tz, Tz)\big), \\ G(Bz, St, St) \end{array} \right\}$$

$$G(t, z, z) \le h\, max \left\{ \begin{array}{c} G(t, z, z), G(z, t, z), G(t, z, z), G(z, z, z), \\ \frac{1}{2}\big(G(t, z, z) + G(z, t, t)\big), G(z, t, z), \\ \frac{1}{2}\big(G(t, z, z) + G(z, t, z)\big), G(t, t, t), \\ \frac{1}{2}\big(G(t, z, z) + G(z, z, z)\big), G(z, t, t) \end{array} \right\}$$

It implies that $t = z$. Therefore t is a unique common fixed point of $A, B, S$and T.

Theorem 3.2: Let(X, G)be a $G-$metric space and $A, B, S, T: X \to X$ be four self-mappings such that:

(1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$;
(2) One of the pairs (A, S) and (B, T) satisfies the property E.A;
(3)

$$G(Ax, By, By) \le h\, max \left\{ \begin{array}{c} G(Sx, Ty, Ty), G(By, Sx, Ty), \\ G(Ax, Ty, Ty), G(By, Ty, Ty), \\ G(By, Ax, Ty), G(Ax, Sx, Sx), \\ G(Ty, Sx, Sx), G(By, Sx, Sx) \\ G(Ax, By, Sx), G(Ax, Sx, Ty), \\ G(Sx, By, By), G(By, By, Ty) \end{array} \right\}$$

for all $x, y \in X$, where $h \in (0,1)$;
(4) One of $A(X), B(X), S(X)$ and $T(X)$ is a complete subset of X

Then the pairs (A, S) and(B, T) have a coincidence point. Further if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point in X.

Proof: Suppose that the pair (B, T) satisfies the property E.A, then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$. Since $B(X) \subseteq S(X)$,there exist a sequence $\{y_n\}$ in X such that $Bx_n = Sy_n$. Hence $\lim_{n\to\infty} Sy_n = t$.
We claim that $\lim_{n\to\infty} Ay_n = t$ on contrary suppose that this is not true. Then from 3^{rd} condition, we have

$$G(Ay_n, Bx_n, Bx_n) \le h\, max \left\{ \begin{array}{c} G(Sy_n, Tx_n, Tx_n), G(Bx_n, Sy_n, Tx_n), \\ G(Ay_n, Tx_n, Tx_n), G(Bx_n, Tx_n, Tx_n), \\ G(Bx_n, Ay_n, Tx_n), G(Ay_n, Sy_n, Sy_n), \\ G(Tx_n, Sy_n, Sy_n), G(Bx_n, Sy_n, Sy_n) \\ G(Ay_n, Bx_n, Sy_n), G(Ay_n, Sy_n, Tx_n), \\ G(Sy_n, Bx_n, Bx_n), G(Bx_n, Bx_n, Tx_n) \end{array} \right\}$$

Taking the limit as $n \to \infty$, we get

$$\lim_{n\to\infty} G(Ay_n, t, t) \le h\, max \left\{ \begin{array}{c} G(t, t, t), G(t, t, t), \\ \lim_{n\to\infty} G(Ay_n, t, t), G(t, t, t), \\ \lim_{n\to\infty} G(t, Ay_n, t), \lim_{n\to\infty} G(Ay_n, t, t), \\ G(t, t, t), G(t, t, t), \\ \lim_{n\to\infty} G(Ay_n, t, t), G(t, t, t), \\ G(t, t, t), \lim_{n\to\infty} G(Ay_n, t, t) \end{array} \right\}$$

It implies that

$\lim_{n\to\infty} G(Ay_n, t, t) \le h \lim_{n\to\infty} G(Ay_n, t, t) < G(Ay_n, t, t)$ as $h \in (0,1)$ is a contradiction. Hence $\lim_{n\to\infty} Ay_n = t$.
Now suppose that $S(X)$ is a complete subset of X, then $t = Su$ for some $u \in X$. now we will show that $Au = Su = t$.

