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Research Article

Crack Propagation Analysis of Fiber-Reinforced Composite Hollow Transmission Shaft

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Abstract

The main purpose of this research was to investigate crack propagation resistance of shaft. Hollow transmission composite shafts were produced and their crack behavior was investigated. Glass fiber-reinforced polyester polymer composite material is used for the hollow transmission shaft of Hidasie H260 helicopter model of Dejen Aviation Engineering Complex, DAEC. The elastic engineering constants of composite transmission shaft are determined using the Classic Laminate Theory, CLT. Linear Elastic Fracture Mechanics, LEFM was used to analyze its crack propagation by taking a through-thickness central crack. The stress at the crack tip is analyzed and it was found that the glass fiber-reinforced polyester polymer composite transmission shaft has good resistance to crack propagation. FEA method was used to crack propagation analyses. The model of shaft is generated using Finite Element Method. Due to the symmetry of the problem, only a quarter part of it is modeled and analyzed. The crack-tip region was meshed using quarter point (singular) 8-node quadrilateral elements to get accurate results and the analysis used a fit of the nodal displacements in the vicinity of the crack tip. The maximum stress is at the crack tip since there is a stress concentration at sharp edges.

Keywords: *crack propagation, composite, hollow transmission shaft, filament winding, LEFM*

1. Introduction

Dejen Aviation Engineering Complex, DAEC, a company in Debrezeyt, tried to have a H260L model helicopter with light-weight by manufacturing all its parts from composite material. In doing so the company tried to make the helicopter carries as much weight as it used to carries and they call it Hidasie helicopter. Most of its components are manufactured from glass fiber-reinforced polyester polymer composite material by reverse engineering and assembled it. However, there are some components of the helicopter which need analysis and simulations before manufacturing by reverse engineering. One of these components is the transmission shaft, which transmits torque from the main gear-box to the intermediate gear-box.

The transmission shaft of the H260L model helicopter is made from 6061 carbon steel. However, DAEC proposed to change it by glass fiber-reinforced polyester polymer composite material. Analytical experiment was performed for the glass fiber-reinforced polyester polymer composite transmission shaft of the helicopter, even using fiber-reinforced composite materials have an advantage in reducing the weight, increase in strength and resistance in crack propagation.

It is known that during operation, rotating shafts are subjected to degenerative effects which may cause initiation of structural defects such as crack and crack-like which finally leads to the catastrophic failure or breakdown of the shaft. Thus, it is important to design for crack propagation and vibration analysis of the shaft. Several methods, such as destructive and non-destructive tests, can be used to monitor the crack and vibration condition of the shaft. Vibration analysis, which can be used to detect structural defects of the shaft such as crack and crack-like, of any structure offers an effective, inexpensive and fast means of non-destructive testing.

Nowadays, the interest of using composite materials for structural purposes in many branch of engineering such as aircraft, turbo machinery, and power plants is increasing. It is due to the reason that composite materials have good characteristics such as high strength-to-weight and stiffness-to-weight ratio, good damping capacity and better resistance to fatigue and crack propagation [Alun F. Liu *et al*, 2005] as compared to monolithic materials such as metals; individuals are interested in composite materials.

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Due to fiber-reinforced composite materials have an advantage in reducing the weight, increase in strength and resistance in crack propagation was chosen over the transmission shaft of the H260L model helicopter made 6061 carbon steel Helicopter. Rotating transmission shaft of the aircraft was made from fiber-reinforced composite materials and it may be one of the potential applications of composite materials.

Analytical analysis on its crack propagation resistance has been made. Linear Elastic Fracture Mechanics, LEFM is used to analyze its crack propagation by taking a through-thickness central crack. The stress at the crack tip is analyzed and it was found that the glass fiber-reinforced polyester polymer composite transmission shaft has good resistance to crack propagation. The fracture mechanics are employed for determining the stress-intensity factors (SIFs) of the cracks in deformed structures. ANSYS finite element software was used to perform the numerical analysis for the transmission shaft. A procedure for the model of crack propagation for cracks was introduced and SIFs have been calculated by using the finite-element method.

1.1. Background

In general, any material consisting of two or more constituents with different properties and distinct boundaries between the constituents can be referred to as a composite material [Valery V. Vasiliev*et al*, 2007]. In addition to the above definition, three other criteria should normally be satisfied in order for a material to be called a composite material. These criteria are:

1) Both constituents have to be present in reasonable proportions.
2) The constituent phases should have distinctly different properties, such that the composite's properties are noticeably different from the properties of the constituents.
3) A synthetic composite is usually produced by deliberately mixing and combining the constituents by various means.

The above definition would encompass bricks, concrete, wood, bone, as well as modern synthetic composites such as fiber-reinforced plastics, FRP. The fiber-reinforced plastics have become increasingly important over the past 50 years, and are now the first choice for fabricating structures where low weight in combination with high strength and stiffness are required [F L Matthews *et al*, 2007]. Fiber-reinforced composite materials are also known as advanced composites. The basic components of these materials are long and thin fibers possessing high strength and stiffness. The fibers in these materials are bounded with matrix whose volume fraction is usually less than 50%.

