International Journal of Current Engineering and Technology E-ISSN 2277 – 4106, P-ISSN 2347 – 5161

Available at http://inpressco.com/category/ijcet

Research Article

Pragmatic Mathematical Perceptions for Judging Role of Diverse Variables during Ferrofluid Based Lubrication of Bearings used in Agricultural Sector

Nitin D. Patel, Murari Lal Gaur* and Manjusha. S. Kulshrestha

Department of Agricultural Meteorology & Agril Engineering, B A College of Agriculture, Anand Agricultural University Anand Gujarat-388120, India

Received 01 Oct 2018, Accepted 02 Dec 2018, Available online 03 Dec, **Vol.8, No.6 (Nov/Dec 2018)**

Abstract

Among various moving constituents of any mechanical system, bearing remains a major element, which governs overall issues related to friction, wear, velocity and mechanical efficiency. Tribologically optimized contacting surfaces plays imperative role to extend the life of bearings as well as materials used in it. This article discusses background practical aspects and relevant mathematical principles for adopting alternative innovative lubricants like ferrofluids on double layered porous rough inclined slider bearing with due consideration of slip velocity and material roughness. The ferrofluid flow model of Jenkins is utilized for assessing its applied aspects to attain tribologically optimized design of bearings with foresighted real applications in agricultural industries. For developing Reynolds's equation, popular models for roughness (Christensen &Tonder) and for slips (Beavers & Joseph) are factored into derivation. Few distinct dimensionless entities like slip parameter, material parameter, standard deviation, variance, skewness, magnetization parameter, porosity of inner layer, porosity of outer layer etc. were found most suitable choices to attempt overall evaluation on their influences on ultimate effectiveness of bearing system. Literature based appraisals on key parameters (mathematical & physical) were cultured by establishing quantified influences of slip velocities, roughness, lubrication etc. on performances of bearing systems with ferrofluid based lubrication. The load carrying capacity gets increased rapidly with increase in film thickness. Variation of load carrying capacity with respect to slip parameter along with skewness, standard deviation, inner & outer layer porosity showed linear effects with parallel shifts for different sets of values. These influences were remained non-linear for material based persuade. Synthetic predictive equations for computing work load factor as a function of magnetization & material parameter, showed satisfactory simulation performances.

Keywords: *Double layered Porosity, Roughness, Pressure, Load, Slider Bearing, Agricultural Equipment.*

1. Introduction

From mechanical point of view a bearing supports the shaft or housing to permit their free motion about an axis of rotation. Load is usually applied to bearings in any two basic directions, firstly the radial loads which act at right angles to the shaft i.e. bearing's axis of rotation, and secondly the axial thrust which acts parallel to the axis of rotation. In recent years increasing attention has been focused on quantification of friction, wear and lubrication for such moving mechanical elements, where these 3 parameters significantly govern the overall efficiency of given machine-driven systems. The magnitude of wastage of resources & energy resulting from high friction & wear remains very high, and thus offering vast potential for saving such losses by proper understanding & analysis of tribological facts. Among various moving constituents of any mechanical system, bearing acts as key portion, which governs & regulates majority of issues related to overall mechanical efficiency.

A variety of bearing are adopted by agro-industrial sector (hydrodynamic bearings, hydrostatic bearings, rolling element bearings etc.) having great influence on reliability, life and power consumption of agricultural machines, tools or equipment's dealing friction, wear, heat generation & its dissipation. It altogether requires a thoughtful apprehension towards modelling of tribological losses, with a right kind of physical & mathematical understandings and suitable deliberations in terms of solutions, which could be properly utilized at the ends of engineers, designers & researchers who so ever is engaged in such industrial interventions.

*Corresponding author's ORCID ID: 0000 0002 1158 3980
DOI: https://doi.org/10.14741/ijcet/v.8.6.11

1.1 Literature based Appraisals

By now, it is well established by relevant researchers that it is roughness of the bearing surfaces that retards the motion of the lubricant, and thus offers adverse results. The random character of the roughness with adopted stochastic approach remains a well-established fact with significant levels of adverse effects (Tzeng and Saibel, 1967). The modified version of above concept (Christensen and Tonder, 1969, 1970) incorporated generalized influences of roughness, by releasing a few salient findings which includes 3 major findings stating that, (i) the influence of surface roughness on the behavior of lubricant film on the load bearing capacity & friction always remains a critical aspect for study of slider bearings, (ii) porous sliders have significant importance in fluid mitigated moving pads, and (iii) there remains a high applications of porous bearings mounted in water pumps, motors, vacuum cleaners, record players, generators, and many general purpose machines in agricultural and non-agricultural sector. Performance evaluation of ferrofluid (as an active lubricating agent) for tackling porous exponential slider bearings under a varied set of experimental conditions (Shah and Bhat, 2004), revealed significant influences of slip velocity. Results altogether advocated the consideration of pooled influences of various issues, while going for an optimum design solution for such kind of bearing systems in any of the general mechanical setup. Similar kind of experimentations and results (Patel and Deheri, 2012) have well established the higher influences of magnetic squeeze film when compared with shapes of bearing elements as well as slip velocity patterns/magnitude. Results derived after comparing 3 magnetic fluid flow models by adopting ferrofluid based rough parallel plate slider bearings (Patel and Deheri, 2016), also supported this phenomenon. Extended tribological based research work on effects of slip velocities by adopting variety of models (Patel and Deheri, 2017) for a bearing set up under ferrofluid based lubrication was a focal point for varied kinds of bearings namely rough porous convex pad slider bearing as well as curved rough porous bearings. Stability analysis of double porous and surface porous layer journal bearing remained another important subject of research to report influences of materials, surfaces and porous layer configurations, both for journal bearings under double layer porous and surface layer porous conditions (Rao et al., 2016) and (Srinivasan, 1977).

1.1.1 Lubrication and hydrodynamics

Lubrication plays an important role to overcome or at least minimize above described losses. From real physics point of view a significant portion of lubricant with high viscous properties allows very smooth relative motion between two moving (sliding/ rotating) surfaces. The recent advancement in tribology has facilitated the use of innovative kinds of lubrication in various mechanized applications in industrial and other sectors. It includes improved fluid film lubrication, hydrostatic lubrication, hydrodynamic lubrication, elasto hydrodynamic lubrication, boundary lubrication and many others. Inventive lubricants too have emerged as a valuable solution for tackling many operational issues of bearings, but still they have their own questions. According to (Mowri, 2011), a prominent US based ball bearing company found that the 54 % cause for their bearing failures remained lubrication-related. Improper bearing lubrication or re-lubrication was reported to accounts even up to 40 to 50 percent of machine failures. It was reported that only across US industries, about $240 billion is lost annually due to downtime and repairs to manufacturing equipment damaged by poor lubrication. Such losses in developing or undeveloped nations dominated with agricultural industries are always expected manifold high, and so India is not an exception. Being an agrarian nation, majority of its population is engaged in art of agriculture and agricultural operations, which has been now a day transformed into a high-tech science and engineering based profession. Variety of electrical & mechanical machineries are being implied in agricultural production & processing chains, which involves bearings, in one or other form.

The annual consumption of lubricants in the world is around 40 million tons out of which more than 60% finishes in the environment with no control. In most cases lubricants remains of mineral origin, toxic and not readily degradable. Moreover, recently it is an encouraging fact that ecological acceptability of lubricants is also being given due importance and concerns, especially in the fields where application of mineral oils can cause damage to environment. Recent trends of research (Stojilkovića and Kolba, 2016) showed that the vegetable oils can well substitute some of mineral oils just because of their good lubricity and biodegradability.

Many inventers (Das, 1998), (Lin et al., 2006) and (Lin and Lu, 2010) have shared their findings on observed optimum load bearing aspects i.e. load bearing capacities for sliders bearings, lubricated with couple of stressed fluids in magnetic field. Ultimate loads and their non-linear variations were found highly sensitive for designing bearing systems having magnetic fluids therein. Such kind of non-linearity and turbulences were considered of high priority by analyzing effects of magnetic fluid-based squeeze films between annual plates and transverse surface, and the influences of overall roughness elements (Snyder, 1963) and (Taylor and Dowson, 1974). The established importance of materials, surfaces, and interfaces provided an in-depth comparison of hydrodynamic principles by adopting ferrofluid lubrication for an inclined rough slider bearing system (Mishra et al., 2018). Importance of design principles, mathematical optimization is being invariably demonstrated to pave a path of futuristic extension of their work. In this connection, some researchers (Agrwal, 1970), (Anwar and Rodkiewicz, 1972), (Chou et al., 2003) and (Christensen and Tonder, 1970) have well reported their salient R&D outcomes by considering magneto hydrodynamic squeeze films, and other hydrodynamic kind of lubrications for slider bearings. Many other important fundamentals of fluid film-based lubrications are already established (Bhat, 2003) and (Hamrock, 1994) for general purpose utilities and understandings on lubrication aspects under magnetic fluids.

