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Research Article

Explicit Solutions of Double-Chain DNA Dynamical System in (2+ 1)-Dimensions

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Abstract

Deoxyribonucleic acid (DNA) holds the genetic information that creatures need to live and reproduce themselves. The nonlinear dynamics of the double chain DNA model is studied. The double chain model is extended to (2+1)-dimensions. The travelling wave solution is found by applying the $\frac{G'}{G}$ *expansion method. According to the different system parameters, two cases are discussed and plotted. The system nonlinear dynamics appears in solitary waves and its anti- solitary waves in the DNA strands.*

Keywords: Deoxyribonucleic acid; Longitudinal and transversal motions; Solitary wave solution.

1. Introduction

Deoxyribonucleic acid (DNA) (Abdelrahman, Zahran *et al*. 2015, Okaly, Mvogo *et al*. 2018, Shapovalov and Obukhov 2018, Dwiputra, Hidayat *et al*. 2019) is the information storage medium dwell in every cell. It holds the genetic information that creatures need to live and reproduce themselves. DNA structure has been extensively studied during the last decades (Watson and Crick 1953, Englander, Kallenbach *et al*. 1980, Peyrard and Bishop 1989, Dauxois, Peyrard *et al*. 1993, Peyrard, Cuesta-López *et al*. 2008, Alatas and Hermanudin 2012, Zdravković, Chevizovich *et al*. 2019). The characteristics multiplicity of DNA, result in the difficulty of purposing DNA by a specific mathematical model. The double helix structure of DNA has been stimulated by the pioneer works of (Watson and Crick 1953). (Englander, Kallenbach *et al*. 1980) studied the open states dynamics of DNA, considering only the rotational motion of nitrogen bases. (Yomosa 1983) offered a dynamics plane-base rotator model. (Takeno and Homma 1984) improved Yomosa's model by considering the degree of freedom, describing base rotations in the plane perpendicular to the helical axis around the backbone structure. The denaturation process of the transverse motions of bases along the hydrogen bond was studied in (Peyrard and Bishop 1989). The transverse motions along the hydrogen bond and longitudinal motions along the backbone direction were suggested in (Muto, Lomdahl *et al*. 1990). These two motions made the main contribution in the DNA denaturation process.

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(De-Xing, Sen-Yue *et al*. 2001) introduced a new double-chain model of DNA. They considered that, DNA consists of two long elastic homogeneous strands, which represent two polynucleotide chains of DNA molecule, joined by an elastic membrane demonstrating the hydrogen bonds between the base pair of these chains. The nonlinear dynamics of this model was considered from different perspectives (Xian-Min and Sen-Yue 2003, Alka, Goyal *et al*. 2011, Ouyang and Zheng 2014). In (Xian-Min and Sen-Yue 2003), The exact solutions of this model were studied using Pickering's truncation expansion and the Conte's Painlevé truncation expansion methods. In (Ouyang and Zheng 2014), the traveling wave solutions of this double-chain DNA model is considered using the method of dynamical systems. The Riccati parameterized factorization method is applied in, (Alka, Goyal *et al*. 2011) to find the solitary wave solution.

Abundant methods are demoralized in studying the nonlinear dynamics for several applications. Some of these methods are exponential function method (Arshed, Biswas *et al*. 2018, Mabrouk and Rashed 2019), singular manifold method (Mabrouk and Rashed 2017, Saleh, Kassem *et al*. 2017, Saleh, Mabrouk *et al*. 2018), Hirota's bilinear method (Cao, Malomed *et al*. 2018, Morales-Delgado, Gómez-Aguilar *et al*. 2018), homogenous balance method (Ji, Wu *et al*. 2010, Mabrouk 2019) , $\frac{G'}{G}$ expansion method (Wang, Li *et al*. 2008, Mabrouk 2019), sine–cosine and tanh–coth methods (Wazwaz 2006).

This work is motivated to extend the double-chain DNA model (De-Xing, Sen-Yue *et al*. 2001, Xian-Min and Sen-Yue 2003) to (2+1) dimensions, then solve analytically this extended model by applying the $\frac{G'}{G}$ expansion method. The paper is planned as follows. The second section, presents the double chain DNA mathematical model. The third section, briefly describes the $\frac{G'}{G}$ expansion method. The method is applied to solve DNA dynamical system in the fourth section. The paper ends with conclusions.