1.2. Objective of the work

The general objective of this research work was to analyze and simulate crack propagation of fiber-reinforced composite transmission shaft. In detail, analytical study was carried out and tried to identify the nature of crack propagation with respect to its crack location and crack direction of the fiber-reinforced composite transmission shaft for Hidasie helicopter. To achieve the objectives, following are needed to study.

- Modeling the crack on the hollow transmission shaft in terms of the type of crack and where crack propagates in reality.
- Apply finite element software to analyze crack propagation analytically.

1.3. Motivation of the research

Works done by various researchers in earlier reveal that there are some finite element and few analytical solutions available for shafts and beams with holes and rarely available for surface cracks on fiber-reinforced composite material shafts. Most of these solutions were dealt with regular shape of cracks. The strength of the fiber-reinforced composite material is influenced by number of factors. These factors include anisotropic and non-homogeneous nature of the material, mechanical incompatibility of the constituent phases, the elastic and plastic behavior of the matrices and reinforcing materials, the volume fraction of the components, the direction of applied load and so on. Due to all these factors it is expected to have crack in or on surface of the shaft. The manufacturing process or assembly can also further induce flaws or cracks in the shaft.

2. Literature review

Application of composite materials in many engineering designs and products such as automobile, aircrafts, turbo machineries, marine, trains, constructions and machine components of power plants has increased dramatically. Nowadays most of components of aircrafts such as helicopter; and marine such as boat are made from composite materials. Therefore, it is important to take care in designing and material selection for the components of the aircrafts and marines. Otherwise, the consequence is very costly and may also cause catastrophic failure in terms of human life and property damage.

Composite materials are used in a wide range of industrial applications due to their lightweight and high specific stiffness (E/ρ) ratio. They have been widely used for components of aerospace and military airplane structures in their earlier practices. Composite materials are recently used in a lot of commercial applications replacing the conventional materials such as metallic alloys and polymers and it is a fact that the composite technologies have had tremendous development comparing with its earlier situation. New matrix materials and various reinforcing components have led to advanced improvement in the composite technology. At the same time, as it is expected, composite materials and its technology are having great challenges with this extremely high progress. Composite materials emerged in the middle of the 20th century as a promising class of engineering materials providing new prospects for modern technology.

[D.J. Bieryla*et al*, 2005] made a research on shaft crack monitoring via torsional vibration analysis. Torsional vibration signature has shown them the potential to detect shaft cracks during normal machinery operations of rotating equipment. The method they used tracks characteristic changes in the natural torsional vibration frequencies that are associated with shaft crack propagation. The method is generally applicable to many types of rotating equipment. A laboratory scale rotor test bed was developed to investigate shaft cracking detection techniques under controlled conditions. They took a sample shaft with a semi-elliptical surface crack, which was propagated in three point bending. The fatigue crack was incrementally grown in nine steps, with depths ranging from approximately 0 – 60% of the shaft diameter. After the crack was grown to each pre-defined depth, the shaft was installed in the rotor test bed and the changes in shaft torsional vibration features observed. The first torsional natural frequency was shown to be sensitive to the shaft crack depth, which for the crack depths tested produced a 2 Hz frequency drop. The relationship between crack depth and torsional natural frequency is nonlinear. The test data show that changes in the torsional shaft frequency in the range of 0.1 to 0.2 Hz. can be detected by a visual inspection. This study points to the potential of using online torsional signature analysis as a diagnostic for shaft crack monitoring in rotating equipment.

[P. R.Baviskar*et al*, 2011] studied on analysis of crack in shaft of blower using finite element analysis and experimental techniques. They came up with addressing inverse method for fault detection in moving parts. They suspected and concluded that one of the failures might be due to the crack initiation and propagation in any of moving parts. They also found that the natural frequency is monitored to access crack location and crack size in the beam because it is susceptible to minute changes. In their theoretical analysis, the crack was simulated by a spring connecting the two segments of the beam. They modeled the beam using finite element method of analysis. The modal analysis of the beam was conducted using the ANSYS software whereas the experimentation was done on Fast Fourier Transformer Analyzer. The results they obtained from ANSYS and experimentation have good agreement.

[Guillermo A. Riveros*et al*, 2006] has studied the numerical evaluation of stress intensity factor, K_I using J-integral approach. He used Costal and Hydraulic Engineering Technical Note, CHETN to describe the numerical evaluation of the stress intensity factors using the J-integral approach. He calculated the stress intensity for semi-infinite plate with an edge crack. He obtained only a difference of 1.25 percent between the closed-form and numerical solution. The CHEIN also helped him to perform detailed three-dimensional meshing of any complicated geometry. The meshing approach was used to generate a 3-D mesh of a hydraulic steel structure with multiple cracks. He also discussed the stress intensity factor for the 3-D problem with multiple cracks.

