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Research Article

Shaking Force and Shaking Moment Balancing of two types of Watt Mechanisms

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Abstract

This paper presents a simple technique for the balancing of two types of watt mechanisms. The mathematical basis for the realization of the method is static and dynamic substitution of distributed masses by concentrated point masses. Shaking force is balanced by the method of redistribution of masses and adding force counterweights. Shaking moment is balanced by mounting geared inertia counterweights. The proposed method is illustrated by a numerical example which proves that better results are produced compared to the results of previous method.

Keywords: *Shaking force balancing, shaking moment balancing, Watt mechanisms.*

1. Introduction

When mechanisms run at high speeds, shaking forces and moments are a dominant cause of mechanism or machine vibrations. To reduce the impact of these vibrations one of the approaches is attaching dampers to the machines. Another approach is dynamic balancing where the mechanism is designed to exert no shaking forces and shaking moments at all. The dynamically balanced mechanisms reduce noise, wear and fatigue (Lowen, *et al*, 1968) and improve the accuracy. The drawbacks of dynamic balancing are the considerable increase of mass, inertia and complexity of the mechanism. To omit the complexity of using the loop equations (Gosselin *et al*, 2004; Wu Y *et al*, 2005; Wu Y *et al*, 2007; Arakelian *et al*, 2008], most studies on parallel mechanisms are involved with balancing of each mechanism link individually. The method of linearly independent vectors (Berkof *et al*, 1969) is most efficient method; however the derivation of the balance conditions is cumbersome and specific for each mechanism. The author (Bagci, 1979) applied the method of linearly independent vectors to derive the equations for four-, six- and eight-bar planar mechanisms. A method different from the complex vector method of (Berkof and Lowen, 1973) was proposed by (Kochev, 1988) which applies ordinary vector algebra and develops the balancing equations in Cartesian form. It was demonstrated (Bagci, 1982) that idle loops could be added to more general four-bar linkages in order to achieve the same result. However, this method enhances the individual forces that the linkage exerts on its supports. Angeles *et al*. (1982) proved that this problem can be solved by force control under redundant actuation. In this way the shaking moment because of these forces is reduced. Ouyng *et al*. (2006) presented a novel approach for the force balancing and the facilitation of the design of controllers of RTC (Real Time Controllable) for the five-bar mechanism.

This approach is called Adjusting Kinematic Parameter (AKP), but it only works for RTC mechanisms. Cheng and Pei (2006) presented a concept of using both a linkage balancer and counterweight disks to reduce shaking force and shaking moment of high speed presses. Sebastien *et al*. (1988) dealt with the problem of shaking force balancing of high speed manipulators, where shaking force reduction was 77% but it involves lengthy derivations. Van der Wijk et.al (2012) developed a concept aiming at low-mass and low-inertia dynamic balancing with duplicate mechanism, counter mass, separate counter-rotation and a counter-rotary counter-mass. But the best compromise for low-mass and low-inertia is duplicate mechanism more over it requires a considerable space. Hong-sen Y and Guo-jhih Yan (2009) proposed an integrated design approach for variable input speed servo four-bar linkages where the dimensions of the links, the counterweights, the input speed trajectory and the controller parameters are considered as the design variables simultaneously but the optimization problem will be more lengthy when the number of links is more than four. Gao Feng (1989) and Arakelian *et al*. (1999) proposed methods for shaking moment balancing by using geared inertia counter-weights. The present research is the extension of the work carried out by authors (Gao Feng 1989; Arakelian *et al*, 1999).

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The paper is organized as follows: section 1 deals with the introduction, section 2 presents shaking force and shaking moment balancing of sub-linkages. Section 3 discusses balancing of Watt mechanism and section 4 gives the conclusions.

2. Shaking force and shaking moment balancing of two types of watt mechanisms

2.1 Articulation dyad

2.1.1 Complete shaking force and shaking moment balancing of an articulation dyad

An open kinematic chain of two binary links and one joint is called a dyad.When two links are articulated by a joint so that movement is possible that arrangement of links is known as articulation dyad.The scheme of complete shaking force and shaking moment balancing of an articulation dyad (Gao Feng 1989, Arakelian *et al*. 1999) is shown in Fig.1.