2. The double chain DNA model

The new double-chain DNA model (De-Xing, Sen-Yue *et al.* 2001) is;

$$u_{tt} - c_1^2 u_{xx} = \lambda_1 u + \gamma_1 uv + \mu_1 u^3 + \beta_1 uv^2 \quad (1)$$
$$v_{tt} - c_2^2 v_{xx} = \lambda_2 v + \gamma_2 u^2 + \mu_2 u^2 v + \beta_2 v^3 + c_0 \quad (2)$$

Where, $u(x, y, t)$ and $v(x, y, t)$ are the difference of the longitudinal displacements and the difference of the transverse displacements; of the bottom and top strands; correspondingly. The constants c_1, c_2, λ_1, λ_2, γ_1, γ_2, μ_1, μ_2, β_1, β_2 and c_0 are defined as;

$$\begin{cases} c_1 = \pm\frac{\epsilon}{\rho},\ c_2 = \pm\frac{F}{\rho}, \lambda_1 = \frac{-2\mu}{\rho\sigma h}(c - l_0) \\ \beta_1 = \beta_2 = \frac{4\mu l_0}{\rho\sigma h^3},\ \ c_0 = \frac{\sqrt{2}\mu(h-l_0)}{\rho\sigma} \\ \lambda_2 = \frac{-2\mu}{\rho\sigma}, \gamma_1 = 2\gamma_2 = \frac{2\sqrt{2}\mu l_0}{\rho\sigma h^2}, \mu_1 = \mu_2 = \frac{-2\mu l_0}{\rho\sigma h^3} \end{cases} \quad (3)$$

ρ is the mass density, σ is the area of the transverse cross-section, ϵ is the Yong's modulus, F, refers to the tension density of the strand, μ, stands for the rigidity of the elastic membrane, h is the distance between the two strands and l_0 is the height of the membrane in the equilibrium position.

This system can be extended to (2+1)-dimensions as;

$$u_{tt} - c_1^2 u_{xx} - c_1^2 u_{yy} = \lambda_1 u + \gamma_1 uv + \mu_1 u^3 + \beta_1 uv^2 \quad (4)$$
$$v_{tt} - c_2^2 v_{xx} - c_2^2 v_{yy} = \lambda_2 v + \gamma_2 u^2 + \mu_2 u^2 v + \beta_2 v^3 + c_0 \quad (5)$$

Now, consider the transformation;

$$v = b_1 u + b_2 \quad (6)$$

Where, equation (4) is reduced to;

$$u_{tt} - c_1^2 u_{xx} - c_1^2 u_{yy} = u^3(\mu_1 + \beta_1 b_1^2) + u^2(\gamma_1 b_1 + 2\beta_1 b_1 b_2) + u(\lambda_1 + \gamma_1 b_2 + \beta_1 b_2^2) \quad (7)$$

Equation (5), is reduced to;

$$b_1 u_{tt} - b_1 c_2^2 u_{xx} - b_1 c_2^2 u_{yy} = u^3(b_1\mu_2 + \beta_2 b_1^3) + u^2(\gamma_2 + \mu_2 b_2 + 3\beta_2 b_1^2 b_2) + u(\lambda_2 b_1 + 3\beta_2 b_1 b_2^2) + \lambda_2 b_2 + \beta_2 b_2^3 + c_0 \quad (8)$$

Equations (7) and (8) are similar for,

$$b_2 = \frac{h}{\sqrt{2}}\ \ ,\ F = \epsilon \quad (9)$$

Finally, the system of equations (4) and (5) is reduced to a single equation;

$$u_{tt} - c_1^2 u_{xx} - c_1^2 u_{yy} - Au^3 + Bu^2 + Cu = 0 \quad (10)$$

Where, $A = \frac{\Psi}{h^3}(4b_1^2 - 2), B = \frac{6\sqrt{2}b_1}{h^2}\Psi, C = \frac{6\Psi}{h} - \frac{2\Psi}{l_0}, \Psi = \frac{\mu l_0}{\rho\sigma}$ $and\ c_1 = c_2$ (11)