[S. K. Bhaumik, *et al*, 2002] studied the cause of fatigue failure of a hollow power transmission shaft in operation. They investigated the crack of the transmission shaft due to the leakage of oil in operation. The crack was through the thickness and had propagated in a helical manner. They revealed that the crack had propagated circumferentially covering about ¾ of the shaft periphery by fluorescent dye penetrator inspection. They cleaned the fracture surface of the cracked hollow power transmission shaft with acetone and observed under a stereo-binocular microscope to determine the fracture origin. It was found that the fracture origin coincided with one of the edges of the keyway. Finally, they concluded that the cause of the failure of the transmission shaft at edge of the keyway was due to fatigue. The fatigue crack has initiated because of stress concentrations resulting from combined effect of rough machining and inadequate fillet radius at the keyway end edges.

3. Materials and methods

3.1. Production of composite transmission shaft

The fiber and resin used for manufacturing of the composite transmission shaft are glass fiber and polyester respectively. A one cubic centimeter of catalyst is also added for every liter of polyester resin as hardener in order to facilitate the mixing process and drying. Particulate-reinforced metals the reinforcement content is usually kept to less than 40% (0.4 volume fraction) owing to processing difficulties and increasing brittleness at higher contents. The volume fraction of the glass fiber to polyester of the composite transmission shaft was used for the experiment is 65%. The manufacturing method used for the composite transmission shaft is filament winding method. The specimens used for experimental testing are manufactured manually using the two filament winding methods together.

The transmission shaft of the helicopter is outside of the fuselage which runs over tail fuselage as shown in Fig. 1. The bearings are inside the bearing holders which are fixed to the beam which runs from the main body of the helicopter to its tail rotor.

Figure 1: H260L model helicopter

Figure 2: Manufacturing of the hollow transmission shaft

The winding machine consists of a turning device for the rotating mandrel, filament guide, resin impregnating bath and bobbin for roving as shown in Fig. 2. The transmission shaft is manufactured by setting different orientation angle on the mandrel by filament winding manually in winding shop of Dejen Aviation Engineering Complex, DAEC. Specimens of different orientation angle and stacking sequence were manufactured in order to decide which orientation angle and stacking sequence is better for the transmission shaft. Torsion testing machine which was used is shown in Fig. 3 given below.

Figure 3: Torsion testing machine with specimen

The procedures to test the torque capacity of the transmission shaft are described as follows:

- Twist is induced in quarter degree increments. After each application, the arm should be balanced and the resulting torque recorded.
- When 8 degrees of twist has been applied, the twist should be increased in 2 degree increments. Again, the arm should be balanced after each step. Record the torque and record if any hysteresis effects are experienced.
- After 30 degrees of twist has been applied, the twist should be increased first to a total twist of 90 degrees. Record the torque and note any hysteresis effects. Next, increment the angle of twist in 90 degree increments until the specimen breaks. After every increment, record the torque. Usually this process needs to be done fairly quickly to ensure minimal error due to relaxation in the specimen.

4. Crack propagation results analysis

The composite transmission shaft is subjected to a torque from the intermediate gear-box of the helicopter. This indicates if the transmission shaft has any flaw it will experience the three type of loading modes of crack. These are Mode-I loading, where the principal load is applied normal to the crack plane tend to open the crack, Mode-II loading, corresponds to in-plane shear loading and tends to slide one crack face with respect to the other and Mode-III loading, refers to out-of-plane shear [T. L. Anderson *et al*, 1994]. The stress-intensity factor was calculated for the specimen analytically.

In the Mode-I, or opening mode, the body is loaded by tensile forces such that the crack surfaces are pulled apart in the y direction as shown in Fig. 4. The deformations are then symmetric with respect to the planes perpendicular to the y axis and the z axis.

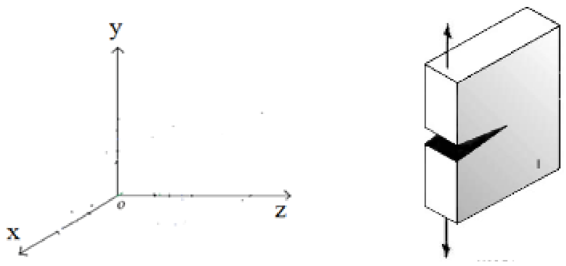

Figure 4: Mode-I, opening mode of loading

In the Mode-II, or sliding mode, the body is loaded by shear forces parallel to the crack surfaces, which slide over each other in the x direction as shown in Fig. 5. The deformations are then symmetric with respect to the plane perpendicular to the z axis and skew symmetric with respect to the plane perpendicular to the y axis [T. L. Anderson *et al*, 1994].

