Research Article

Mathematical Modelling of Linear Complex Mechanical Systems by Inspection Method based on Force/Torque-Voltage Analogy

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Received 20 Feb 2018, Accepted 25 April 2018, Available online 28 April 2018, Vol.8, No.2 (March/April 2018)

Abstract

This paper introduces an easiest method for modeling mechanical systems, which will be helpful for control engineers dealing with higher order mechanical systems because the method is more beneficial for higher order systems. This method is a continuation of inspection method of network solution based on KCL. The method is perfectly fit for SISO systems. Even though it can be used for modeling MIMO systems by considering one input and one output at a time. In this method, entire system static parameters and dynamic parameters have to be transformed into Laplace domain such that complete system will be represented in transformed domain. Transfer function Matrix can be derived from the transformed system by considering each mass as independent nodes. Transfer function matrix of the system relates all inputs and all outputs of the system and selecting proper combination of input and output any one can find the corresponding transfer function by using Crammers' Rule. In this way, the obtained transfer function will be exactly same as differential equation method. One limitation is the method is not suitable for such systems with nonlinear nature and system states and state trajectory are to be taken into consideration.

Keywords: KCL-Kirchhoff's Current Law, SISO-Single input single output systems, MIMO-Multi input multi output systems, Transfer Function, System states, Crammer's Rule, Transfer function matrix.

1. Introduction

This method is a continuation of inspection method used for obtaining the mathematical model of electrical networks in circuit theory. Mainly there are two popular technics available for finding out the mathematical model of any network, which are Mesh-Current analysis and Node-Voltage analysis. A is mechanical-translational system completely characterized by the system parameters like mass in kg, viscous friction coefficient in N-s/m, stiffness of the spring in N/m and there are equivalent circuits parameters of all three there in electrical system, which are resistance in ohm, inductance in henry and capacitance in farad. Among this group, the one-one equivalency depends on the type of analogy selected for the modeling. Similarly a mechanical rotational system can also be modeled with its parameters like moment of inertia, rotational viscous friction coefficient and rotational stiffness of the spring. This paper explains the transfer function modeling of complex systems by force/torque-voltage analogy.

2. Formation of system Dynamics

Force-voltage analogy applies the principles of nodevoltage analysis. In mesh-current analysis, apply KCL at each node to find out the linear node equations and represent in the form of matrix and the modeling is followed by the solution. In the beginning step obtain the system in transformed domain. If velocity (v) is the variable and is equivalent to current in electrical circuit, the opposing force offers by

Mass (M) =
$$M \frac{dv}{dt} N$$
.

Viscous friction coefficient (B) = Bv N.

Spring stiffness (K) = $K \int v dt$. N. and corresponding transform equivalents are

Mass (M) = MSv N. (S-is the Laplace Operator)

Viscous friction coefficient (B) = Bv N.

Spring stiffness (K) = K(v/S) N.

In the mechanical systems, the mass can be treated as each node and all other elements can be treated as circuit elements connected to it. In conventional methods, it is very difficult and time consuming to calculate the transfer function of a higher order mechanical system. First step is to formulate the differential equation then rearrange it and convert it

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^{*}Corresponding author's ORCID ID: 0000-0000-0000 DOI: https://doi.org/10.14741/ijcet/v.8.2.39

into Laplace domain. All these inter mediate steps can be eliminated and strait way write down the transfer function matrix from the transformed circuit.

3. Methodology outline

- a) Redraw the system with transformed parameters.
- b) Identify the no of masses in the system and determine the order of transfer function matrix and the order of the model.
- c) Number the input variables and output variables.
- d) Construct the transfer function matrix.
- e) Co-relate necessary input and output variables and calculate corresponding transfer function by using crammers rule.

4. Illustration

Consider the mechanical system in Fig.1 as an example.



Fig.1 Mechanical system

In the example the time varying quantities are represented by small letters and constants are represented by capital letters. Two forces applied to two masses are f1 and f2, and corresponding velocities in each mass are v1 and v2. The system can be represented in transformed domain as follows.



Fig.2 Laplace Transformed Mechanical system

There are two masses in the system and each mass is multiplies with a first degree term and spring stiffness is divided with a first degree term. So the order of the overall system may four and the transfer function matrix has an order of two. Exact order will be determined by the values different parameters. Consider mass M1 as the first mesh, then the

T11 term of transfer function matrix (T)=M1S+(B1+B12)+K1/S.

Since the friction in between two masses opposing the movement of each other so,

T12=-(B12) =T21.

Consider M2 as the second mesh, T22=M2S+B12+K2/S and there are two inputs f1 and f2, two outputs v1 and v2.The transfer function matrix (T) can be formed as follows,

$$\begin{bmatrix} f_1(s) \\ f_2(s) \end{bmatrix} = \\ \begin{bmatrix} M1S + (B1 + B12) + K1/S & -(B12) \\ -(B12) & M2S + B2 + K2/S \end{bmatrix} \begin{bmatrix} v1(s) \\ v2(s) \end{bmatrix}$$

From the above matrix any transfer function can be easily found out. For instance if v2=0, then two possible transfer functions are v1(s)/f1(s) and v2(s)/f1(s).Let us see how to calculate these two.

Solution v1(s)= $\Delta 1/\Delta$ (Crammers rule)

$$\Delta = \begin{vmatrix} M1S + (B1 + B12) + K1/S & -(B12) \\ -(B12) & M2S + B2 + K2/S \end{vmatrix}$$
$$\Delta 1 = \begin{vmatrix} f1(s) & -(B12) \\ 0 & M2S + B2 + K2/S \end{vmatrix}$$

Therefore,

$$v1(s)/f1(s) = \frac{M2S+B2+K2/S}{(M1S+(B1+B12)+K1/S)(M2S+B2+K2/S)-(B12)}$$

Displacement(x) = $\int v dt$, and x(s)=v(s)/S and hence

$$x1(s)/f1(s) = \frac{M2S^2 + B2S + K2}{(M1S^2 + (B1 + B12)S + K1)(M2S^2 + B2S + K2) - (B12S)^2}$$

Solution v2(s)= $\Delta 2/\Delta$ (Crammers rule)

$$\Delta = \begin{vmatrix} M1S + (B1 + B12) + K1/S & -(B12) \\ -(B12) & M2S + B2 + K2/S \end{vmatrix}$$
$$\Delta 2 = \begin{vmatrix} M1S + (B1 + B12) + K1/S & f1(s) \\ -(B12) & 0 \end{vmatrix}$$

Therefore,

$$v2(s)/f1(s) = \frac{B12}{(M1S+(B1+B12)+K1/S)(M2S+B2+K2/S)-(B12)^2}$$

Displacement(x)= $\int v dt$, and x(s)=v(s)/S, and hence

$$x2(s)/f1(s) = \frac{B12s}{(M1s^2 + (B1 + B12)s + K1)(M2s^2 + B2s + K2) - (B12s)^2}$$

Conclusion

Two transfer functions are obtained from a single model. Once developed the mathematical model, formulation of transfer function repeatedly in between any variable is easy so that repeated manipulations of

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differential equations can be avoided. The method will correctly workout with rotational systems also. The method will be more beneficial while modeling very large systems. The transfer function matrix (T) obtained is a symmetrical matrix. So the method will fit perfectly for symmetrical systems. If a particular control system is symmetrical in nature then it must be a linear system. Further study over the nature of the transfer function matrix will leads to astonishing results regarding the system modeling method and the obtained transfer function will be exactly same as the one which will be given by differential equation method.

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