Magnetic fluids are often classified as ferrofluids which remains a colloidal suspensions of single-domain magnetic particles, with typical dimensions of about 10 nm, dispersed in a liquid carrier (Neuringer and Rosensweig, 1927). Stability of a ferrofluid in terms of various forces and torques on the magnetic particles is of diverse nature, and it is a bit different from usual magneto-rheological fluids used for dampers, brakes and clutches, formed by micron sized particles dispersed in oil. In such applications the magnetic field causes an enormous increase of the viscosity, so that, for strong enough fields, they may behave like a solid. On the other hand, a ferrofluid often keeps its fluidity even if subjected to strong magnetic fields (even>10 Kg). Porous slider bearings and their ultimate efficiencies in functioning remained solely governed by friction and lubrication aspects. At present, the literature talks of less about double layer porous medium with roughness and ferrofluid lubrication in the bearing systems (Kumar, 1980) and (Murti, 1974). Practically important observations on transformations & micro polar fluid-based lubrication for sets of porous sider bearings were revealed by (Neminath and Gudadappagouda, 2008) where basic Reynolds equation was utilized under a dynamic situation. This study aims to analyses the performance of combined effect of slip velocity and surface roughness on the ferrofluid based squeeze film lubrication in double layered porous inclined slider bearing.

1.1.2 Role of loads and its computations

Slider bearings are mainly designed for supporting the transverse load in any of the engineering system. The performance characteristics of bearing systems have been analyzed by researchers taking various film shapes into consideration (Agrwal, 1986) and (Andjharia et al., 1998). To improve upon the bearing performance many investigations made use of the couple stress fluid model regarding the fluid film lubrication. The fact that use of couple stress fluid increased the load carrying capacity and extended the response time of squeeze film action was well established in the studies reported by (Lin, 2001). Later he analyzed the couple stress effect on the steady state performance of a wide parabolic shaped slider bearing and found that the couple stress effect resulted in an improvement in the steady state performance. The performance analysis of a ferrofluid lubricated plane inclined slider bearing by (Agrawal, 1986), proved that results were relatively better than the corresponding traditionally lubricated bearing. Findings from researchers on exponential slider bearing with a ferrofluid lubricant suggested that the magnetic fluid lubricant caused increased load carrying capacity while the friction remained almost unchanged. Their results revealed that the magnetic fluid sharply increased the load carrying capacity for a squeeze film performance between porous annular disks, confirming the positive impact of magnetic fluid lubrication on the steady state performance of a porous composite slider bearing.

1.1.3 Applications Aspects

The research field of magnetic fluids is a multi-disciplinary area, because Ferrofluids have ample applicability in industrial domain. Researchers have established plenty of ways to synthesize stable magnetic fluids, motivated by the perspective of many and important technological uses. Majority of applications are based on key properties of ferrofluids, like, (i) how it reaches to a location where magnetic field is strongest and how it remains stayed there, (ii) how it absorbs electromagnetic energy at convenient frequencies and get heats up, (iii) how its physical properties gets change with the application of a magnetic field. Researchers have provided greater emphasis on using evolutionary algorithms with optimized design parameters of bearings, which always makes a difference in final performances. Applying mathematical concepts to arrive at optimum materials & design parameters happens to be an important notion for such engineering based versatile product. Friction and load carrying capacity remains two most important aspects for deciding overall effectiveness of bearings, where an optimum lubrication remains inevitable to ensure their satisfactory long operation. An excess of lubrication can be as damaging as a lack of lubrication.

Majority of agricultural machines remains reliant on periodic lubrication of bearings to keep them away from wearing or seizing. Major interventions remain to support rotating machinery, shafts & components for instance wheels on tractors, rolls on hay balers, disk on tillage equipment and gears in transmission. More recently (Patel and Deheri, 2017) analyzed the ferrofluid lubrication of a plane inclined slider bearing with velocity slip. The flow of the ferrofluid was based on Shliomis model, showing that the magnetization could reduce the adverse effect of surface roughness up to some extent when suitable values of slip parameter were in place. Lubrication occurs in many agricultural engines & machines to reduce friction between the moving plates. The basic functions of a lubricant are (i) Friction reduction by maintaining a film of lubricant between surfaces moving with respect to each other and thus preventing damage, and (ii) Heat removal because lubricant acts as a coolant, removing heat generated The scope of the tribological system is of indispensable value in bearing selection for most of the agricultural machines and equipment's with several practical based considerations for assessing magnitude of collective stress which indeed is influenced by factors like nature of the load, nature of the motion, prevailing temperatures & time factors. Abrasive wear in agricultural machines and equipment's is a burning issue, as most of the working components use to be of ground engaging nature having relatively higher roughness and thus requiring smoother rotations in various active parts of the machines. This altogether becomes the domain of bearing and lubrication. Hardness of tillage tool, grain structure and its chemical composition are also the influential factors in determination of wear rate. Agricultural machineries may be considered as a first choice for using such biodegradable lubricants because the machinery is used directly in the environment where the lubricant can easily meet soil, water and plants. Tractors as well as other agricultural machinery usually work in highly specific conditions including extremely high or low temperatures, in different position and slopes, under the influence of thick dust and exposed to different chemical agents (plant protection agents, mineral fertilizers) and often work many hours under full load. In some of the developing countries, the universal tractor transmission oil (UTTO) is highly functional oil used as lubricating agent to deliver functions like, (i) lubrication of gearbox, rear axle and gears, (ii) power transfer and hydraulic system lubrication, and (iii) providing adequate cooling and friction wet brakes.

We use 10 point Cambria Font and the font size must not be changed. The authors are required to format their article according to this.

2. Methodological Portrayal

Lubricants with variable viscosity are assuming importance for their applications in bearing-based operations in machines, which often subjected to high speeds, loads, increasing mechanical shearing forces and continually swelling pressures. Optimum mathematical designs and physics-based understanding of such configurations are discussed below, by accommodating basic issues that truly governs bearing performances.

2.1 Optimal Design versus Conventional Designs

To fully understand the fundamental differences between conventional and optimal design, major issue remains the proper dimensioning (sizing) of bearing components for a given set of utility scenarios/purposes in any machine. What we essentially need is to identify, opt & solve a proper mathematical equation that governs the real working physics at one hand and describes the equality between a stress and an allowable stress at other hand. These stresses are invariably related with net or overall load on specific types of machine/s for which bearings are to be designed and optimized. Common mathematical equations have a certain number of unknowns (usually geometric dimensions) and many potential solutions. To solve such an equation the designer is usually required to choose only one of these unknowns (main unknown) and consider as known all the others (giving them concrete values using its own experience and indications in the literature) or to express them as a function of the main unknown. Often the unknown what is meant to be removed is expressed as a product of the main unknown and a coefficient for which there are indications (within limits, sometimes very large) in the literature. In this way, a single-unknown equation is obtained and solved without any difficulty. Unfortunately, such an approach represents only the solving of the initial problem within a hyper-plane of the solutions space of the equation with several unknowns. The important constraints that one must take into consideration refer to the economic, technological, assembling, material aspects of the design problem. It should be also discussed another aspect of the problem. In recent years there has been an explosive development of CAD software tools that enable a thorough analysis of the state of stresses and strains occurring indifferent designed parts. It is because of many factors that governs the overall matrix of choices in this regard. These factors include entities like friction, roughness, lubrication, dimensional analysis & design and several such factors.

2.2 Mathematical & Physical basics on Slider Bearings

From mathematical point of view the ultimate momentum equations for ferrofluids requires special care as it exhibits significant magnetic stresses. Equally vital remains the velocity profiles, pressure differential equation, and relevant boundary conditions, which needs to be made equivalent to Reynolds equation.

2.2.1 Basics of slider bearing

Slider bearings are often encountered in all engineering applications, including agricultural machines and tools. They support and guide the parts movably opposed to each other as well as absorb and transfer the occurring forces. Variety of tractors, farm machineries, irrigation pumps & motors, mechanical & electrical gadgets in crop processing & food processing plants and industries; adopts such bearings in one or other form. These can be more popularly seen in shafts of motors and pulleys and other power transmissions. Most usually such type of bearing consists two plates for which the upper late is inclined to the lower plate at a very small angle. These plates are to be sees as having infinite length so that flow in between these can be considered as a horizontal flow remaining dependent upon the vertical distance alone. Bearings are broadly categorized into two types, fluid film and rolling contact type. In fluid film bearing entire load of shaft is carried by a thin film of fluid present between the rotating & non-rotating elements, while in rolling contact bearings, the rotating shaft load is carried by a series of balls/rollers placed between rotating & non-rotating elements. Fluid film bearings may be sliding contact type, journal bearing, thrust bearing, or slider bearing, while rolling contact type bearings may be either ball bearing or roller bearing.