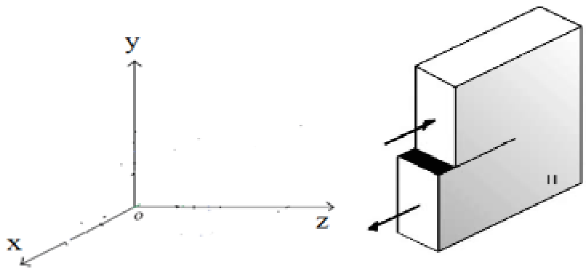

Figure 5: Mode-II, sliding mode of loading

Finally, in the Mode-III, or tearing mode, the body is loaded by shear forces parallel to the crack front the crack surfaces and the crack surfaces slide over each other in the z direction as shown in Fig. 6. The deformations are then skew-symmetric with respect to the plane perpendicular to the z and the y axis [[T. L. Anderson *et al*, 1994].

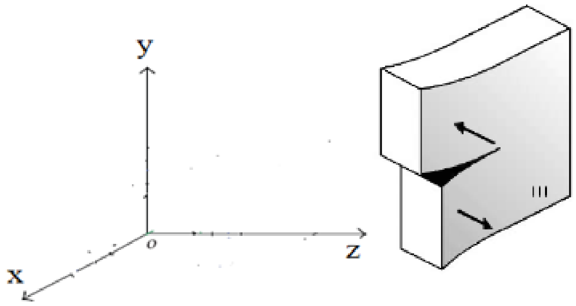

Figure 6: Mode-III, tearing mode of loading

For each of the above modes, crack extension may take place only in the direction of the x axis as it is the original orientation (direction) of the crack. In a more general situation, we may typically find a mixed mode situation. For such cases a superposition of the modes is the best solution that has to be taken.

In a linear elastic mixed mode situation, the principle of stress superposition states that the individual contributions to a given stress component are additive. So if σ_{ij}^{I}, σ_{ij}^{II} and σ_{ij}^{III} are the stress components associated to the modes-I, II and III respectively, then the stress component σ_{ij} is given by the following expression [M. Janssen *et al*, 2004].

$$\sigma_{ij} = \sigma_{ij}^{I} + \sigma_{ij}^{II} + \sigma_{ij}^{III} \quad (1)$$
$$for\ i, j = x, y$$

The transmission shaft of the helicopter is subjected to torsion as it is described in equation (2-4). It is considered to be torsional problem when the applied moment or torque twists the transmission shaft about its longitudinal axis as shown in Fig. 7. The maximum shear stress that can be developed due to the applied torque was obtained analytically using following expressions [Robert L. Norton *et al*, 2006; Arthur P. Boresi*et al*, 2003; S. Timoshenko *et al*, 1951].

$$\tau = \frac{Tr}{J} \quad (2)$$

$$\tau_{r\theta} = G\gamma_{r\theta} = G\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right) \quad (3)$$

$$\tau_{z\theta} = G\gamma_{z\theta} = G\frac{\partial v}{\partial z} \quad (4)$$

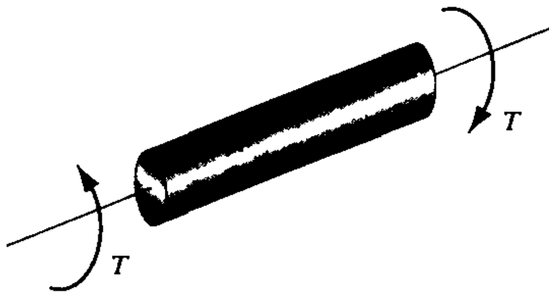

Figure 7: Transmission shaft subjected to torque

A small area was taken from the transmission shaft where the maximum shear stress is obtained. In that small area, the shear stress was taken as a remote tensile stress for a central crack through-thickness as shown in Fig. 8. This remote tensile stress will have an opening mode of fracture on the central crack. Even though, they are not that much significant, it is obvious that there is also a sliding and tearing mode of fracture in the transmission shaft. But let's assume that we have only an opening mode of fracture.

It is assumed that the crack propagation is going to be analyzed using Linear Elastic Fracture Mechanics, LEFM. It is also assumed that the transmission shaft is subjected to plane strain in order to obtain the stress intensity factor, SIF.

For a through-thickness central crack size of $2a$, width of b and a remote tensile stress of σ as shown in Fig. 8, stress intensity factor, K_I can be obtained analytically using the following expression [Alun F. Liu *et al*, 2005].

$$K_I = C\sigma\sqrt{\pi a} \tag{5}$$

Where $C = (1-0.1\eta^2 + 0.96\eta^4)\sqrt{1/\cos(\pi\eta)}$

$$\eta = \frac{a}{b}$$

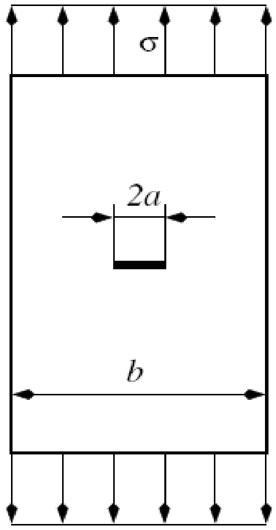

Figure 8: Through-thickness central crack

Consider a finite glass fiber-reinforced polyester polymer composite plate with width, $b = 0.2m$ and half crack size, $a = 0.02m$ so the stress intensity factor, K_I of the opening mode of fracture is $19\ MPa\sqrt{m}$.