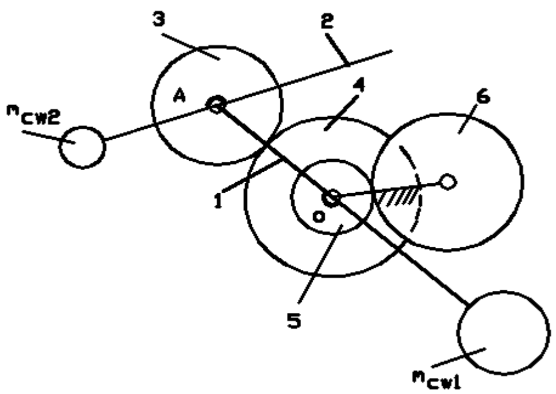

Fig.1 Complete shaking force and shaking moment balancing of an articulation Dyad

For shaking force balancing link 2 is dynamically replaced by two point masses. A counterweight $m_{cw2} = (m_2 l_{AS2}) / r_{cw2}$ is added to link 2 which permits the displacement of the center of mass of link 2 to joint A. Then, by means of a counterweight with mass $m_{cw1} = [(m_2 + m_{cw2}) l_{OA} + m_1 l_{OS1}] / r_{cw1}$ a complete balancing of shaking force is achieved. A complete shaking moment balance is realized through four gear inertia counter weights 3-6, one of them being of the planetary type Gao Feng (1989) and mounted on link 2.

2.1.2 Complete shaking force and shaking moment balancing of an articulation dyad by gear inertia counterweights mounted on the base

The scheme used in the present work [Fig.2] is distinguished from the earlier scheme by the fact that gear 3 is mounted on the base and is linked kinematically with link2 through link $1'$.

To prove the advantages of such a balancing, the application of the new system with the mass of link $1'$ not taken into account is considered. In this case (compared to the usual method Fig.1), the mass of the counter weight of link 1 will be reduced by an amount

$$\delta m_{cw_1} = m_3 l_{OA} / r_{cw_1} \quad (1)$$

where, m_3 is the mass of gear 3,

l_{OA} is the distance between the centers of hinges O and A, r_{cw_1} is the rotation radius of the center of mass of the counter weight.

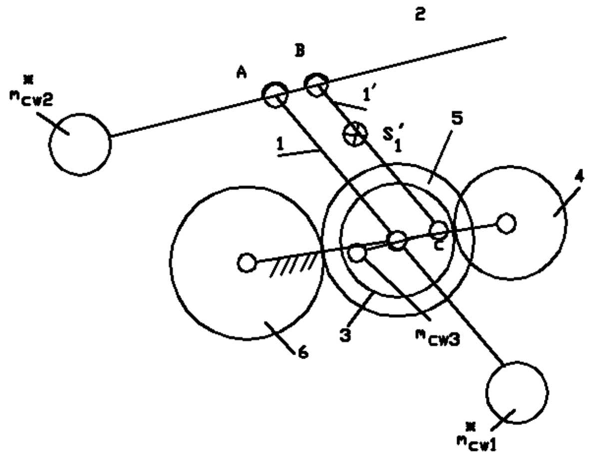

Fig. 2 Complete shaking force and shaking moment balancing of an articulation dyad by gear inertia counterweights mounted on the base

It is obvious that the moment of inertia of the links is correspondingly reduced. If the gear inertias are made in the form of heavy rims in order to obtain a large moment of inertia, the moments of inertia of the gear inertia counter weights may be presented as

$I = \frac{m_i D_i^2}{4}$ (i=3… 6).

Consequently, the mass of gear 6 will be reduced by an amount

$$\delta m_6 = 4(m_3 l_{OA}^2 + \delta m_{cw_1} r_{cw_1}^2) \frac{T_6}{D_6^2 T_5} \quad (2)$$

Where,

T_5 *and* T_6 are the numbers of teeth of the corresponding gears. Thus, the total mass of the system will be reduced by an amount

$$\delta m = \delta m_{cw_1} + \delta m_6 \quad (3)$$

Here the complete shaking force and shaking moment balancing of the articulation dyad with the mass and inertia of link $1'$ taken into account is considered. For this purpose initially, statically replace mass m_1' of link $1'$ by two point masses m_B and m_c at the centers of the hinges B and C

$$m_B = m_{1'} l_{CS_{1'}} / l_{BC}$$
$$m_C = m_{1'} l_{BS_{1'}} / l_{BC} \quad (4)$$

Where,

l_{BC} is the length of link 1,

$l_{CS_{1'}}$ and $l_{BS'_1}$ are the distances between the centers of joints C and B and the center of mass S'_1 of link $1'$, respectively.

After such an arrangement of masses the moment of inertia of link $1'$ will be equal to

$$I^*_{S'_1} = I_{S'_1} - m_{1'} l_{BS'_1} l_{CS'_1} \quad (5)$$

where,

$I_{S'_1}$ is the moment of inertia of link $1'$ about the center of mass S'_1 of the link.