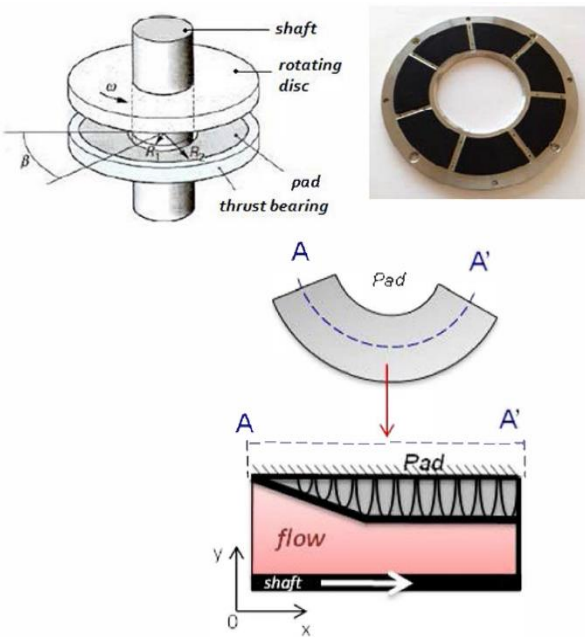
2.2.2 Lubricants

Slider bearings necessitate multiple kinds of fluids as lubricants, where key property of lubricant remains its viscosity. To prevent undesirable viscosity, change with temperature, the use of electrically or mechanically conducting fluid has received a great attention in recent times, which have a higher thermal conductivity, but lower viscosity than convectional lubricants, conducting away the generated heat. But such low viscous property would yield a disadvantageous situation in terms of reduced load-carrying capacity, which can be easily improved by applying external electromagnetic field. Motion of electrically conducting lubricant across electromagnetic field induces electrical field intensity, results in current density which interacts with magnetic field to produce Lorentz force acting on the lubricant. This force may produce a component opposite to direction of motion by properly orienting applied magnetic field. As a result, film pressure is increased. Physio-mathematical principles are analyzed on ferrofluid based squeezed lubrication in slider bearings.

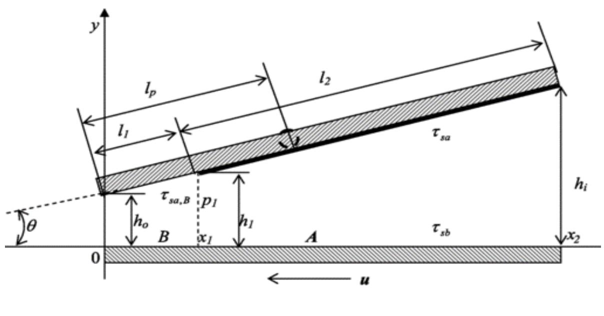
Classically it was who first time carried out extensive experimental investigation and showed the dependence of friction on viscosity of lubricant, load and dimensions of the journal bearing. His experimental investigations form the background of the hydrodynamic theory. Later, some researcher conducted experiments and published the findings in the form of present day hydrodynamic theory of lubrication. Corresponding mathematical equation known as Reynolds' equation is given below, in which 'U' is surface speed of the wedge (in x-direction), 'p' is pressure (at any point in the film in x, z direction), μ is absolute viscosity of the lubricant, and 'h' is film thickness (measured in y-direction).

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial z}\right)=\frac{U}{2}\frac{\partial h}{\partial x}$$

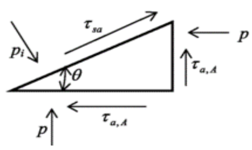
The left-hand side of the equation represents flow under the pressure gradient, while the right-hand side a pressure generation mechanism. In this equation it was assumed that the lubricant is incompressible & Newtonian, the wedge shape to be a straight profile., bearing is to be very long in the Z direction, and variation of pressure are in X and Z direction. In modern industry, mechanical parts are subjected to friction and wear, leading to heat generation, which affects the reliability, life and power consumption of machinery. To overcome the tribological losses due to friction and wear, a significant portion of lubricant with high viscous properties allows very smooth relative motion between two sliding surfaces. Advancement in modern tribology has facilitated the use of applying variety of lubricants as against the traditional ones. It includes options in terms of ferrofluid based lubrication and many innovative solid lubricant additives having high viscous thin film formation between the sliding surfaces which can adequately wet and adhere to a work surface. MoS¬2, graphite, and boric acid are some of the examples as used by design engineers in industries.

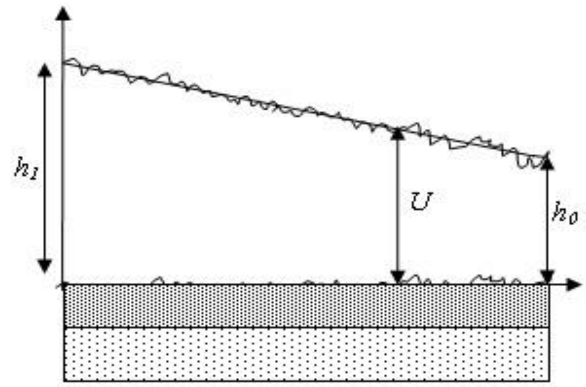


(A) Elementary active physical components

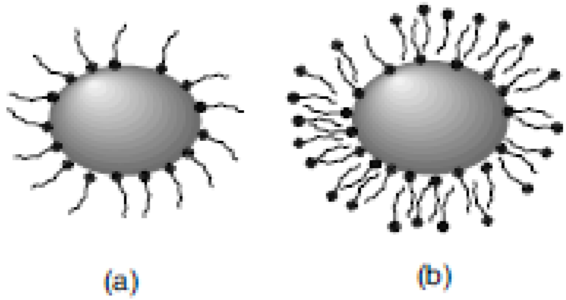


(a) Configuration of the bearing





(B) Active force ingredients



(C) Surfacted ferrofluid grains : 'a' single-layered grains; 'b' double-layered grains

Fig.1 (A, B, C) Physical & mathematical attributes during ferrofluid based lubrication in slider bearings

3. Results and Discussion

Some of the key results which has close resemblance with theme of present article are made part of this segment, along with a detailed physico-mathematical perception and the derivation of governing equations with dimensional treatments for obtaining various results to compare many such dimensionless entities to quantify their ultimate influences on load carrying capacities under different sets of conditions.

3.1 Relevant Clues/Results from Literature

Plethora of real as well as numerical experimentations are conducted by researchers to offer variety of results which altogether revealed positive influences of ferrofluid based bearing systems, offering better performance in terms of capabilities to bear higher loads at varied speeds, maximum film pressure (about 60 – 70 % more than that of conventional lubricant based bearing system), reduced temperature rise, and many other factors. Moreover, the physics & mathematics of' bearing system with smarter lubricants' remains a rising issue, inviting appropriate understanding towards its relevant physic-mathematical contexts.

3.1.1 Physico-mathematical perspective

Looking upon the applicability of fluid dynamics in general bearing systems (without magnetic lubricates), fundamentally there remains only 3 forces viz. (a) pressure gradient (b) gravity force and (c) viscous force, which regulate or define equation of motion in a manner that sum of gradients of all such forces remains equal to rate of change of velocity multiplied by material density. However, in case of ferro-hydrodynamics the magnetic body force too acts including four interparticle forces viz. magnetic attraction, vander waals attraction, steric repulsion and electric repulsion. From published results it was learned that magnetic body force can be taken care by making consideration that it gets originated from interaction of magnetic field with magnetization of fluid and thus being highly important. Functional aspects of such lubricants are taken care by famous R. E. Rosensweig model. Reviewed results from other researchers have well exposed the fact that performance of bearing system can be amply improved by opting appropriate values for magnetization parameter and slip coefficient under skewed roughness, hence ferrofluid based lubrication as a well-established option for effective functioning of bearings. The geometry and configuration of the ferrofluid lubricant revealed the facts that pressure distribution and distribution of load carrying capacity in bearings are solely governed by many parameters & characteristics from material and mathematical points of view.

3.1.2 Alternative materialistic options

The most important task of any lubricant remains to keep separate the parts which are moving relative to one another (balls or rollers and raceways) and thus minimize friction and prevent wear. From material point of views, there remain many innovative options to attain a thin film between the contact areas in a bearing. They often affect maximum running speed, temperature, torque level, noise level and, ultimately, the life of bearing. Reviewed results on use of bio-degradable as well as magnetized materials as a lubricant along with certain aspects of hydrodynamic lubrications in bearing related physics, are found to have great untapped potential. For agricultural machineries (tractors, farm equipment, tools, pumps, motors etc.) and other industrial based applications, use of lubricant remains extremely imperative issue, as its ultimate physicochemical properties governs the overall success of machine efficiencies (highest load carrying with least frictional losses & wears). More recently (Stojilkovića and Kolba, 2016) researched certain vegetable oils that are emerging as an effective lubricant, meeting all the due levels of desired properties.

Table 1 Desired Properties

Physical-chemical features	Unit	Alternate Lubricant Sources		
		Rapeseed oil	Soybean oil	Sunflower oil
Density at 20⁰C	g/cm³	0.92	0.92	0.92
Kinematic viscosity, at 40⁰C	mm²/s	34.07	32.59	35.6
Kinematic viscosity at 100⁰C	mm²/s	7.84	7.66	7.75
Index viscosity	-	213	217	196
Flash point	⁰C	322	326	328
Pour point	⁰C	-13	-8	-14
Iodine number	mgKOH /g	118.41	126.2	131.2
Contents of fatty acids	-	Varied widely	Varied widely	Varied widely

3.2. Resulting Governing Equations

Major governing equations that deals the concept of conservation of mass and momentum, adopts two fundamental propositions, firstly the mass can neither be created nor destroyed i.e. mass of fluid is conserved, and secondly the flow is continuous i.e. empty spaces do not occur between particles which are in contact. In case of bearing and its lubrication fluid is assumed to be incompressible fluid i.e. density is constant. Equation of momentum mathematically guides the principle of conservation of momentum by applying Newton's second law of motion to a mobile element of lubricant fluid, where net rate of momentum flow is taken as equal to the net sum of forces acting on the fluid. Considering above basic postulates, for ferrofluid based squeeze film lubrication (in double layered porous inclined slider bearing), roughness & slip characteristics are visualized in in Figure 1, where physical configurations of all such forces & elements of bearing system are illustrated.