3. Mathematical method

In this section, the (G'/G)-expansion method, which is excessively used in finding traveling wave solutions of nonlinear equations (Wang, Li *et al.* 2008, Parkes 2010, Naher and Abdullah 2014), is briefly summarized in the following steps;

i. For a partial differential equation (PDE);

$$P(u, u_x, u_y, u_t, u_{xx}, u_{xy}, \ldots) = 0 \quad (12)$$

Where p, is a polynomial in u and its partial derivatives.

ii. Suppose the solution of the partial differential equation (12) is in the form;

$$u(x, y, t) = u(\xi)\ ,\ \xi = x + y - \delta t. \quad (13)$$

The constant δ, is the wave velocity. The PDE (12), is reduced to a nonlinear ordinary differential equation (ODE); which can be integrated many times -If possible- with setting the constants of integration equal to zero, for simplicity.

iii. Assume the solution of the reduced ODE, is in the form;

$$u(\xi) = \sum_{i=0}^{m} a_i \left(\frac{G'}{G}\right)^i \quad (14)$$

Where $G = G(\xi)$, satisfies the second order linear ODE;

$$G''(\xi) + \eta\, G'(\xi) + \theta G(\xi) = 0 \quad (15)$$

Where, $G' = \frac{dG}{d\xi}, G'' = \frac{d^2G}{d\xi^2}$, a_i , η and θ, are real constants to be determined. The positive integer m, is determined through balancing, the highest order linear and nonlinear terms' derivatives, appearing in the ODE. Substitute (13) and (15), into the final ODE, then collect all terms, with the same order of (G'/G) and set each coefficient to zero, yield a set of algebraic equations for a_i, η, δ and θ

4. Explicit solution of (2+1)-dimensional double chain DNA dynamical system

This section is motivated to find the explicit solutions of the dynamical system (4), (5). First, use equation (13) into equation (10), get;

$$Au'' - Bu^3 - Cu^2 - Du = 0 \quad (16)$$

Where, $A = -(\delta + 2c_1^2)$ (17)

The balance between u'' and u^3, reveals that $m = 1$, and the solution of equation (16) is;

$$u(\xi) = a_0 + a_1\left(\frac{G'}{G}\right) \quad (18)$$

Where, a_0 and $a_1 \neq 0$, substitute (18) using (15) into (16) yields;

$$(2\mathrm{A}a_1 - \mathrm{B}a_1^3)\left(\frac{G'}{G}\right)^3 + (3\mathrm{A}\eta a_1 - 3\mathrm{B}a_0 a_1^2 - \mathrm{C}a_1^2)\left(\frac{G'}{G}\right)^2 + (Aa_1\eta^2 + 2\mathrm{A}a_1\theta - 3\mathrm{B}a_1 a_0^2 - 2Ca_0 a_1 - \mathrm{D}a_1)\left(\frac{G'}{G}\right) + (Aa_1\eta\theta - \mathrm{B}a_0^3 - Ca_0^2 - \mathrm{D}a_0) = 0$$
(19)

Collect the terms with the same order of (G'/G), and set each coefficient to zero yields; a set of algebraic equations for a_0, a_1, A, η and θ.

$$\begin{cases} 2Aa_1 - Ba_1^3 = 0 \\ 3A\eta a_1 - 3Ba_0 a_1^2 - Ca_1^2 = 0 \\ Aa_1\eta^2 + 2Aa_1\theta - 3Ba_1 a_0^2 - 2Ca_0 a_1 - Da_1 = 0 \\ Aa_1\eta\theta - Ba_0^3 - Ca_0^2 - Da_0 = 0 \end{cases} \quad (20)$$

Solutions of this system of equations result in;

$$a_1 = \sqrt{\frac{2A}{B}},\ a_0 = \sqrt{\frac{A}{2B}}\eta - \frac{C}{3B},\ \theta = \frac{3B}{2A}a_0^2 + \frac{C}{A}a_0 + \frac{D}{2A} - \frac{1}{2}\eta^2$$
and
$$A = \frac{Ba_0^3 + Ca_0^2 + Da_0}{a_1\eta\theta} \quad (21)$$

The function $G(\eta)$, is found from solving equation (15), in the following two cases;