Thus a new dynamic model of the system is obtained, where the link $1'$ is represented by two point masses m_B, m_C and has a moment of inertia $I^*_{S'_1}$.

This fact allows for an easy determination of the parameters of the balancing elements as follows:

$$m^*_{CW2} = (m_2 l_{AS2} + m_B l_{AB}) / r_{CW2} \quad (6)$$

where,

m_2 is the mass of link 2

l_{AB} is the distance between the centers of the hinges A and B

l_{AS_2} is the distance of the center of hinge A from the center mass of S_2 of link 2

r_{CW_2} is the rotation radius of the center of mass of the counterweight with respect to A and

$$m^*_{CW1} = [(m_2 + m_{CW2} + m_B) l_{OA} + m_1 l_{OS1}] / r_{CW1} \quad (7)$$

where,

m_1 is the mass of link 1,

l_{OS_1} is the distance of the joint center O from the center of mass S_1 of link 1.

Also, $m_{CW_3} = m_C l_{OC} / r_{CW_3}$ (8)

where,

$l_{OC} = l_{AB}$,

r_{CW_3} is the rotation radius of the center of mass of the counterweight.

Taking into account the mass of link $1'$ brings about the correction in Eq.(3) in this case,

$$\delta m = \delta m_{CW_1} + \delta m_6 - \delta m'_1 \quad (9)$$

where,

$\delta m'_1$ is the value that determines the change in the distribution of the masses of the system links resulting from the addition of link $1'$.

2.2 Asymmetric link with three rotational pairs

A link with three nodes is called ternary link, where nodes are points for attachment to other links. In the earlier research by Gao Feng (1989) relating to balancing of linkages with a dynamic substitution of the masses of the link by three rotational pairs shown in Fig.3 two replacement points A and B are considered. This results in the need to increase the mass of the counter weight. However, such a solution may be avoided by considering the problem of dynamic substitution of link masses by three point masses. Usually the center of mass of such an asymmetric link is located inside a triangle formed by these points.

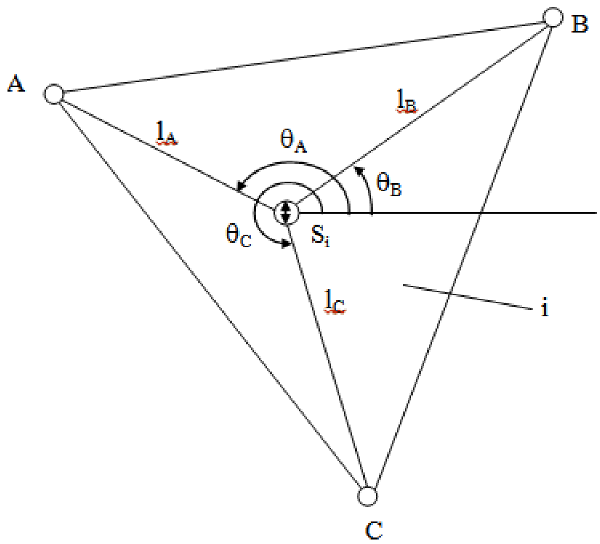

Fig.3 Dynamic substitution of the masses of the link by three rotational pairs

The conditions for dynamic substitution of masses are the following:

$$\begin{bmatrix} 1 & 1 & 1 \\ l_A e^{i\theta_A} & l_B e^{i\theta_B} & l_C e^{i\theta_C} \\ l_A^2 & l_B^2 & l_C^2 \end{bmatrix} \begin{bmatrix} m_A \\ m_B \\ m_C \end{bmatrix} = \begin{bmatrix} m_i \\ 0 \\ I_{S_i} \end{bmatrix}$$

Where,

m_A, m_B, and, m_C are point masses,

l_A, l_B and l_C are the moduli of radius vectors of corresponding points,

θ_A, θ_B and θ_C are angular positions of radius vectors;

m_i is the mass of link,

I_{S_i} is the moment of inertia of the link about an axis through S_i (axial moment of inertia of link).

From this system of equations the masses are obtained

$$m_A = D_A/D_i \,; m_B = D_B/D_i \,; m_C = D_C/D_i \qquad (10)$$

where, D_A, D_B, D_C and D_i are determinants of the third order obtained from the above system of equations.