The film thickness for above bearing system was considered as that of [5], by adhering to following mathematical shape,

$$\bar{h} = a-(a-1)X\ ,\quad a=\frac{h_1}{h_0}\ ,\ \bar{h}=\frac{h}{h_0}\ ,\ X=\frac{x}{A}$$

Where, $\bar{h}$: Uniform fluid film thickness (mm), h_0 : Fluid film thickness at x=0, X : coordinate of the center of pressure, and A : Length of the bearing.

The porous regions are assumed to be homogeneous & isotropic, and the lubricant as an incompressible Newtonian fluid. Slip model of [4] was adopted, while model given by [11] was considered to describe the flow of a magnetic fluid. Accordingly, the equation for equation of these models for steady flow were taken as follows,

$$\rho(\bar{q}.\nabla)\bar{q}=-\nabla p+\eta\nabla^2\bar{q}+\mu_0(\bar{M}.\nabla)\bar{H} \qquad (1)$$

$$\rho(\bar{q}.\nabla)\bar{q}=-\nabla p+\eta\nabla^2\bar{q}+\mu_0(\bar{M}.\nabla)\bar{H}+\frac{\rho\alpha^2}{2}\nabla\left[\frac{\bar{M}}{M}\{(\nabla\bar{q})\bar{M}\}\right] \qquad (2)$$

$$\nabla.\bar{q}=0 \qquad (3)$$

$$\nabla\times\bar{H}=0 \qquad (4)$$

$$\bar{M}=\bar{\mu}\bar{H} \qquad (5)$$

$$\nabla.(\bar{H}+\bar{M})=0 \qquad (6)$$

Using equations (4) and (5) the form of equation (1) can be altered as follows,

$$\rho(\bar{q}.\nabla)\bar{q}=-\nabla\left(p-\frac{\mu_0\bar{\mu}H^2}{2}\right)+\eta\nabla^2\bar{q}+\mu_0(\bar{M}.\nabla)\bar{H} \qquad (7)$$

Keeping in view the popular Neuringer - Roseinweig model for magnetic fluid flow and the stochastic averaging model of [8] along with their inherent assumptions on hydromantic lubrication; the ultimate Reynolds' type equation for pressure distribution is arrived as follows,

$$\rho(\bar{q}.\nabla)\bar{q}=-\nabla\left(p-\frac{\mu_0\bar{\mu}H^2}{2}\right)+\eta\nabla^2\bar{q}+\mu_0(\bar{M}.\nabla)\bar{H} \qquad (8)$$

Where

$$K(h)=\left\{\begin{matrix} h^3+3\alpha h^2+3(\alpha^2+\sigma^2)h+ \\ 3\sigma^2\alpha+\alpha^3+\varepsilon+12\phi_1h_1+12\phi_2h_2\end{matrix}\right\}\frac{(4+sh)}{(2+sh)} \qquad (9)$$

The magnitude of the magnetic field (H^2) was derived by using a simpler but effective relationship i.e. $H^2=KA^2\sin(\pi X)$.

3.2.1 Dimensional analysis

Studying relationships between physical quantities with the help of their dimensions and units of measurements, was taken as a sound base for attaining superior solutions of above derived governing equation/s. It was well achieved by adhering to fundamental units and principle of dimensional homogeneity that defy analytical solution and be solved via numerical experimentations. In many of the studies on bearing and related mechanical aspects, accomplishing non-dimensional forms of respective governing equations is established as a good practice to study the relative influences of multiple variables. In present study it is achieved by selecting certain characteristic quantities and then substituting suitable entities to make them dimensionless. About 12 such distinct dimensionless quantities were worked out in this study, whose elaborative expressions are presented provided below,

$X = \dfrac{x}{A}$	$\mu^* = \dfrac{K\mu_0\bar{\mu}h_0^2 A}{\eta U}$	$\bar{s} = sh_0$
$\beta = \dfrac{h_0^3}{2\dot{h}_0 A}$	$\bar{\alpha}'^2 = \dfrac{\rho\alpha^2\bar{\mu}A\sqrt{K}}{2\eta}$	$\bar{\psi}_2 = \dfrac{\phi_2 h_2}{h_0^3}$
$\bar{\varepsilon} = \dfrac{\varepsilon}{h_0^3}$	$P_{mn} = \dfrac{h_0^3 p}{\eta A^2\dot{h}_0}$	$\bar{\alpha} = \dfrac{\alpha}{h_0}$
$\bar{\sigma} = \dfrac{\sigma}{h_0}$	$W_{mn} = \dfrac{h_0^3 w}{\eta A^4\dot{h}_0}$	$\bar{\psi}_1 = \dfrac{\phi_1 h_1}{h_0^3}$

Appropriate boundary conditions were carved of, which remained as follows,

$$P_{mn}(1) = P_{mn}(\alpha) = 0 \tag{10}$$

Using above relationship, the equation (8) gets its outcomes in following form,

$$\frac{d}{dx}\left[P_{mn} - \frac{1}{2}\mu^* X(1-X)\right] = \left\{\frac{6}{K(\bar{h})}\left(T_1\frac{(2+\bar{s}\bar{h})}{(1+\bar{s}\bar{h})}\bar{h} - \beta^{-1}X + T_2 Q\right)^* \left(1-\bar{\alpha}'^2\sqrt{X(1-X)}\right)\right\} \tag{11}$$

where,

$$T_1 = 3\left(\bar{\alpha}^2 + \bar{\sigma}^2\right),$$

$$T_2 = \left(3\bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^3 + \bar{\varepsilon} + 12\bar{\psi}_1 + 12\bar{\psi}_2\right),$$

$$K(\bar{h}) = \left\{\bar{h}^3 + 3\bar{\alpha}\bar{h}^2 + T_1 + T_2\right\}\frac{(4+\bar{s}\bar{h})}{(2+\bar{s}\bar{h})}$$

By integrating equation (11) and then adopting the boundary conditions (10), the realistic expression for the dimensionless pressure distribution was found in following form,

$$P_{mn} = \frac{1}{2}\mu^*\sin(\pi X) + \left[\left\{6\int_0^1 \frac{1}{K(\bar{h})}\left(T_1\frac{(2+\bar{s}\bar{h})}{(1+\bar{s}\bar{h})}\bar{h} - \beta^{-1}X + T_2 R\right)\right\}^* \left(1-\bar{\alpha}'^2\sqrt{X(1-X)}\right)dX\right] \tag{12}$$

Where,

$$R = -\int_0^1 \frac{\frac{1}{K(\bar{h})}\left(T_1\frac{(2+\bar{s}\bar{h})}{(1+\bar{s}\bar{h})}\bar{h} - \beta^{-1}X\right)\left(1-\bar{\alpha}'^2\sqrt{X(1-X)}\right)dX}{\frac{1}{K(\bar{h})}\left(1-\bar{\alpha}'^2\sqrt{X(1-X)}\right)dX}$$

Ultimately the mathematical formulation for load carrying capacity (W_{mn}) in its dimensionless form was attained as follows,

$$W_{mn} = \frac{2\mu^*}{\pi} - \left[\left\{6\int_0^1 \frac{X}{K(\bar{h})}\left(T_1\frac{(2+\bar{s}\bar{h})}{(1+\bar{s}\bar{h})}\bar{h} - \beta^{-1}X + T_2 R\right)\right\}^* \left(1-\bar{\alpha}'^2\sqrt{X(1-X)}\right)dX\right] \tag{13}$$

A rigorous numerical analysis was attempted to visualize the eventual influences of various dimensionless mathematical entities (i.e. independent variables) on the level of load bearing capacity attained. Many specific permutation and combinations were synthesized for visualizing the individual influences of certain key variables like μ^* , $\bar{s}$, $\bar{\alpha}$, $\bar{\psi}_1$, $\bar{\psi}_2$ and $\bar{\beta}$ under diverse sets other associated variables. The crystal-clear scenario in regards to these combinations or conditions is well reflected in Figures 2 to 5 with pictorial illustrations on achieved numerical ranges adopted therein. The detailed arguments and analysis of results are presented in below given segment of manuscript, which revealed that the slip and material-based attributes remains most sensitive and effective to govern the ultimate load carrying capacities of bearing systems as adopted in this piece of research work. It provides a strong line of sight for futuristic design considerations of various kinds of bearings and lubrication systems where utilities of innovative lubricants could be further enhanced by attaining highest load carrying capacities with least losses in terms of frictions, heat or slippage.