Again from 3^{rd} condition, we have

$$G(Au, Bx_n, Bx_n) \le h\, max \left\{ \begin{array}{c} G(Su, Tx_n, Tx_n), G(Bx_n, Su, Tx_n), \\ G(Au, Tx_n, Tx_n), G(Bx_n, Tx_n, Tx_n), \\ G(Bx_n, Au, Tx_n), G(Au, Su, Su), \\ G(Tx_n, Su, Su), G(Bx_n, Su, Su) \\ G(Au, Bx_n, Su), G(Au, Su, Tx_n), \\ G(Su, Bx_n, Bx_n), G(Bx_n, Bx_n, Tx_n) \end{array} \right\}$$

Taking the limit as $n \to \infty$, we get

$$G(Au, t, t) \le h\, max \left\{ \begin{array}{c} G(t, t, t), G(t, t, t), G(Au, t, t), G(t, t, t), \\ G(t, Au, t), G(Au, t, t), G(t, t, t), G(t, t, t) \\ G(Au, t, t), G(Au, t, t), G(t, t, t), G(t, t, t) \end{array} \right\}$$

It implies that

$$\lim_{n\to\infty} G(Au, t, t) \le h \lim_{n\to\infty} G(Au, t, t) < G(Au, t, t)$$

Hence $Au = Su$ (1)

Therefore u is a coincidence point of the pair (A, S), the weak compatibility of A and S implies that $ASu = SAu$ and hence $AAu = ASu = SAu = SSu$.

Since $A(X) \subseteq T(X)$, there exists $v \in X$ such that

$Au = Tv$ (2)

Again from 3^{rd} condition, we have

$$G(Au,Bv,Bv) \leq h\,max\left\{\begin{matrix} G(Su,Tv,Tv), G(Bv,Su,Tv), \\ G(Au,Tv,Tv), G(Bv,Tv,Tv), \\ G(Bv,Au,Tv), G(Au,Su,Su), \\ G(Tv,Su,Su), G(Bv,Su,Su) \\ G(Au,Bv,Su), G(Au,Su,Tv), \\ G(Su,Bv,Bv), G(Bv,Bv,Tv) \end{matrix}\right\}$$

From (1) and (2) we have

$$G(Tv,Bv,Bv) \leq h\,max\left\{\begin{matrix} G(Au,Au,Au), G(Bv,Tv,Tv), \\ G(Tv,Tv,Tv), G(Bv,Tv,Tv), \\ G(Bv,Tv,Tv), G(Au,Au,Au), \\ G(Tv,Tv,Tv), G(Bv,Tv,Tv), \\ G(Tv,Bv,Tv), G(Tv,Tv,Tv), \\ G(Tv,Bv,Bv), G(Bv,Bv,Tv) \end{matrix}\right\}$$

It implies that $Tv = Bv$

Thus $Au = Su = Tv = Bv = t$ (3)

Now we take $x = t$ and $y = v$ then from 3^{rd} condition, we have

$$G(At,Bv,Bv) \leq h\,max\left\{\begin{matrix} G(St,Tv,Tv), G(Bv,St,Tv), \\ G(At,Tv,Tv), G(Bv,Tv,Tv), \\ G(Bv,At,Tv), G(At,St,St), \\ G(Tv,St,St), G(Bv,St,St) \\ G(At,Bv,St), G(At,St,Tv), \\ G(St,Bv,Bv), G(Bv,Bv,Tv) \end{matrix}\right\}$$

On using (3), we obtain $At = t$.

Since the pair (A,S) is weakly compatible,
therefore $ASu = SAu = At = St$.
Hence we have $At = St = t$.

The same result can be obtained for the pair (B,T).

For Uniqueness: Suppose that there exist another fixed point z of A, B, S and T such that $x = t$ and $y = z$. Then by condition 3^{rd}, we have

$$G(At,Bz,Bz) \leq h\,max\left\{\begin{matrix} G(St,Tz,Tz), G(Bz,St,Tz), \\ G(At,Tz,Tz), G(Bz,Tz,Tz), \\ G(Bz,At,Tz), G(At,St,St), \\ G(Tz,St,St), G(Bz,St,St) \\ G(At,Bz,St), G(At,St,Tz), \\ G(St,Bz,Bz), G(Bz,Bz,Tz) \end{matrix}\right\}$$

$$G(t,z,z) \leq h\,max\left\{\begin{matrix} G(t,z,z), G(z,t,z), \\ G(t,z,z), G(z,z,z), \\ G(z,t,z), G(t,t,t), \\ G(z,t,t), G(z,t,t) \\ G(t,z,t), G(t,t,z), \\ G(t,z,z), G(z,z,z) \end{matrix}\right\}$$

It implies that $t = z$. Therefore t is a unique common fixed point of $A, B, S$and T.