Case 1; $\eta^2 - 4\theta > 0$

The solution of the DNA system (4), (5) is;

$$u = \sqrt{\frac{A}{2B}}\eta - \frac{C}{3B} + \sqrt{\frac{2A}{B}}\left[\frac{-\eta}{2} + \frac{\sqrt{\eta^2-4\theta}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right) + C_2 cosh\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right)}{C_1 cosh\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right) + C_2\sinh\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right)}\right)\right] \quad (22)$$

$$v = b_1\left(\sqrt{\frac{A}{2B}}\eta - \frac{C}{3B} + \sqrt{\frac{2A}{B}}\left[\frac{-\eta}{2} + \frac{\sqrt{\eta^2-4\theta}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right) + C_2 cosh\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right)}{C_1 cosh\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right) + C_2\sinh\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right)}\right)\right]\right) + b_2 \quad (23)$$

Which can be simplified for $C_1 = 0$ and $C_2 = 1$, to;

$$u_1 = -\frac{C}{3B} - \sqrt{\frac{A}{2B}}\left(\sqrt{(\eta^2 - 4\theta)}\coth\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right)\right) \quad (24)$$
$$v_1 = b_1\left(-\frac{C}{3B} - \sqrt{\frac{A}{2B}}\sqrt{(\eta^2 - 4\theta)}\coth\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right)\right) + b_2 \quad (25)$$

The longitudinal motion u_1 and the transversal motion v_1 are plotted in Fig.1 and Fig.2, for $C = 9$, $B = 1$, $a = b = 1$, $z=1$, $t=3$ and $\eta^2 - 4\theta = 2$. The periodic solution is clear for the longitudinal motion and its anti-periodic solution appears for the transverse motion.

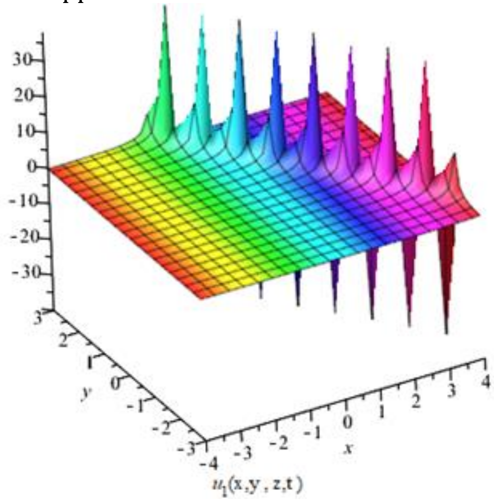

Fig.1 Traveling wave of longitudinal motion for DNA system, at C=9, B=1, a=b=1, z=1, t=3 and $\eta^2 - 4\theta = 2$.

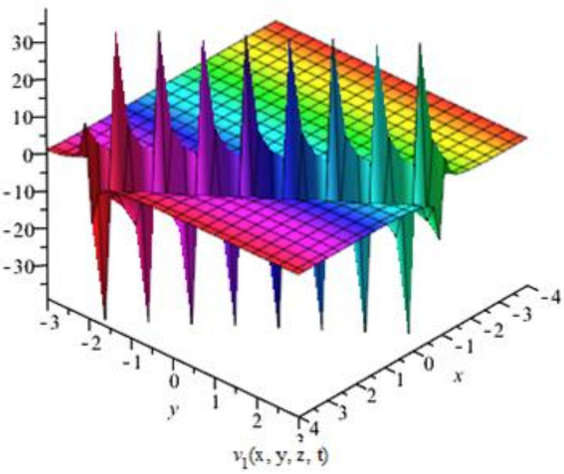

Fig.2 Anti-traveling wave of transversal motion for DNA system, at C=9, B=1, a=b=1, z=1, t=3 and $\eta^2 - 4\theta = 2$.

System solution for $C_1 = 1$ and $C_2 = 0$ is;

$$u_2 = -\frac{C}{3B} - \sqrt{\frac{A}{2B}}\sqrt{(\eta^2 - 4\theta)}\tanh\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right) \quad (26)$$
$$v_2 = b_1\left(-\frac{C}{3B} - \sqrt{\frac{A}{2B}}\sqrt{(\eta^2 - 4\theta)}\tanh\left(\frac{\sqrt{\eta^2-4\theta}}{2}\xi\right)\right) + b_2 \quad (27)$$

Which is plotted in Fig.3 and Fig.4, showing kink and anti-kink solutions for longitudinal and transversal motions.