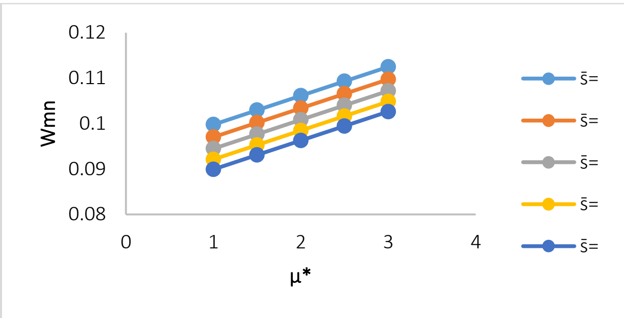
3.3 Influence of diverse parameters on load bearing capacity of bearings

Though there might be many other indicators to establish the success and usefulness of bearing systems with innovative lubrication agents like ferrofluids, we have concentrated our efforts to visualize the quantified influences of plethora of independent variables on load carrying capacities as attained with various sets of configurations. Some of the major results are discussed below,

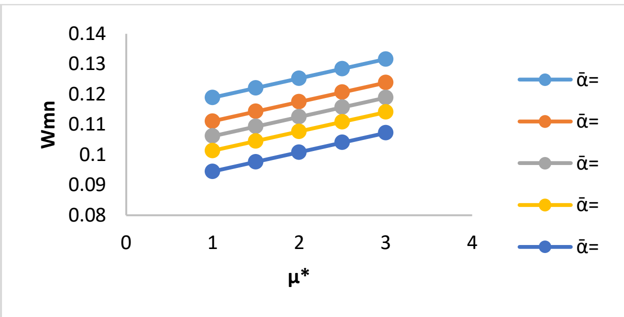
In general, it was observed from the equation of load carrying capacity that as compared to traditional lubrication there is a load increase of $\frac{2\mu^*}{\pi}$ due to Ferrofluid lubrication. The linearity of the expression in with respect to μ^* tends that the load will gets increase with increasing values of μ^*.

3.3.1 Variation of load carrying capacity with respect to μ^*

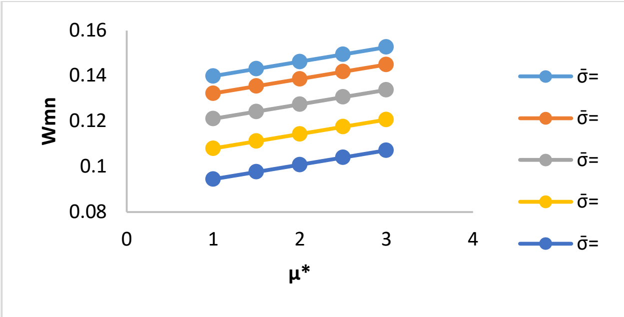
The variation of load carrying capacity with respect to magnetization parameter displayed in Figures 2 indicates that the more magnetization tends to increase the load carrying capacity significantly. When an overall variation of 66.7% was given in magnetization parameter along with above 12% variations in slip velocity, the net variations in load carrying capacity comes to the tune of 18%. The pictorial trends of these variations across various values are depicted in part ‘a’ of Figure 2. For the same set of magnetization parameter when the variance values were changed by 10.7%, the resultant increased in load carrying capacity was attained 16%. Similarly, when a net variation of 9.5 and 13% were imposed on standard deviation and outer layer porosity, the increase in magnitude of load carrying capacity were remained as 19 and 14% respectively. In case of material parameter a net variation of about 9% was given, which yielded about 14% higher load carrying capacity. The overall graphical illustrations on above cited results are visible in Figure 2 (a-e), which are self-explanatory to reflect magnitudes as well as ranges of numerical values as adopted under different configurations of analysis.



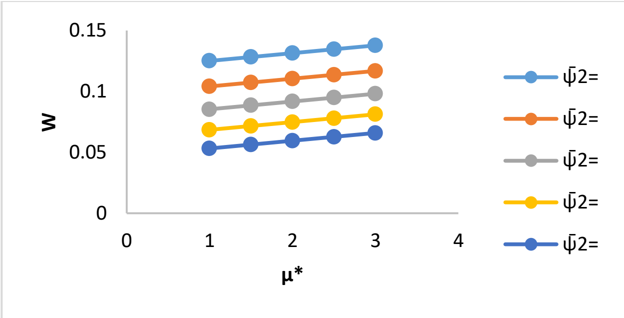
(a) Under varied values of $\bar{s}$



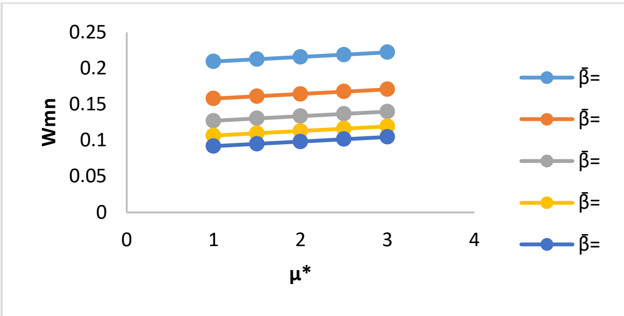
(b) Under varied values of $\bar{\alpha}$



(c) Under varied values of $\overline{\sigma}$



(d) Under varied values of $\overline{\psi}_2$

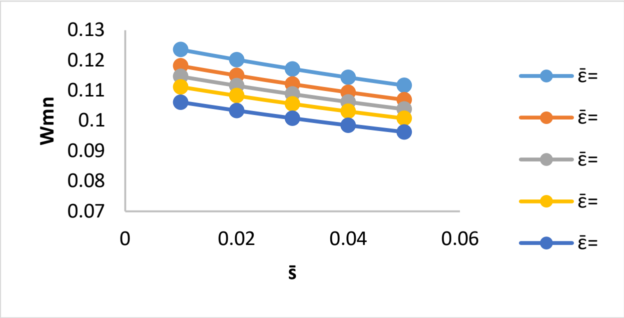


(e) Under varied values of $\overline{\beta}$

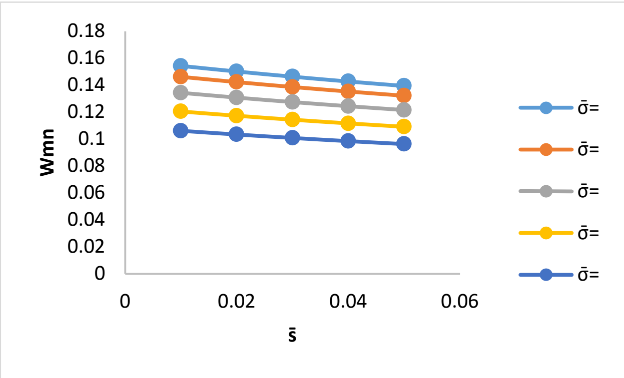
Fig.2 Variation of load carrying capacity with respect to μ^* under varied ranges of five most relevant variables as used in governing mathematical equations

3.3.2 Variation of load carrying capacity with respect to $\overline{s}$

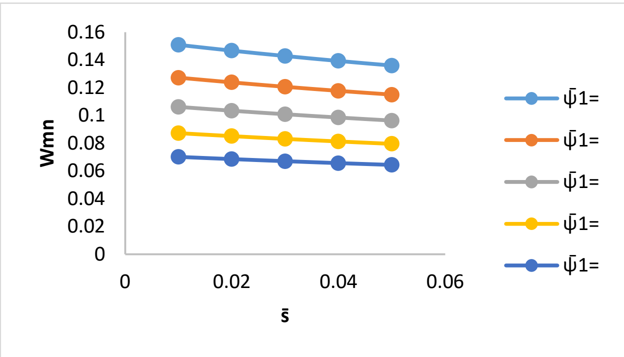
When slip velocity was taken as primary independent variable along with four others (skewness, standard deviation, inner & outer layer porosity) to evaluate net effects on dependent parameters i.e. load carrying capacity; the influences were more likely linear with parallel shifts for different sets of values. Results revealed that for a net variation of about 80% in slip velocity, the corresponding disparity in above cited four additional parameters were kept as 10.4, 10.5, 10.2 and 10.2%. The overall influences of these variations were found as about 12% reductions in load carrying capacity. The exact trends, values and relativity of these changes are well reflected in Figure 3(a-d).