3. Balancing of Watt mechanisms

3.1 Watt mechanism with three fixed points

Watt mechanism consists of six links out of them two are ternary and the remaining four are binary links. In Watt mechanism two ternary links are directly connected to one another. This mechanism is obtained when one of the ternary links in the basic Watt chain is fixed. This is a simple mechanism as the radii of path curvature of all motion transfer points are known. This mechanism is used in steam engines and is also used to oscillate the agitator in some washing machines. In the Watt mechanism with three fixed points shown in Fig.4, link 1 and 3 are ternary links and all other links are binary links. The balanced Watt mechanism with three fixed points is shown in Fig.5

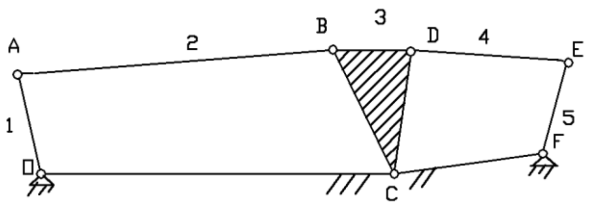

Fig.4 Watt mechanism with three fixed points

3.1.1 Shaking force balancing of the mechanism

For shaking force balancing link 3 is dynamically replaced by three point masses m_{B3}, m_{C3} and m_{D3} and then considered the problems of sub-linkages OAB and DEF.

The dynamic conditions for link 3 to be replaced by three point masses are

$$\begin{bmatrix} 1 & 1 & 1 \\ l_B e^{i\theta_B} & l_C e^{i\theta_C} & l_D e^{i\theta_D} \\ l_B^2 & l_C^2 & l_D^2 \end{bmatrix} \begin{bmatrix} m_{B3} \\ m_{C3} \\ m_{D3} \end{bmatrix} = \begin{bmatrix} m_3 \\ 0 \\ I_{S_3} \end{bmatrix}$$

$$m_{B3} = D_B/D_3 \,; m_{C3} = D_C/D_3 \,; m_{D3} = D_D/D_3 \,; \qquad (11)$$

where,

l_B, l_C, l_D are the moduli of radius vectors of corresponding points, $\theta_B, \theta_C, \theta_D$ are the angular positions of radius vectors, m_3 is the mass of link 3

I_{S_3} is the mass moment of inertia link 3 about its center of mass, D_B, D_C, D_D and D_3 are the third order determinants obtained from the system of equations

For sub-linkage DEF link 4 is dynamically replaced by two point masses m_{D4} and m_{P4} and then kinematically linked link 4 and its corresponding gear inertia counterweight 7 by link 5′ and link 5′ is statically replaced by two point masses m_G and m_H and attached a counterweight m_{CW_4} against link 4.Then link 5 has been dynamically replaced by two point masses m_{E5}, m_{P5} and attached a counterweight m_{CW_5} against it.

For link 4 to be dynamically replaced by two point masses the condition to be satisfied

$$k_4^2 = l_{DS_4} l_{P_4S_4}$$

where,

k_4 is the radius of gyration of link 4 about its center of mass.

l_{DS_4} is arbitrarily fixed and

$l_{P_4S_4}$ is obtained from the above condition.

$$m_{D4} = m_4 l_{P_4S_4} / (l_{DS_4} + l_{P_4S_4})$$

$$m_{P4} = m_4 l_{DS_4} / (l_{DS_4} + l_{P_4S_4})$$

For link 5 to be dynamically replaced by two point masses the condition to be satisfied is

$$k_5^2 = l_{ES_5} l_{P_5S_5}$$

where

k_5 is the radius of gyration of link 5 about its center of mass

l_{ES_5} is arbitrarily fixed and

$l_{P_5S_5}$ is obtained from the above condition.

$$m_{E5} = m_5 l_{P_5S_5} / (l_{ES_5} + l_{P_5S_5})$$

$$m_{P5} = m_5 l_{ES_5} / (l_{ES_5} + l_{P_5S_5})$$

and counterweight mass against 'G' is equal to

$$m_{CW_7} = m_G l_{FG} / r_{CW_7}$$

$$m_G = {m'_5 l_{HS_5}}/{l_{GH}}$$
$$m_H = {m'_5 l_{GS_5}}/{l_{GH}}$$
$$I'^{*}_{S_5} = I'_{S_5} - m'_5 l_{GS_5} l_{HS_5} \qquad (12)$$
$$m_{CW_4} = {(m_4 l_{ES_4} + m_{D3} l_{DE} + m_H l_{EH})}/{r_{CW_4}}$$
$$m_{CW_5} = {((m_4 + m_{D3} + m_H + m_{CW_4}) l_{EF} + m_5 l_{FS_5})}/{r_{CW_5}}$$

Where $r_{CW_4} = (l_{P_4S_4} - l_{ES_4})$, is the radius of rotation of counterweight m_{CW_4} and $r_{CW_5} = (l_{P_5S_5} - l_{FS_5})$ is the radius of rotation of counterweight m_{CW_5}.