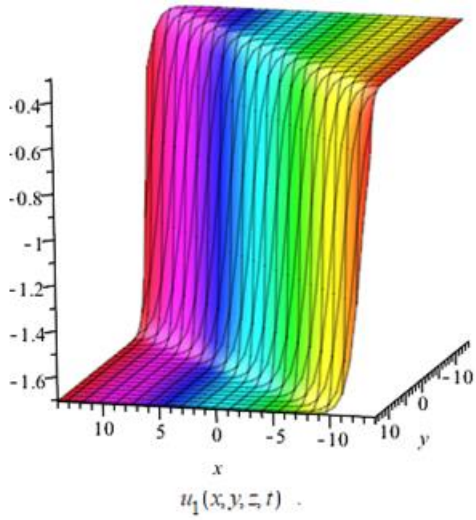

Fig.3 Kink solution of longitudinal motion for DNA system, at C=9, B=1, a =b =1, z=1, t=2 and $\eta^2 - 4\theta = 2$.

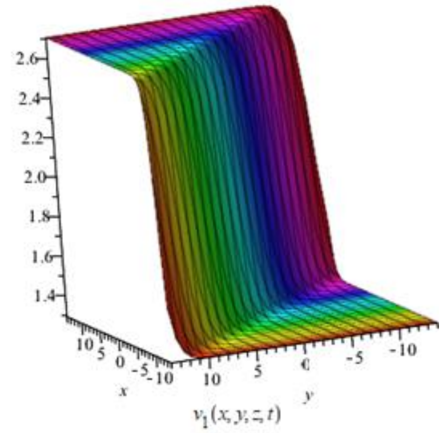

Fig.4 Anti-kink solution of transversal motion for DNA system, at C=9, B=1, a =b=1, z=1, t=2 and $\eta^2 - 4\theta = 2$.

Case 2; $\eta^2 - 4\theta < 0$

The solution of the DNA system (4), (5) is;

$$u = \sqrt{\frac{A}{2B}}\eta - \frac{C}{3B} + \sqrt{\frac{2A}{B}}\left[\frac{-\eta}{2} + \frac{\sqrt{4\theta-\eta^2}}{2}\left(\frac{C_1 \sin\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right) + C_2 \cos\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right)}{C_1 \cos\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right) + C_2 \sin\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right)}\right)\right] \quad (28)$$

$$v = b_1\left(\sqrt{\frac{A}{2B}}\eta - \frac{C}{3B} + \sqrt{\frac{2A}{B}}\left[\frac{-\eta}{2} + \frac{\sqrt{4\theta-\eta^2}}{2}\left(\frac{C_1 \sin\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right) + C_2 \cos\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right)}{C_1 \cos\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right) + C_2 \sin\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right)}\right)\right]\right) + b_2 \quad (29)$$

Which can be simplified for $C_1 = 0$ and $C_2 = 1$, to;

$$u_3 = -\frac{C}{3B} - \sqrt{\frac{A}{2B}}\sqrt{4\theta-\eta^2}\cot\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right) \quad (30)$$

$$v_3 = b_1\left(-\frac{C}{3B} - \sqrt{\frac{A}{2B}}\sqrt{(4\theta-\eta^2)}\cot\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right)\right) + b_2 \quad (31)$$

The longitudinal motion u_3 and the transversal motion v_3 are plotted in Fig.5 and Fig.6, for $C = 9$, $B = 1$, $a = b = 1$, $z=1$, $t=3$ and $4\theta - \eta^2 = 2$.

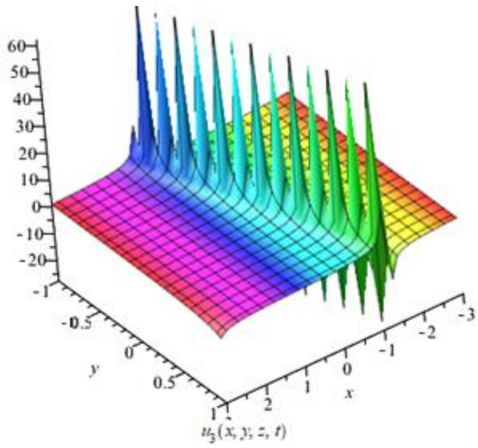

Fig.5 Periodic solution for longitudinal motion at C = 9, B = 1, a = b = 1, z=1, t=3 and $4\theta - \eta^2 = 2$.