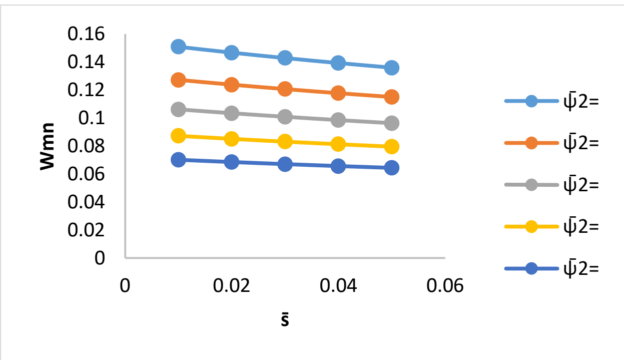
(a) Under varied values of $\overline{\varepsilon}$



(b) Under varied values of $\overline{\sigma}$



(c) Under varied values of $\overline{\psi}_1$

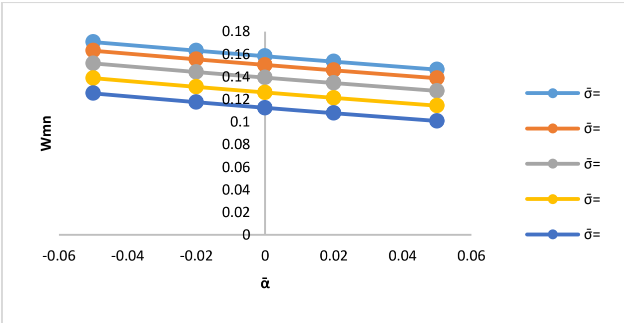


(d) Under varied values of $\overline{\psi}_2$

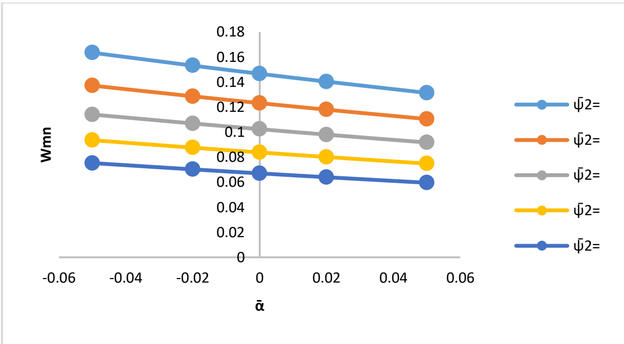
Fig.3 Variation of load carrying capacity with respect to $\bar{s}$ under varied ranges of four most relevant variables as used in governing mathematical equations

3.3.3 Variation of load carrying capacity with respect to $\overline{\alpha}$

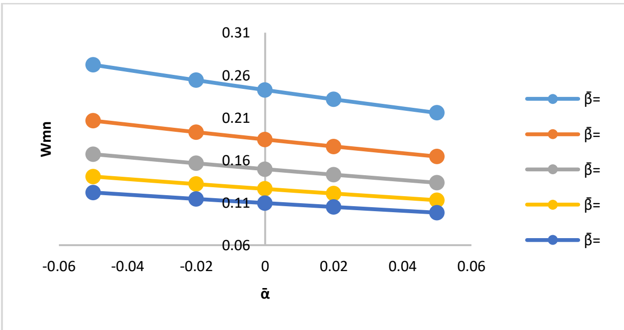
The variance was found to be an effective indicator to influence load carrying capacity of bearing system as adopted in present study. A net variations of about 200% was imposed by considering it as a primary independent variable along with other three (standard deviation, outer layer porosity and material parameter). Resultant effects on load carrying capacity were found equally significant but negative in nature to provide reductions of 8, 10 and 10% respectively. The exact effects and trends of deviations are illustrated in Figure 4 (a-c).



(a) Under varied values of $\overline{\sigma}$



(b) Under varied values of $\overline{\psi}_2$

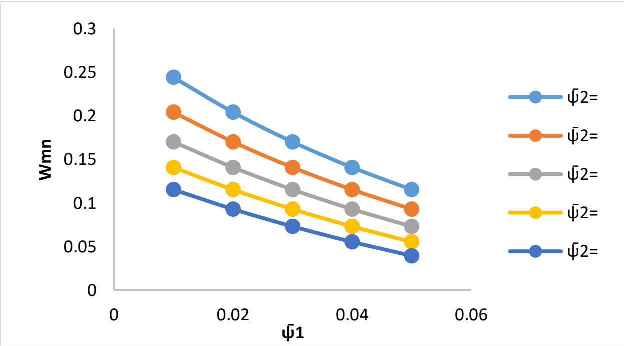


(c) Under varied values of $\overline{\beta}$

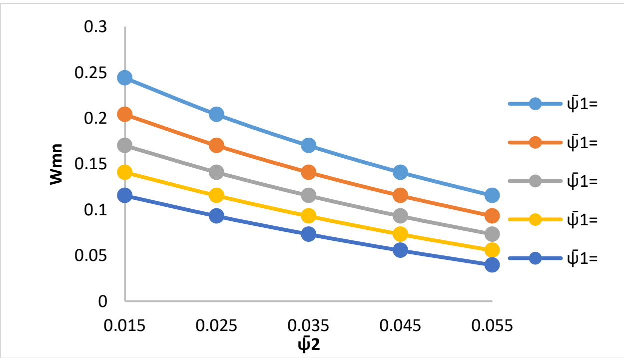
Fig.4 Variation of load carrying capacity with respect to $\overline{\alpha}$ under varied ranges of three most relevant variables as used in governing mathematical equations

3.3.4 Variation of load carrying capacity with respect to $\overline{\psi}_1$, $\overline{\psi}_2$ and $\overline{\beta}$.

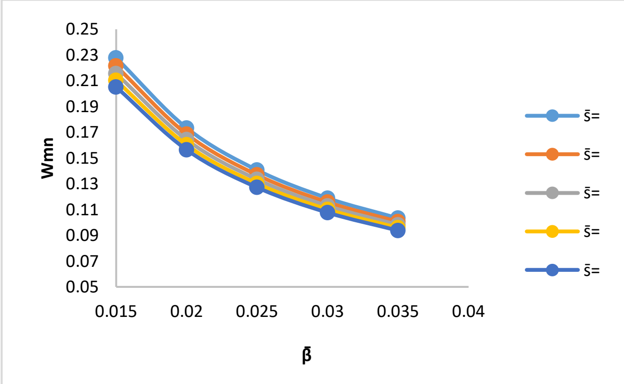
Three vital entities namely inner layer porosity, outer layer porosity and material parameter, happens to be another important group of variables to govern the ultimate load carrying capacity. Results showed that the behavior of these specific attributes was relatively nonlinear in contrast to sets of variables considered in preceding descriptions. Considering this fact the effects of these three variables was assessed by taking them as individual independent parameters of primary nature, with only one additional independent parameter. Results showed that when inner layer porosity was given overall variations of 80% along with 133% variations in outer layer porosity the net effect on load carrying capacity were found to be declined by about 71%. Similarly when a variation of about 73% was adopted in outer layer porosity along with different values of inner layer porosity having net variation of 133% the corresponding load carrying capacity was reduced by 78%. In third case when material parameter values were given a net variation of about 57% along with 120% variations in slip velocity, the equivalent effects on load carrying capacity emerged to the tune of about 95% reductions. Figure 5(a-c) provides a detailed scenario in regards to above described changes.



(a) Load carrying capacity with respect to $\overline{\psi}_1$ under varied values of $\overline{\psi}_2$



(b) Load carrying capacity with respect to $\overline{\psi}_2$ under varied values of $\overline{\psi}_1$



(c) Load carrying capacity with respect to $\overline{\beta}$ under varied values of $\overline{s}$

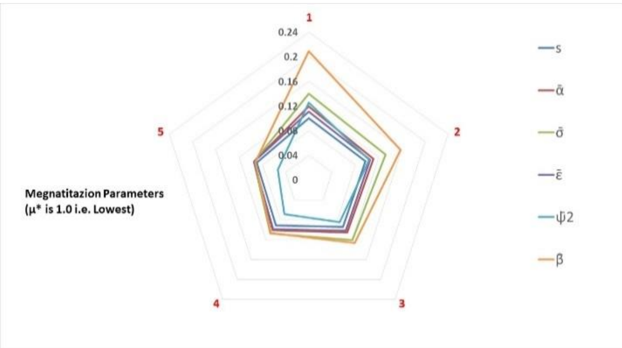
Fig.5 Variation of load carrying capacity with respect to $\overline{\psi}_1$, $\overline{\psi}_2$ and $\overline{\beta}$ under one most relevant individual variable as used in governing mathematical equations

3.3.5 Relative variability of load carrying capacity (Wmn) with pooled influences

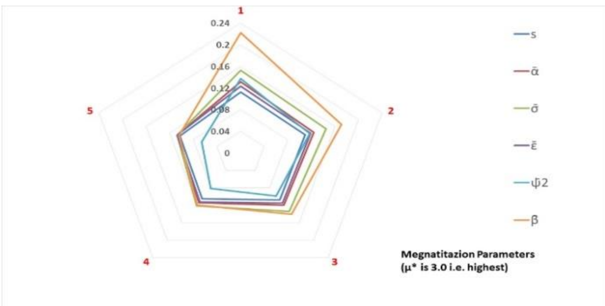
The results described above more or less considered the individualistic influences of various sets of independent parameters on load characteristics for bearing systems as considered in present study. Moreover, an additional effort was made to assess the pooled influence of six different variables for five is specific combinations (shown as notions 1-5) in Figure 6. Two specific conditions were adopted where minimum and maximum values of magnetization parameter (1.0 & 3.0) were considered as constant. For each situation, values of six prime variables (slip velocity, variance, standard deviation, skewness, outer layer porosity and material parameter) were adopted in a pooled manner with pre-decided sets of numerical values as reflected in above mentioned figure. Results reveals that set of conditions in notion 1, gave largest range of variability in Wmn, followed by notion 2 and notion 3. This all together reflects that quantum of net variations under various ranges of variables adopted here in, delivered almost identical trends as well as ranges of changeability for both the conditions (i.e. minimum or maximum value of magnetization parameter).