For sub-linkage OAB link 2 is dynamically replaced by two point masses m_{B2}, m_{P2} and then kinematically linked link 2 and its corresponding gear inertia counterweight 11 by link 1′ and link 1′ is statically replaced by two point masses m_I, m_J and attached a counterweight m_{CW_2} against link 2. Then link 1 is dynamically replaced by two point masses m_{A1}, m_{P1} and attached a counterweight m_{CW_1} against it.

For link 2 to be dynamically replaced by two point masses the condition to be satisfied is $k_2^2 = l_{BS_2} l_{P_2S_2}$

where,

k_2 is the radius of gyration of link 2 about its center of mass,

l_{BS_2} is arbitrarily fixed and $l_{P_2S_2}$ is obtained from the above condition.

$$m_{B2} = {m_2 l_{P_2S_2}}/{(l_{BS_2} + l_{P_2S_2})}$$
$$m_{P2} = {m_2 l_{BS_2}}/{(l_{BS_2} + l_{P_2S_2})}$$

For link 1 to be dynamically replaced by two point masses the condition to be satisfied is

$$k_1^2 = l_{AS_1} l_{P_1S_1}$$

where,

k_1 is the radius of gyration of link 1 about its center of mass,

l_{AS_1} is arbitrarily fixed and

$l_{P_1S_1}$ is obtained from the above condition.

$$m_{A1} = {m_1 r_{P_1S_1}}/{(l_{AS_1} + l_{P_1S_1})}$$
$$m_{P1} = {m_1 l_{AS_1}}/{(l_{AS_1} + l_{P_1S_1})}$$

$$m_I = {m'_1 l_{IS_5}}/{l_{IJ}}$$
$$m_J = {m'_1 l_{JS_5}}/{l_{IJ}}$$
$$m_{CW_{11}} = {m_I l_{OI}}/{r_{CW_{11}}}$$
$$I'^{*}_{S_1} = I'_{S_1} - m'_1 l'_{JS_1} l_{IS_1}$$
$$m_{CW_2} = {(m_2 l_{AS_2} + m_{B3} l_{AB} + m_J l_{AJ})}/{r_{CW_2}}$$
$$m_{CW_1} = {((m_2 + m_{B3} + m_J + m_{CW_2}) l_{OA} + m_1 l_{OS_1})}/{r_{CW_1}}$$

Where, $m_{CW_{11}}$ is the counterweight attached against point mass m_I

$r_{CW_2} = (l_{P_2S_2} - l_{AS_2})$, is the radius of rotation of counterweight m_{CW_2}, and $r_{CW_1} = (l_{P_1S_1} - l_{OS_1})$ is the radius of rotation of counterweight m_{CW_1}

3.1.2 Shaking moment balancing of the mechanism

The shaking moments generated by links 1, 2, 4 and 5 are given in eq. (13).The links 2 and 4 are not directly connected to the frame, the geared inertia counterweights required to balance the shaking moments of these two links are mounted on the base of the mechanism, by kinematically linking them to the corresponding links by links of known mass and center of mass.

The shaking moment generated by the linkage is determined by the sum

$$M^{int} = M_1^{int} + M_5^{int} + M_2^{int} + M_4^{int}$$
$$M_1^{int} = (I_{S_1} + m_1 l_{OS_1}^2 + (m_{CW_2} + m_J + m_2 + m_{B3}) l_{OA}^2 + m_{CW_1} r_{CW_1}^2 + I'^{*}_{S_1} + m'_1 l_{OS_1}^2)\alpha_1$$
$$M_5^{int} = (I_{S_5} + m_5 l_{FS_5}^2 + (m_{CW_4} + m_H + m_4 + m_{D3}) l_{EF}^2 + m_{CW_5} r_{CW_5}^2 + I'^{*}_{S_5} + m'_5 l'^2_{FS_5})\alpha_5$$
$$M_2^{int} = (2 m_I l_{OI}^2)\alpha_2$$
$$M_4^{int} = (2 m_G l_{FG}^2)\alpha_4 \qquad (13)$$

where, M_1^{int}, M_5^{int} are the shaking moments of rotating links 1 and 5 respectively

I_{S_1}, I_{S_5} are mass moments of inertia of links 1 and 5 about their centers of masses respectively

$I'^{*}_{S_1}, I'^{*}_{S_5}$ are the changed moments of inertia of links 1′,5′ respectively

$\alpha_1, \alpha_2, \alpha_4, \alpha_5$ are the angular accelerations of links 1,2,4 and 5 respectively

For shaking moment balancing 8 gear inertia counterweights are used, four at F and four at O.