Now consider any of the three modes we have introduced. Within the scope of the theory of linear elasticity, a crack introduces a discontinuity in the elastic body such that the stresses tend to infinity as one approach the crack tip. Using the semi-inverse method of Westerngaard, Irwin related the singular behavior of the stress components to the distance to the crack tip r. The relation he obtained can be written in a simplified form as [E.E. Gdoutos*et al,* 2005].

$$\sigma \cong \frac{K_I}{\sqrt{2\pi r}} \tag{6}$$

The parameter K, the stress intensity factor, plays a fundamental role in fracture mechanics, as it characterizes the stress field in this region. So the stress that will be developed near the crack tip due to the discontinuity can be approximated using equation (6) and tabulated in Table 1 as a function of the distance r from the crack tip. As shown in Table 1 the stress increases as the distance r decreases. This indicates there will be maximum stress just at the tip of the crack.

Table 1: Stress near the crack tip of a through-thickness central crack

Distance, r from the crack tip in meter	Stress, σ in MPa
0.01	75
0.009	80
0.007	91
0.005	107
0.003	138
0.002	170
0.001	240
0.0009	253
0.0007	287

Henceforth, we have considered the problem of a cracked glass fiber-reinforced composite plate in a plane stress situation, which means that Mode-III situations will be disregarded. And later on, we consider only the two mode of fracture. This will help to analyze the general and the real world situation of the transmission shaft of the helicopter.

Let's consider a static crack in a glass fiber-reinforced polyester polymer composite plate which is in a plane stress situation in order to find the stress in general. It was assumed that the crack surfaces are free of stress and that the crack is positioned along the negative x-axis, as shown in the Fig. 9.

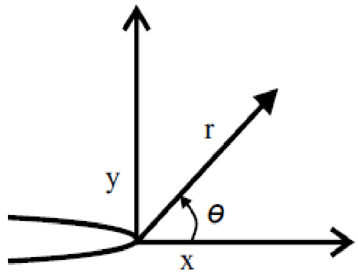

Figure 9: Crack tip polar coordinate

Then the distribution of the stresses in the region near the tip of the crack may be derived as an interior asymptotic expansion [Nestor Perez *et al*, 2004]. In polar coordinates, we have the stresses near the tip of the crack as:

$$\sigma_{ij}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^{I}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{ij}^{II}(\theta) + \sigma_{ij}^{0} \tag{7}$$

The above equation is for $r \to 0$ and $i,j = x,y$ and σ_{ij}^0 indicates the finite stresses at the crack tip. Here the normalizing constants for the symmetric and anti-symmetric parts of the stress field of K_I and K_{II} represent respectively the stress intensity factors for the corresponding modes I and II. These are defined by the following expressions [Nestor Perez *et al*, 2004].

$$K_I = \lim_{r \to 0} \sqrt{2\pi r}\,\sigma_{yy}(r,0) \tag{8a}$$

$$K_{II} = \lim_{r \to 0} \sqrt{2\pi r}\,\sigma_{xy}(r,0) \tag{8b}$$

The stress varies as the function of polar angle, θ indicted in Fig. 9 above. The angular variation functions for Mode-I are given respectively by the following expression [Nestor Perez *et al*, 2004].

$$f_{xx}^{I}(\theta)=\cos\left(\frac{\theta}{2}\right)\left(1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right) \tag{9a}$$

$$f_{yy}^{I}(\theta)=\cos\left(\frac{\theta}{2}\right)\left(1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right) \tag{9b}$$

$$f_{xy}^{I}(\theta)=\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right) \tag{9c}$$

While the equivalent functions for Mode-II are the followings.

$$f_{xx}^{II}(\theta)=-\sin\left(\frac{\theta}{2}\right)\left(2+\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\right) \tag{10a}$$

$$f_{yy}^{II}(\theta)=\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right) \tag{10b}$$

$$f_{xy}^{II}(\theta)=\cos\left(\frac{\theta}{2}\right)\left(1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right) \tag{10c}$$

It is also possible to find the corresponding displacement field near the crack tip of the glass fiber-reinforced polyester polymer composites plate, which is discontinuous over the crack. This displacement field is obtained using the following expression [Nestor Perez *et al*, 2004].

$$u_i(r,\theta)=u_i^0+\frac{K_I}{G}\sqrt{\frac{r}{2\pi}}f_i^{I}(\theta)+\frac{K_{II}}{G}\sqrt{\frac{r}{2\pi}}f_i^{II}(\theta) \tag{11}$$