3.3.6 Predictive equations for computing work load as a function of μ^* & $\overline{s}$ and μ^* & $\overline{\beta}$

As an extended utility of results arrived in this analysis, another effort was made to synthesize some of the simplistic predictive equations for estimating the influence of most sensitive variables as emerged from plethora of results described in preceding segments in first attempt. The pair of magnetization parameter and slip velocity was identified for this purpose, as it gave sizable influences on load carrying capacity as shown in Figure 7 (a). Using preidentified numerical values for varied matrix of & parameters (shown in embedded tabular contents inside the figure). By conducting curve fitting exercise using these two-independent variables simultaneously, a synthetic predictive equation was developed for computing load carrying capacity of bearing (dependent variable). The exact architecture of this specific predictive equation is shown in same figure which was suitably evaluated for its simulation performances. The comparison of original and simulated values of load carrying capacities are illustrated in Fig 5(a), showing a balanced prediction adhering to the trends of variations. Another exercise was performed for developing 2nd such equation by accommodating alternative pair of independent parameter (μ^* & $\bar{\beta}$). The above described end results for this are shown in Fig 5 (b) with equation. The comparison of simulation performance of both the equations were of nature but the magnitude of dispersals was not identical. Dispersals were more in 2nd case showing sensitivity of specific pairs opted herein.



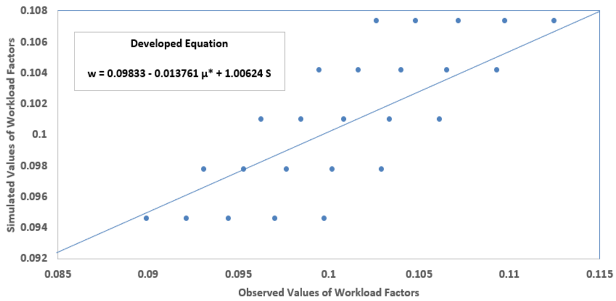
(a) For Lowest Value of Magnetization Parameters (μ^*)



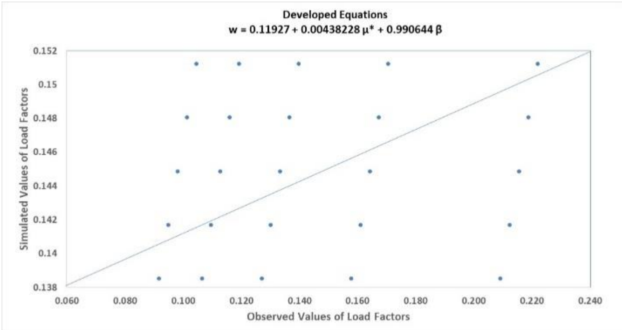
(b) For Highest Value of Magnetization Parameters (μ^*)

Notion	Set of Conditions					
1	$\bar{s}=0.01$	$\bar{\alpha}=-0.05$	$\bar{\sigma}=0.1$	$\bar{\varepsilon}=-0.05$	$\psi\bar{\Sigma}=0.015$	$\bar{\beta}=0.015$
2	$\bar{s}=0.02$	$\bar{\alpha}=-0.02$	$\bar{\sigma}=0.2$	$\bar{\varepsilon}=-0.02$	$\psi\bar{\Sigma}=0.025$	$\bar{\beta}=0.020$
3	$\bar{s}=0.03$	$\bar{\alpha}=0.00$	$\bar{\sigma}=0.3$	$\bar{\varepsilon}=0.00$	$\psi\bar{\Sigma}=0.035$	$\bar{\beta}=0.0125$
4	$\bar{s}=0.04$	$\bar{\alpha}=0.02$	$\bar{\sigma}=0.4$	$\bar{\varepsilon}=0.02$	$\psi\bar{\Sigma}=0.045$	$\bar{\beta}=0.030$
5	$\bar{s}=0.05$	$\bar{\alpha}=0.05$	$\bar{\sigma}=0.5$	$\bar{\varepsilon}=0.05$	$\psi\bar{\Sigma}=0.055$	$\bar{\beta}=0.035$

Fig.6 Relative Variability of Working Load Factors (W_{mn}) as influenced by 6 Different Parameters (cited in Legends) under Varied Range of Magnetization Parameters (μ^*)



(a) Magnetization Parameters (μ^*) & Slip Velocity ($\bar{s}$)



(b) Magnetization Parameters (μ^*) and Material Parameters ($\bar{\beta}$)

Fig.7 Simulation Performance of 'Predictive Equation' Synthesized to Depict Combined Influences of 2 Specific Sets of Parameters ' μ^* & $\bar{s}$ ') and ' μ^* & $\bar{\beta}$ 'on Working Load Factors

4. Summary & Conclusion

In recent years increasing attention has been focused on the quantification of friction, wear and lubrication in various moving components of entire mechanical systems. Bearings remain one of the major constituent in this regard, which is widely adopted by agro-industrial sector in its varied forms like hydrodynamic bearings, hydrostatic bearings, rolling element bearings etc. Improving reliability, life and power consumption of machines largely gets influenced by dominating factors like friction, heat generation and its dissipation inside the bearing system. Literature based appraisals on some of the key parameters (mathematical as well as physical) were reviewed and cultured by establishing the quantified influences of certain physically dominant parameters (slip velocities, roughness, lubrication etc.) on performances of bearing systems having ferrofluid as an active lubricating agent. While assessing the lubrication and hydrodynamics of specified bearing systems, the non-linearity and turbulence aspects for hydrodynamic bearings were found of greater importance and relevance. It becomes inevitable to opt for a correct design configuration which in turn could facilitate decision on proper combinations of bearings and its lubrication system. Suitability of a ferrofluid for bearing systems (in terms of various forces and torques on the magnetic particles) exhibits a diverse nature, being different from the usual magneto rheological fluids used for dampers, brakes and clutches, formed by micron sized particles dispersed in oil. Slider bearings using ferrofluids are found as a better option for supporting the transverse load in any of the engineering system. Role of loads and its computation for various situations is demonstrated to judge the ultimate performance of bearing systems.

Researchers have established plenty of ways to synthesize stable magnetic fluids, motivated by the perspective of many and important technological uses. Majority of such applications remained influenced on key properties of ferrofluids, like, (i) how it reaches to a location where magnetic field is strongest and how it remains stayed there, (ii) how it absorbs electromagnetic energy at convenient frequencies and get heats up, (iii) how its physical properties gets change with the application of a magnetic field. Applicability of ferrofluid based lubrication in bearing systems as used in agricultural industries was assessed, which showed a vast potential to reduce friction between the moving elements by maintaining a film of lubricant between surfaces moving with respect to each other and thus preventing damage, and heat removal. In recent years, increasing attention has been focused on the ferrofluids, as an alternative lubricant, which has its own distinction to offer better lubrication. From physical architectural point of view such fluids are stable colloidal suspension of very fine magnetic particles in a carrier fluid. Physical & mathematical attributes of ferrofluid based lubrication in slider bearings is articulated by presenting basic constituents of relevant hydrodynamic theory of lubrication. Salient results and discussions emerged from this article remained as follows,

- Plethora of real as well as numerical experimentations as conducted by researchers in past have offered variety of results to reveal positive influences of ferrofluid based bearing systems, ensuring better performance in terms of capabilities to bear higher loads at varied speeds, bearing maximum pressure (about 60 – 70 % more than that of conventional lubricant-based bearing system), reducing temperature rise etc.
- From physico-mathematical perspectives the applicability on dynamics of magnetic fluid in bearing systems needs to be considered appropriately not to govern only 3 basic forces (pressure gradient, gravity force, viscous force) but also magnetic force to make a balanced scenario among resultant sum of forces, velocities, material densities.
- There exists many innovative options to attain a thin film between the contact areas in a bearing, which ultimately affect maximum running speed, temperature, torque level, noise level and, ultimately, the life of bearing. Use of bio-degradable oils, magnetized materials etc. can play a big role from lubrication point of view.
- Major governing equations needs to be properly developed and applied before arriving at end designs of bearings related mechanical systems, adhering to prevailing fundamental propositions on physics of bearing & lubricant both.
- Dimensional analysis offers advantageous inferences while studying relationships for attaining advanced solutions derived governing equation/s with proper dimensional homogeneity that defy analytical solution and be solved via numerical experimentations. Some of the distinct dimensionless entities like slip parameter, material parameter, standard deviation, variance, skewness, magnetization parameter, porosity of inner layer, porosity of outer layer etc. were found to be most suitable choices to attempt overall evaluation on their influences on ultimate effectiveness of any bearing based mechanical system. These variables were found to put upon sizeable influences on load carrying capacities of bearings. These influences were remained of linear nature for majority of variables, except material based persuade.
- Variation of load carrying capacity with respect to magnetization parameter displayed that more magnetization tends to increase the load carrying capacity significantly. When an overall variation of 66.7% was given in magnetization parameter along with above 12% variations in slip velocity, the net variations in load carrying capacity comes to the tune of 18%. For the same set of magnetization parameter when the variance values were changed by 10.7%, the resultant increased in load carrying capacity was attained 16%. Similarly, when a net variation of 9.5 and 13% were imposed on standard deviation and outer layer porosity, the increase in magnitude of load carrying capacity were remained as 19 and 14% respectively. In case of material parameter, a net variation of about 9% was given, which yielded about 14% higher load carrying capacity.
- Variation of load carrying capacity with respect to slip velocity along with four other dimensionless variables (skewness, standard deviation, inner & outer layer porosity) showed linear effects with parallel shifts for different sets of values.
- 'Variance' was found to be an effective indicator to influence load carrying capacity of bearing system as adopted in present study. Resultant effects on load carrying capacity were found significant but negative in nature.
- Variation of load carrying capacity with respect to 3 vital entities namely inner layer porosity, outer layer porosity and material parameter, happens to be another important group of variables to govern ultimate load carrying capacity. Results showed that behavior of these specific attributes were relatively nonlinear.
- Synthetic predictive equations for computing work load factor as a function of 'μ* S' and also 'μ* S' gave effective simulation results with higher efficiencies, and deduced that the influence of material parameter dominates in the process.