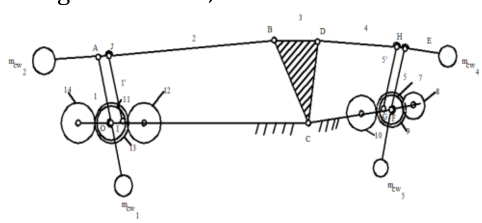

Fig.5 Balanced Watt mechanism with three fixed points

3.2 Watt mechanism with two fixed points

The Watt mechanism with two fixed points is obtained when one of the binary links in the basic Watt chain is fixed. This mechanism is generally used in steam engines. In the Watt mechanism with two fixed points shown in fig.6, links 2and 3 are ternary links and all other links are binary links. The balanced Watt mechanism with two fixed points is shown in fig.7

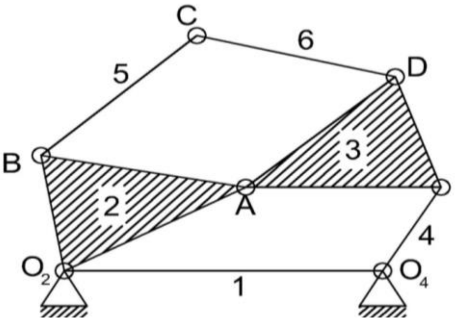

Fig.6 Watt Mechanism with two fixed points

3.2.1 Shaking force balancing of the mechanism

Here the link 2 is dynamically replaced by three point masses $m_{A_2}, m_{B_2}, m_{O_2}$ by using the following conditions

$$\begin{bmatrix} 1 & 1 & 1 \\ l_{O_2}e^{i\theta_{O_2}} & l_A e^{i\theta_A} & l_B e^{i\theta_B} \\ {l_{O_2}}^2 & {l_A}^2 & {l_B}^2 \end{bmatrix} \begin{bmatrix} m_{O_2} \\ m_{A_2} \\ m_{B_2} \end{bmatrix} = \begin{bmatrix} m_2 \\ 0 \\ I_{S_2} \end{bmatrix}$$

$$m_{O_2} = \frac{D_{O_2}}{D_2}, m_{A_2} = \frac{D_{A_2}}{D_2}, m_{B_2} = \frac{D_{B_2}}{D_2} \qquad (14)$$

Where l_{O_2}, l_A, l_B are the moduli of radius vectors of corresponding points.
$\theta_{O_2}, \theta_A, \theta_B$ Are the angular positions of radius vectors
m_2 Is mass of link 2
I_{S_2} Is the mass moment of inertia of link 2 about its centre of mass
$D_{O_2}, D_{A_2}, D_{B_2}$ And D_2 are the third order determinants obtained from the system of equations

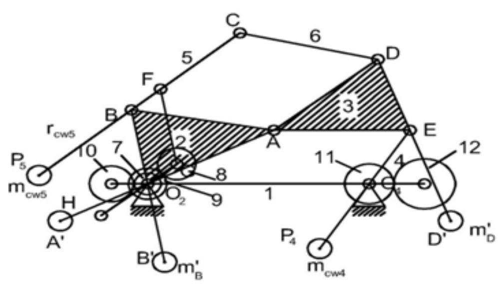

Fig.7 Balanced Watt mechanism with two fixed points

For link 6 to be statically replaced by the point masses m_{C_6} and m_{D_6}

$$m_{C_6} = \frac{m_6 l_{DS_6}}{l_{CD}}$$

$$m_{D_6} = \frac{m_6 l_{CS_6}}{l_{CD}}$$

Changed mass moment of inertia $I'^{*}_{S_6} = I'_{S_6} - m_6 l_{DS_6} l_{CS_6}$

For link 5 to be dynamically replaced by two point masses m_{C_5} and m_{P_5} the condition to be satisfied is ${k_5}^2 = l_{CS_5} l_{P_5S_5}$

Where l_{CS_5} is arbitrarily taken and $l_{P_5S_5}$ is obtained from the above condition

$$m_{C_5} = \frac{m_5 l_{P_5S_5}}{(l_{P_5S_5} + l_{CS_5})}$$

$$m_{P_5} = \frac{m_5 l_{CS_5}}{(l_{P_5S_5} + l_{DS_5})}$$

After link 5 is dynamically replaced by two point masses it is kinematically connected to its corresponding gear inertia counter weight 8 by link 2' more over link 2' is statically replaced by two point masses m_G *and* m_F

$$m_G = \frac{m'_2 l'_{FS_2}}{l_{FG}}$$

$$m_F = \frac{m'_2 l'_{GS_2}}{l_{FG}}$$

Counterweight m_{CW_5} can be obtained as

$$m_{CW_5} = \frac{(m_{C6} l_{BC} + m_F l_{BF} + m_5 \, l_{BS_5})}{r_{CW_5}} \qquad (15)$$