Where $i=x,y$ and for $r\to 0$ and here, u_i^0 are the crack tip displacements. The angular variation functions here are now given by

$$f_x^{I}(\theta)=\cos\left(\frac{\theta}{2}\right)\left(\frac{1-\upsilon}{1+\upsilon}+\sin^2\left(\frac{\theta}{2}\right)\right) \tag{12a}$$

$$f_y^{I}(\theta)=\sin\left(\frac{\theta}{2}\right)\left(\frac{2}{1+\upsilon}-\cos^2\left(\frac{\theta}{2}\right)\right) \tag{12b}$$

$$f_y^{II}(\theta)=\sin\left(\frac{\theta}{2}\right)\left(\frac{2}{1+\upsilon}+\cos^2\left(\frac{\theta}{2}\right)\right) \tag{12c}$$

$$f_x^{II}(\theta)=\cos\left(\frac{\theta}{2}\right)\left(-\frac{1-\upsilon}{1+\upsilon}+\sin^2\left(\frac{\theta}{2}\right)\right) \qquad 12d)$$

The formulas we have presented allow us to have a characterization of the stresses and the displacements in the vicinity of a crack tip of the glass fiber-reinforced polyester polymer composite plate with through-thickness central crack.

4.1. Finite element analysis

ANSYS 12.0 Classic is used to develop the required 3-D finite element model of the transmission shaft. This section explains in detail the geometry and material property of laminated composite material, step wise procedure to develop the composite finite element model, and the boundary and loading conditions applied on the model.

4.1.1. Modeling of the composite shaft

A full length Finite Element, FE model is constructed for the 200 mm long hollow circular composite shaft under static torsion load. 3-D 8 nodes SOLID46 element was used to develop the required 3D composite shaft model. In the 3-D composite shaft model, each layer in the laminate is mapped meshed separately in thickness direction with different element coordinate system to represent a composite layer arrangement. This means, each layer on the hollow transmission shaft is modeled as a separate volume and meshed using SOLID46 elements. SOLID46 is a layered version of 8-node structure solid designed to model layered thick solids. This element allows up to 250 different material layers with different orientations and orthotropic material properties in each layer. If more than 250 layers are required, a user-input constitutive matrix option is available. The element may also be stacked as an alternative approach. The element has three degrees of freedom, DOF at each node and translations in the nodal x, y, and z directions. The layers were assumed perfectly bonded. The mapping mesh technique was used for the entire domain as shown in Fig. 10.

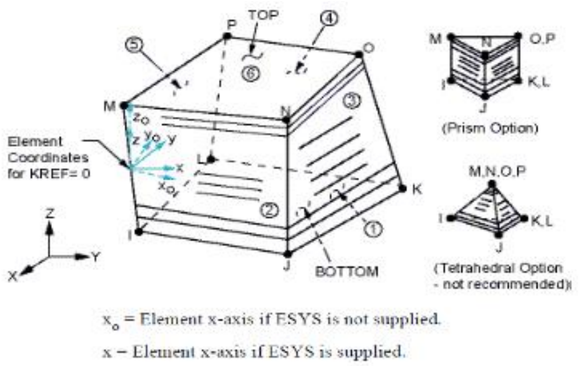

Figure 10: Solid46 DOF and element type

The standard ANSYS procedure (material property 3D 8 nodes SOLID 46 elements, preprocessor of material model, boundary condition of transmission shaft, volume mapped meshing, 3 element coordinate system, second layer in the Sub-laminate was created and finally solution menu and postprocessor) was used to create the 3-D finite element model of the composite hollow transmission shaft and generate mesh.

4.1.2. Material properties

The elastic engineering constants were found using Classic Laminate Theory, CLT for composite materials. The properties were used with conventional laminate theory to calculate the theoretical effective properties of the orthotropic monolithic model. The material used for the composite laminate is E-glass fiber-reinforced polyester polymer laminate. The glass fiber-reinforced polyester polymer layers were modeled with homogenized linear elastic orthotropic materials. The unidirectional layer orthotropic properties from equations (13) – (15) for the transmission shaft material are given as:

$$E_1 = 49\ GPa \quad E_2 = E_3 = 21\ GPa \tag{13}$$

$$v_{12} = v_{23} = v_{13} = 0.215 \tag{14}$$

$$G_{12} = G_{23} = G_{13} = 3GPa \tag{15}$$

Where E_1, E_2 and E_3 are the Young's moduli of the composite lamina along the material coordinates. G_{12}, G_{23} and G_{13} are the Shear moduli and v_{12}, v_{23} and v_{13} are Poisson's ratio with respect to the 1-2, 2-3 and 1-3 planes, respectively as shown in Fig. 11.The engineering constants for laminate [Dr. Markus Milwich*et al*, 2009] in the global coordinate of the glass fiber-reinforced polyester polymer composite transmission shaft are obtained from the elements of compliance matrix using following equation (16).