- The load carrying capacity increases rapidly with increase in the film thickness ratio. The overall food for thought from this study remains that methods and principals of newly emerging science of tribology have great potential to improve the efficiencies and extend service life of bearing materials, and many other parts of same nature. The most critical need remains is tribologically optimized contacting surfaces by (i) identifying critical factors influencing the tribo-system and (ii) identifying solutions to improve efficiency and reducing wear which includes many factors like using friction & wear optimized materials, optimizing material pairings, selecting & using the correct lubricants, and (d) arriving at designs that have a beneficial impact on overall tribo-system performance. Being stable colloidal suspension of very fine magnetic particles in a carrier fluid the ferrofluids has their own distinguish-ness to offer better lubrication. The outcomes of present piece of research may pave a path for a better and closer integrations of applied and advanced mathematical applications to real world problems on variety of bearings and bearing systems.

References

V .K. Agrawal (1986). Magnetic fluid based porous inclined slider bearing, *Wear*, 107,133-139.

V .K. Agrawal (1986).Inertia effects in hydro magnetic inclined slider bearing. *Japanese Journal of applied physics*, 9, 820-824.

P. I. Andharia, J. L. Gupta and G. M. Deheri (1998). Effect of longitudinal surface roughness on hydrodynamic lubrication of hyperbolic slider bearings. *PROC. NAT. ACAD .SCI.* India, 68(A), III.

M. I. Anwar and C. M. Rodkiewicz (1972). Inertial and convective effect in hydrodynamic lubrication of a slider bearing, *ASME journal of lubrication technolo*gy, 93, 313-315

S. K. Basu, S. N. Sengupta and B. B. Ahuja (2005). Fundamentals of Tribology, *Prentice-Hall of Indian Private Limited*, New Delhi.

G. S. Beavers and D. D. Joseph (1967). Boundary conditions at a naturally permeable wall, *Journal of Fluid Mechanics,* 30(1), 197-207.

M. V. Bhat. (2003). Lubrication with a magnetic fluid. Team Spirit (India) Pvt. Ltd.

T. L. Chou, J. W. Lai and J. R. Lin (2003). Magneto hydrodynamic squeeze film characteristics between a sphere and a plane surface, *Journal of Marine science and technology*, 11, 174 –178.

H. Christensen and K. C. Tonder (1969). Tribology of rough surfaces: parametric study and comparison of lubrication*, SINTEF Report No. 22/69-18*.

H. Christensen. and K. C. Tonder (1970). The Hydrodynamic lubrication of rough bearing surfaces of finite width, *ASME-ASLE lubrication conference, Paper No. 70. Lub.*

N. C. Das (1998). A study of optimum load-bearing capacity for slider bearings lubricated with couple stress fluids in magnetic field, *Tribology international,* 31, 393-400.

B. J. Hamrock (1994). Fundamentals of fluid film lubrication, *New York: McGraw-Hill*.

J. T. Jenkins (1972). A magnetic fluid, *Arch. Ration., Mech. Anal*, 46(1), 42.

V. Kumar (1980). Friction of a plane porous slider of optimal profile, *Wear,*. 62, 417-418.

S. Laghrabli, M. El Khlifi, M. Nabhani and B. Bou-Saïd (2017). Ferrofluid lubrication of finite journal bearings using Jenkins model*. Lubrication Science*, 29, 441–454.

J. R. Lin and R. F. Lu (2010). Dynamic characteristics for magneto hydrodynamic wide slider bearing with an J. R. Lin exponential film profile, *Journal of Marine Science and Technology*, 18, 268-276.

J. R. Lin, R. F. Lu and C. R. Hung (2006). Dynamic characteristics of wide exponential film-shape slider bearings lubricated with a non-Newtonian coup stress fluid, *Journal of Marine Science and Technology*, 14, 93-101.

J. R. Lin (2001). Linear Stability Analysis of Rotor Bearing System: Couple Stress Fluid Model. *Computers & Structures*, 79, 801-809.

S. R. Mishra, M. Barik and G. C. Dash (2018). An analysis of hydrodynamic ferrofluid lubrication of an inclined rough slider bearing, *Tribology - Materials, Surfaces & Interfaces*, 12(1), 17-26.

M. Mowry (2011). The True Cost of Bearing Lubrication*, igus white paper*, 1-8.

P. K. Murti (1974). Analysis for porous slider bearing, *Wear*, 28, 131-134.

B. N. Neminath andB. M. Gudadappagouda (2008). Dynamic Reynolds equation for micropolar fluid lubrication of porous slider bearings, *Journal of Marine Science and Technology*, 16, 182-190.

J. L. Neuringer and R. E. Rosensweig (1964). Magnetic fluids, *Physics of fluids*, 7(12), 1927.

J. R. Patel and G. M. Deheri (2016). Performance of a Ferrofluid Based Rough Parallel Plate Slider Bearing: A Comparison of Three Magnetic Fluid Flow Models. *Advances in Tribology*, 2016, Article ID 8197160, 9.

J. R. Patel and G. M. Deheri (2017). A Comparison of Magnetic Fluid Flow Models on the Behavior of a Ferrofluid Squeeze Film in Curved Rough Porous Circular Plates Considering Slip Velocity, *Iran J Sci Technol Trans Sci* (2017). https://doi.org/10.1007/s40995-017-0365-z.

N. D. Patel and G. M. Deheri (2012). A Ferrofluid Lubrication of A Rough, Porous Inclined Slider Bearing With Slip Velocity, *Journal of Mechanical Engineering and Technology*, 4(1), 15-34.

N. Petroff (1883). Friction in Machines and the effect of lubricant (in Russian), *J. St. Petersburg*, 17, 228,377 and 530.

O. Reynolds. (1886). On the theory of lubrication and its application to Mr Beauchamp Tower's experiments, including an experimental determination of viscosity of Olive Oil, *Phil. Trans*., 177, 157-234.

R. C. Shah and M. V. Bhat (2004). Ferrofluid lubrication of a slider bearing with circular convex pad. *Journal of the National Science Foundation*, 32, 139-148

T. W. Snyder (1963). The non-linear hydrodynamic slider bearing, *ASME journal of lubrication technology*, 62, 52-54.

U. Srinivasan (1977). The analysis of a double-Layered porous slider bearing, *Wear*, 42, 205- 215.

M. Stojilkovića and M. Kolba (2016). Tribological Properties of Bio degradable Universal Tractor Transmission Oil, *Tribology in Industry*, 38(2), 229-234.

T Rao, T. V. V. L. N., A. M. A. Rani, M. Awang, T. Nagarajan and F. M. Hashim (2016). Stability analysis of double porous and surface porous layer journal bearing, *Tribology - Materials, Surfaces & Interfaces*, 10(1), 19-25.

C. M. Taylor and D. Dowson (1974). Turbulent lubrication theory – application to design, *ASME Journal of Lubrication Technology*, 96, 36-47.

S. T. Tzeng and E. Saibel (1967). Surface roughness effect on slider bearing lubrication. *Trans. ASME, J. Lub. Tech.* 10, 334-338.

Nomenclature

Symbol	Expression	Symbol	Expression
$\dot{h}$	Normal velocity of bearing surface	μ^*	Magnetization parameter in non-dimensional form
p	Pressure distribution(N/mm2)	$\bar{\alpha}$	Dimensionless variance
s	Slip parameter	$\bar{\varepsilon}$	Dimensionless skewness
w	Load carrying capacity (N)	μ_0	Permeability of free space (N/A2)
H	Magnitude of the magnetic field	$\bar{\sigma}$	Dimensionless standard deviation
A	Length of the bearing	$\bar{\mu}$	Magnetic susceptibility
P_{mn}	Dimensionless pressure	U	Velocity of slider
H_1	The thickness of the inner layer of the porous plate (mm)	H_2	The thickness of the outer layer of the porous plate (mm)
W_{mn}	Dimensionless load carrying capacity	ϕ_1	The permeability of inner layer (col2kgm/s2)
$\bar{X}$	X coordinate of the center of pressure	ϕ_2	The permeability of outer layer (col2kgm/s2)
$\bar{H}$	External magnetic field	$\bar{\psi}_1$	Porosity of inner layer
$\bar{s}$	Dimensionless slip parameter	$\bar{\psi}_2$	Porosity of outer layer
$\bar{\beta}$	Material Parameter	η	Viscosity of the suspension
σ	Standard deviation	$\bar{q}$	Fluid velocity
α	Variance	$\bar{M}$	Magnetization vector
ε	Skewness		