Where $r_{CW_5} = l_{P_5S_5} - l_{CS_5}$ is radius of rotation of counterweight m_{CW_5}

Link 3 is dynamically replaced by three point masses m_{A3}, m_{D3}, m_{E3} by using the following conditions

$$\begin{bmatrix} 1 & 1 & 1 \\ l_A e^{i\theta_{A_3}} & l_A e^{i\theta_D} & l_B e^{i\theta_E} \\ {l_A}^2 & {l_D}^2 & {l_E}^2 \end{bmatrix} \begin{bmatrix} m_{A_3} \\ m_{D_3} \\ m_{E_3} \end{bmatrix} = \begin{bmatrix} m_3 \\ 0 \\ I_{S_3} \end{bmatrix}$$

$$m_{A3} = \frac{D_{A3}}{D_3}, m_{D3} = \frac{D_{D3}}{D_3}, m_{E3} = \frac{D_{E3}}{D_3} \qquad (16)$$

Where l_A, l_D, l_E are the moduli of radius vectors of corresponding points
$\theta_{A3}, \theta_D, \theta_E$ are the angular positions of radius vectors
m_3 is the mass of link 3
I_{S3} Is the mass moment of inertia of link 2 about its centre of mass
D_{A3}, D_{D3}, D_{E3} And D_2 are the third order determinants obtained from the system of equations.
Counterweight against point B of link 2 can be obtained as

$$m'_B = (m_{CW_5} + m_F + m_5 + m_{C6}) l_{O_2B} \Big/ l'_{O_2B}$$

Where l'_{O_2B} is arbitrarily fixed

Counterweight against point A of link 3 can be obtained as

$$m_A^{'} = \frac{(m_{A2} + m_{A3}) l_{O_2A}}{l_{O_2A}^{'}}$$

Where $l_{O_2A}^{'}$ is arbitrarily chosen

Counterweight against point D of link 3 can be obtained as

$$m_D^{'} = \frac{(m_{D3} + m_{D6}) l_{DE}}{l_{DE}^{'}}$$

Where $l_{DE}^{'}$ is arbitrarily chosen

For link 4 to be dynamically replaced by two point masses m_{E4}, m_{P4} the condition to be satisfied is $k_4^2 = l_{ES_4} l_{P_4S_4}$, where l_{ES_4} is arbitrarily is chosen and $l_{P_4S_4}$ is obtained from the above condition

$$m_{E4} = \frac{m_4 l_{P_4S_4}}{(l_{P_4S_4} + l_{ES_4})};$$

$$m_{P4} = \frac{m_4 l_{ES_4}}{(l_{P_4S_4} + l_{ES_4})}$$

Counterweight against link 4 can be obtained as

$$m_{CW_4} = \frac{(m_{E3} + m_{D6} + m_D^{'}) l_{O_4E}}{r_{CW_4}}$$

Where $r_{CW_4} = l_{P_4S_4} - l_{O_4S_4}$ is the radius of rotation of counterweight m_{CW_4}

3.2.2 Shaking moment balancing of the mechanism

The shaking moments generated by links 2, 4 and 5 are given in eq. (17).

The shaking moment generated by the mechanism can be determined by the sum

$$M^{\text{int}} = M_2^{\text{int}} + M_5^{\text{int}} + M_4^{\text{int}} \tag{17}$$

Where

$$M_2^{\text{int}} = (I_{S2} + I_{S2}^{'*} + m_2^{'} l_{GS2}^{'} l_{FS2}^{'} + (m_{A2} + m_{A3}) l_{O2A}^2 + (m_{CW5} + m_F + m_5 + m_{C6}) l_{O2B}^2 + m_B^{'} l_{O2B}^{'2}) \alpha_2$$

$$M_4^{\text{int}} = (I_{S4} + m_4 l_{O4S4}^2 + m_{CW4} r_{CW4}^2 (m_D^{'} + m_{D3} + m_{D6} + m_{E3}) l_{O4E}^2) \alpha_4$$

$$M_5^{\text{int}} = (2 m_G l_{O2G}^2) \alpha_5$$

$M_2^{\text{int}}, M_4^{\text{int}}, M_5^{\text{int}}$ are the shaking moments of rotating links 2,4 and 5 respectively

I_{S_2}, I_{S_4} are the mass moment of inertia of links 2 and 4 respectively

$\alpha_2, \alpha_4, \alpha_5$ are the angular accelerations of links 2,4,and 5 respectively

For shaking moment balancing 6 gear inertia counterweights are used four at O_2 and two at O_4.