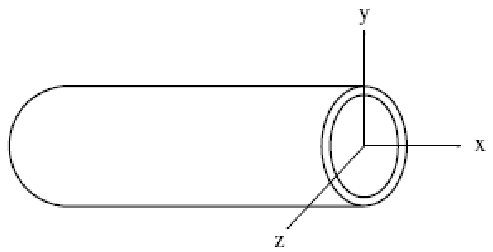

Figure 11: Global coordinate of the transmission shaft

$$E_{x,lam} = \frac{1}{a_{11}}, \quad E_{y,lam} = \frac{1}{a_{22}}, \quad G_{xy,lam} = G_{yx,lam} = \frac{1}{a_{33}}, \quad v_{xy,lam} = \frac{a_{21}}{a_{11}}, \quad v_{yx,lam} = \frac{a_{12}}{a_{22}} \tag{16}$$

4.1.3. Boundary conditions

The aim to model the transmission shaft in FEA was subjected to pure torsion. So for this condition, one end of the transmission shaft is assumed fixed with all the DOF and the other end is subjected to applied torque which is a distributed forces in tangential direction to the outside of the fixture of the hollow transmission shaft. The distributed forces can be calculated by converting the applied torque to the tangential force and multiplying by outside diameter and dividing the same by number of nodes on the side of the fixture of the transmission shaft model.

In order to restrict the movement of the nodes in the radial direction at the end at which the force is applied, the DOF in r-direction is protected. The nodes are to be rotated along cylindrical coordinate system so that the applied forces in nodal Θ-direction are tangential to the perimeter of the shaft. No cantilever effect will be formed since the forces will deform the shaft about its axis by pure twisting.

4.1.4. Analytical crack model

The crack propagation of the transmission shaft on ANSYS can be analyzed by taking an infinitesimal part from it using LEFM. Since the transmission shaft is subjected to shear stress, it was assumed an opening mode of fracture mechanics on a plate with through-thickness central crack. A sharp central crack is assumed since it is the worst in crack propagation. It develops large stress concentration at its tip point and makes the structure fail. Model was prepared using ANSYS shown in Fig. 12 (a) and (b).

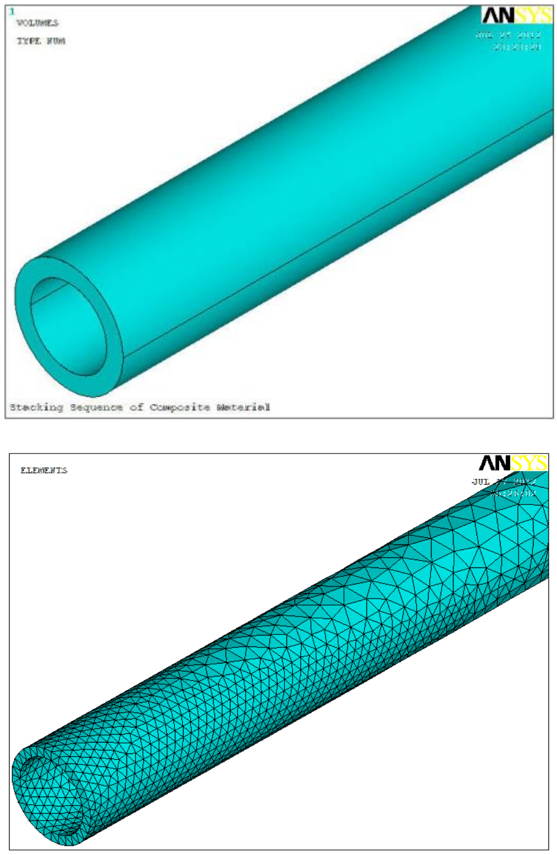

Figure 12: ANSYS model of a hollow transmission shaft (a) The zoomed 3D model (b) Zoomed Meshed model

The following points must be considered due to the above assumption in order to analyze the transmission shaft in ANSYS software.

- Since the LEFM assumption is used, the Stress Intensity Factors, SIFs at a crack tip may be computed using the ANSYS's KCALC command. The analysis used a fit of the nodal displacements in the vicinity of the crack tip. The shear stress is applied as a remote stress to the infinitesimal part of the transmission shaft.

- Due to the symmetry of the problem, only a quarter part of it is modeled and analyzed. So the boundary condition for the model is a symmetry boundary condition at the left and bottom side.
- The crack-tip region is meshed using quarter point (singular) 8-node quadrilateral elements to get accurate results. The meshed quarter part of plate with central crack is shown in Fig. 13.