Conclusion

In this Chapter, we use the notion of the property E.A, to prove common fixed point theorems for weakly compatible maps in G −metric space. Our result extends the result of Vishal G. and Raman D. (2015). This can be further extended.

References

Aamri M. and El Moutawakil D. (2002). Some New Common Fixed Point Theorems Under Strict Contractive Conditions. Journal of Mathematical Analysis and Applications, vol. 27, pp. 181 – 188.

Ali Syed Shahnawaz, Jain Jainendra, Sanodia P.L. & Jain Shilpi (2016). Extension of Sub-Compatible and Sub-Sequential Continuous Maps in G-Metric Space. International Journal of Current Engineering and Technology, vol.6(6), pp. 2017-2019.

Di Bari and Vetro C. (2008). Common Fixed Point Theorems for Weakly Compatible Maps satisfying a General Contractive Condition. International J Math. Math.sci, Article ID 891375.

Gahler S. (1962). Zur Geometric 2 −Metrische Raume. Reev. Roum. de Math. Pure.et Appl., vol. 11, pp. 664–669.

Gahler S. (1962). 2 −Metrische Raume und ihre topologische Struktur. Mathematische Nachrichten, vol. 26, pp. 115–148.

Gopal D., Imdad M. and Vetro C. (2011). Common Fixed Point Theorems for Mappings Satisfying Common Property E.A in Symmetric Spaces, Filomat, vol. 25, pp 59 – 78.

Gopal D., Imdad M. and Vetro C. (2011). Impact of Common Property E.A on Fixed Point Theorems in Fuzzy Metric Spaces, Fixed Point Theory and Applications, Article ID 297360.

Jungck G. (1976). Commuting Mappings and Fixed Point. Amer. Math. Monthly, vol. 83, pp 261 – 263.

Manro S. (2013). A Common Fixed Point Theorem for Two Weakly Compatible Pairs in G −Metric Spaces using the Property E.A. Fixed Point Theory & Applications, Article ID 201341, 09 pages.

Manro S., and Bhatia (2010). Expansion Mapping Theorems in G −Metric Spaces, International Journal Contemp. Math. sciences, vol. 5, pp. 2529-2535.

Mustafa Z and Sims B. (2004). Some Remarks Concerninig D −Metric Spaces, in Proceedings of the International Conference on Fixed Point Theory and Applications, Yokohama (Japan), pp. 189–198.

Mustafa Z and Sims B. (2006). A New Approach to Generalized Metric Spaces. Journal of Nonlinear Convex Anal, vol.7, pp. 289–297.

Mustafa Z and Sims B. (2009). Fixed Point Theorems for Contractive Mappings in Complete G −Metric Spaces. Fixed Point Theory & Applications, Article ID 917175, 10 pages.

Mustafa Z., Obiedat H. and Awawdeh F. (2008). Some Common Fixed Point Theorem for Mappings on Complete G −Metric spaces, Fixed Point Theory and Applications, Article ID 189870, 12 pages.

Saadati R., Rhoades B.E. and Vaezpour S.M. (2010). Fixed Points of Four Mappings in Generalized Partially Ordered G −Metric Spaces. Math. Comput. Model., vol. 52, pp. 797 – 801.

Shatanawi W, Mustafa Z and Bataineh M. (2009). Existence of Fixed Point Results in G −Metric spaces. International

Journal of Mathematics and Mathematical sciences, Article ID 283028, 10 pages.

Shatanawi W. (2010). Fixed Point Theory for Contractive Mappings Satisfying ϕ −maps in G −Metric Spaces. Fixed Point Theory and Applications, Article ID 181650, 09 pages.

Shatanawi W., Mustafa Z. and Awawdeh F. (2010). Fixed Point Theorem for Expansive Mappings in G −Metric spaces. International J Contemp. Math. sciences, vol. 5(50), pp. 2463 – 2472.

Vishal G. and Raman D. (2015). Some Fixed Point Theorems in G −Metric and Fuzzy Metric Spaces using E.A Property. Journal of Information and Computer Science, vol. (10), pp. 083-089.

Vishal G. and Tripathi A. K. (2012). Common Fixed Point Theorems in G −Metric Space. International Journal of Mathematics and Computer Application Research, vol. 2(3), pp. 1 – 4.