Shaking force of the mechanism by the proposed method:

$$F_{Proposed} = -(m_2 A_{G2} + m_3 A_{G3} + m_4 A_{G4} + m_5 A_{G5} + m_6 A_{G6} + m_2^{'} A_{G2}^{'})$$

Shaking moment of the mechanism by the proposed method:

$$M_{proposed}^{\text{int}} = M_2^{\text{int}} + M_4^{\text{int}} + M_5^{\text{int}}$$

Shaking force of the mechanism by Gao Feng's method:

$$F_{Gaofeng} = -(m_2 A_{G2} + m_3 A_{G3} + m_4 A_{G4} + m_5 A_{G5} + m_6 A_{G6} + m_{G8} A_{G8})$$

Shaking moment of the mechanism by Gao Feng's method:

$$M_{Gaofeng}^{\text{int}} = M_2^{\text{int}} + M_4^{\text{int}} + M_5^{\text{int}} + (I_{S8} + 2 m_{G8} l_{FG}^2) \alpha_2$$

Numerical example: The Watt mechanism with two fixed points shown in Fig.6 has the following parameters:

$m_2 = 2kg, k_2 = 0.1198m, m_3 = 1.8kg,$
$k_3 = 0.1178m,$
$m_4 = 7kg, k_4 = 0.237m, m_5 = 2.8kg,$
$k_5 = 0.934m, m_6 = 3kg, k_6 = 0.369m,$
$l_{A3} = 3.7m, l_E = 5.8m, l_D = 5.6m, \theta_A = 0^0,$
$\theta_B = 117^0,$
$\theta_{O_2} = 262^0, l_{O_2} = 5m, l_A = 2.6m,$
$l_B = 3.7,$
$l_{O_2B} = 5, l_{O_2A} = 2.3m, l_{AB} = 9m, l_{BC} = 8m,$
$l_{CD} = 6m, l_{BF} = 2.1m,$
$l_{O_4E} = 9m, l_{DE} = 7m, l_{AE} = 5m, l_{AD} = 2.5m,$
$\theta_E = 0^0, \theta_{A\#} = 208^0, \theta_D = 147^0, m_2' = 0.5kg,$
$\omega_2 = 10rad/s, \alpha_2 = 10rad/s^2$

3.2.3 Comparison between the results of Proposed and Gao Feng methods

The results of shaking force and shaking moment by Proposed method and Gao Feng method for Watt mechanism with two fixed points are shown in tables 1 and 2

Table 1 Shaking force comparison of Watt mechanism with two fixed points

Crank angle(deg)	Shaking force generated in proposed method N	Shaking force generated in Gao Feng's method N
0	2726.43	15285.24
90	1840.32	14399.16
180	791.96	13350.82
270	923.45	13482.31
360	2726.43	15285.24

Table 2 Shaking moment comparison of Watt mechanism with two fixed points

Crank angle(deg)	Shaking moment generated in proposed method *10^5 N-m	Shaking moment generated in Gao Feng's method*10^5 N-m
0	-468.22	-468.19
90	-5.23	-5.18
180	33.17	33.23
270	39.41	39.46
360	-468.22	-468.19

The shaking forces in Watt mechanism with two fixed points are determined at intervals of 90^0. At all positions better results are produced by Proposed method. Shaking force of the mechanism is maximum, 2726.43 N, at 0^0, and minimum, 791.96 N, at 180^0 in the proposed method. The shaking force gradually decreases from maximum at 0^0 to minimum at 180^0 and again gradually increases to maximum at 360^0 .The shaking moment of Watt mechanism with two fixed points is maximum, 468.2x10^5 N-m, at 0^0, and minimum,-5.23x10^5 N-m, at 90^0.The shaking moment gradually decreases from 0^0 to 90^0 and again increases to maximum at 360^0 .

Conclusions

It can be observed from the numerical example that shaking forces by proposed method are very much less at all intervals of crank angle, than that of by Gao Feng's method. As there is only one planetary gear 8 to be mounted on the base of the mechanism, there is a little improvement in the shaking moment balancing, but the shaking forces have been substantially reduced. Though the results of a numerical example are not available in the literature to make a comparison in tables 1 & 2, the balanced mechanisms of both the proposed and Gao Feng methods can be compared construction wise. It can be observed that the balanced mechanism of proposed method is constructively more efficient, compact and occupies less space.

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