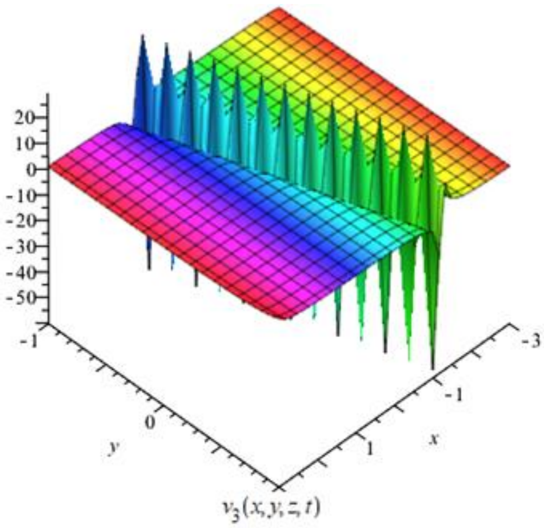

Fig.6 Anti-periodic solution for transversal motion at C = 9, B = 1, a = b = 1, z=1, t=3 and $4\theta - \eta^2 = 2$.

The solution for $C_1 = 1$ and $C_2 = 0$, is;

$$u_4 = -\frac{C}{3B} - \sqrt{\frac{A}{2B}}\sqrt{(4\theta-\eta^2)}\tan\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right) \quad (32)$$

$$v_4 = b_1\left(-\frac{C}{3B} - \sqrt{\frac{A}{2B}}\sqrt{(4\theta-\eta^2)}\tan\left(\frac{\sqrt{4\theta-\eta^2}}{2}\xi\right)\right) + b_2 \quad (33)$$

This solution is plotted in Fig.7 for longitudinal motion and in Fig.8 for transversal motion.

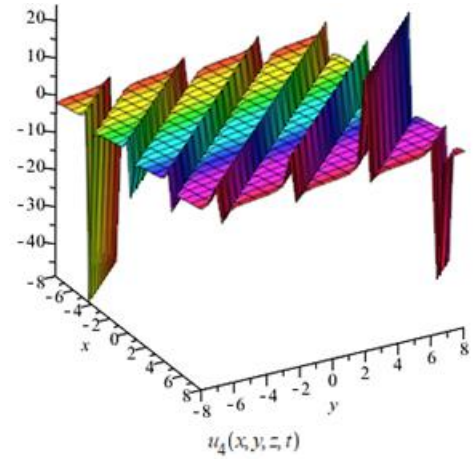

Fig.7 multi-soliton solution for longitudinal motion at C = 9, B = 1, a = b = 1, z=1, t=3 and $4\theta - \eta^2 = 2$.

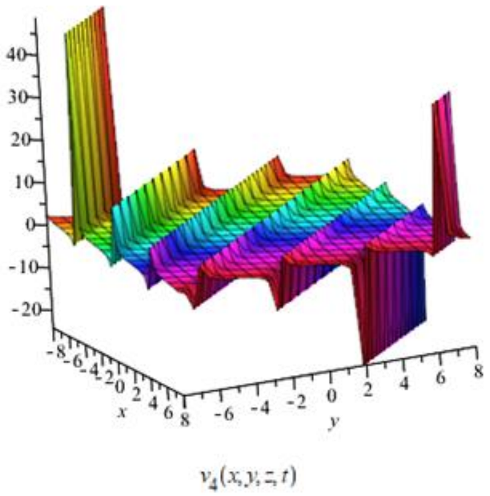

Fig.8 Anti-multi-soliton solution for transversal motion at C=9, B=1, a=b = 1, z=1, t=3 and $4\theta - \eta^2 = 2$.

Conclusions

Double chain DNA dynamical system is analytically solved. Explicit solutions for longitudinal and transversal motions are discussed and plotted. The traveling wave solutions appear in solitary wave for the longitudinal motion and its anti-solitary wave for transversal motion. Two cases are discussed and plotted. Different solitary wave solutions are cleared in, soliton, kink, periodic and multi-soliton waves. The $\frac{G'}{G}$ expansion method is effective in detecting the explicit wave solutions of the Double chain DNA dynamical system.

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