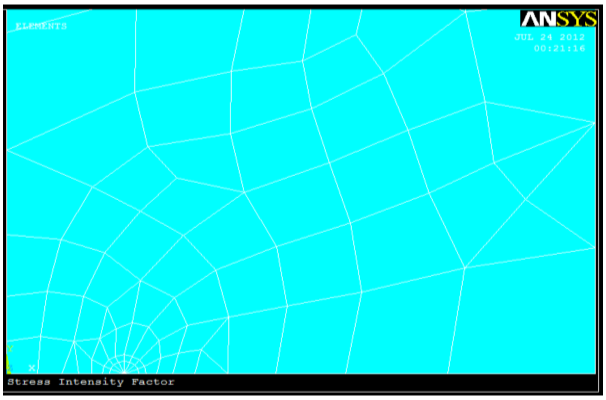

Figure 13: Meshed quarter part of plate with central crack

The deformed shape of the infinitesimal plate is shown in Fig. 14. The stress intensity factor of the plate is obtained. This figure is taken from ANSYS word pad text as a picture. Stress intensity factor calculated from ANSYS is shown in Fig. 15.

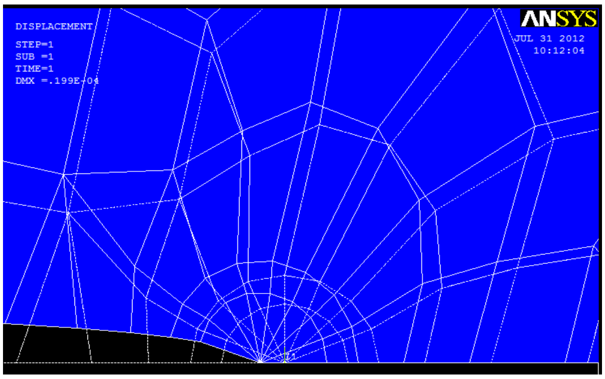

Figure 14: Deformation of the central symmetry crack

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**** CALCULATE MIXED-MODE STRESS INTENSITY FACTORS ****
 ASSUME PLANE STRAIN CONDITIONS
 ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)
 EXTRAPOLATION PATH IS DEFINED BY NODES:      2     15     14
  WITH NODE      2 AS THE CRACK-TIP NODE
 USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
  EX =   0.38000E+06    NUXY =    0.32000      AT TEMP =   0.0000
**** KI =   19.901    ,  KII =    0.0000    ,  KIII =    0.0000      ****
```

Figure 15: Stress intensity factor from ANSYS

The maximum stress found at the crack tip since there is a stress concentration at sharp edges. The Von Misses stress from ANSYS also reveals this fact as shown in Fig. 16.

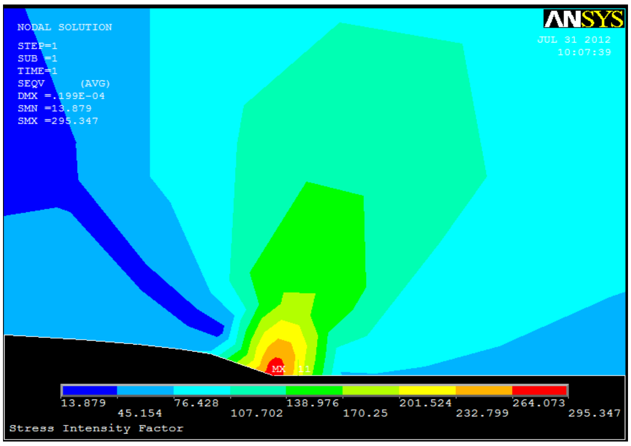

Figure 16: Von Misses stress of the cracked plate

4.1.5. Solution techniques

The analytical analysis of crack propagation of the transmission shaft is done based on the theories of crack on rotating shaft. The rotating glass fiber-reinforced polyester polymer hollow transmission shaft was also analyzed using finite element analysis software by modeling it on ANSYS. The torque capacity of the transmission shaft is also analyzed on ANSYS software until its maximum shear stress becomes equal to the yielding shear stress of the fiber-polyester composites. The torque capacity of the fiber-reinforced composite hollow shaft was also tested on torsional testing machine. Finally, the results obtained from the analytical analysis and ANSYS software are compared. Similarly, the torque capacities of the transmission shaft obtained from ANSYS and from the testing machine are compared. The maximum shear stress developed due to the applied torque on the transmission shaft is 75MPa, which is very small to cause failure. Even the stress at the crack tip is 287MPa which is less than yielding stress of the glass fiber-polyester resin composite material.

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Conclusions

From the crack analysis results, it can be seen that the glass fiber-reinforced polyester polymer composite transmission shaft has good crack extension resistance and is capable to carry the torque applied from the engine of the helicopter. There was no breakage observed from the transmission shaft tests except small delamination takes place. This phenomenon took place only in low load torsion test. At high load, catastrophic failure occurred.

The maximum shear stress developed due to the applied torque on the transmission shaft is 75MPa, which is very small to cause failure. Stress at the crack tip is 287MPa which is less than yielding stress of the glass fiber-polyester resin composite material.

Result confirms that maximum stress occur at the tip of the crack. The center cracks show the highest K_I . The Von Misses stress from ANSYS also reveals this fact.

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