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Research Article

Experimental and Numerical Buckling Analysis for Zig-Zag Model Composite Materials of Clamed-Clamped Rectangular Plates

Hussam Hussein Ali* **and Majid Habeeb Faidh-Allah**

Mechanical Department, College of Engineering, University of Baghdad

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Abstract

The main objective of this research is to determine the experimental and numerical critical buckling load of the composite materials clamped-clamped rectangular laminated plates for a zig-zag model. The first step of experimental analysis involved manufacturing the specimens of a zig-zag model from E-glass/polyester, carbon/polyester and hybrid/polyester materials to find the mechanical properties at room temperature, such as modulus of elasticity, ultimate tensile strength and Poisson's ratio. Also, the second step of experimental part is testing the shear stress of these zig-zag composite materials specimens to measure the separation of the layers in the beginning. And the third step of experimental study involved the buckling test of three types of zig-zag composite materials laminated plates with the different aspect ratios(a/b = 1, 1.5 and 2) to find the critical load that cause buckling. The numerical part deals with the finite element modeling of composite materials laminated plate for a zig-zag model, built with ANSYS software version 15.0. ANSYS element type called (shell 181). The main conclusion of this study is that the presence of the zig-zag fibers in the plate is improve the mechanical properties and the amount of critical buckling load. The layers delaminating for three types of composite materials laminated plates for a zig-zag model is not happened because the maximum shear stress results for these composite are greater than the ultimate buckling stress results. The results showed the critical buckling load is decrease when the aspect ratio is increase. This paper showed a good agreement between the experimental and numerical results, the maximum absolute percentage errors are 17.26%, 20.35% and 19.72% for E-glass/polyester, carbon/polyester and hybrid/polyester respectively.

Keywords: *Buckling analysis, composite materials model zig-zag, clamped-clamped boundary conditions, rectangular plates, ANSYS 15.0*

1. Introduction

Many sorts of failures in designing structures, some of them incorporate fatigue, creep, substitute stresses, bending, buckling and so on. buckling happens in segments, plates, shells, and different structures of standard or unpredictable geometry, in this venture just buckling of covered composite materials plates has been investigated.

The critical buckling load was called for the balance is bothered at which the minimum load, laminated composite plates are getting to be in auxiliary applications and in an extensive assortment of structures including aviation, marine and common framework progressively utilized owing in light of their high particular strength (failure stretch/unit weight) and particular stiffness (stiffness/unit weight) (Reddy J.N, 2007).

Kumar M. M. analyzed the woven glass epoxy laminated composite plates under buckling stress. It was noticed that diverse length to thickness proportion influenced the basic buckling load.

Al Humdany A. discussed the theoretical and numerical analysis for buckling of antisymmetric of simply supported laminated plates under uniaxial loads.

Ahmed N. H. concluded experimental and numerical realization of thermal and mechanical loads of composite laminated plates buckling.

Ealavarasan T., and Selvarasu. S. proposed and analyzed the buckling stress of laminated composite plates made from woven glass epoxy.

Lengvarský P. et. al., studied the buckling analysis of the composite plates with different orientations of layers.

Ugo Icardi investigated the dynamics analysis of third order zig-zag plates model for zero transverse shearing stress at lower and upper surfaces.

Cho M., and Kim J.S. analyzed the higher order zig-zag model of composites materials under compressive loads by using global-local technique.

While in this work, the buckling analysis for zig-zag model of three types composite materials clamped-

*Corresponding author's ORCID ID: 0000-0001-9027-6251,
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clamped rectangular plates is studied experimentally and numerically.

2. Theoretical part

Fundamental presumptions for laminated plates (thin plates) is given by Kirchhoff for the Classical Plate Theory (CPT). The suppositions are for the isotropic materials. These suppositions are listed in the following (Reddy J.N, 2004):

a)The plate is homogeneous.
b)The plate is thin.
c)The plate material is subjected to Hooke's law.
d)The state of plate is at plane stress.
e)The ordinary and shear strains are low. Because of these presumptions, the displacements can be depicted completely regarding the deformation of the mid-surface plane. **Figure (1)** displays the u, v and w displacements, as the following **(**Reddy J.N, 2004):

$$u(x.y)= u_o(x.y) - z\frac{\partial w_0}{\partial x} \tag{1a}$$
$$v(x.y)= v_o(x.y) - z\frac{\partial w_0}{\partial y} \tag{1b}$$
$$w(x.y) = w_o(x.y) \tag{1c}$$

Where $\frac{\partial w_0}{\partial x}$ and $\frac{\partial w_0}{\partial y}$ denote the rotations about y and x axis respectively.

The strain-displacement matrix relations take the form:

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} + z * \begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} = \begin{Bmatrix}\frac{\partial u_o}{\partial x}\\ \frac{\partial v_o}{\partial y}\\ \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x}\end{Bmatrix} + z\begin{Bmatrix}-\frac{\partial^2 w_o}{\partial x^2}\\ -\frac{\partial^2 w_o}{\partial y^2}\\ -2\frac{\partial^2 w_o}{\partial x\,\partial y}\end{Bmatrix} \tag{2}$$

The membrane strains which are $(\varepsilon_{xx}^{(0)}.\varepsilon_{yy}^{(0)}.\gamma_{xy}^{(0)})$ and the flexural (bending) strains are$(\varepsilon_{xx}^{(1)}.\varepsilon_{yy}^{(1)}.\gamma_{xy}^{(1)})$, known as the curvatures.

The relations of stress-strain transformation of an orthotropic lamina in a plane stress are; for $\bar{Q}_{ij}$, :

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_k = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_k \begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} \tag{3}$$

The forces N_{xx}, N_{yy} and N_{xy} with moments M_{xx}, M_{yy} and M_{xy} are following up on an overlay are gotten by coordination of the stress in each layer or lamina through the cover thickness. Knowing the stress as far as the uprooting, it can get thein-plane force resultant resultants N_{xx}, N_{yy}, N_{xy}, M_{xx}, M_{yy} and M_{xy}, are defined as:

$$\begin{Bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{Bmatrix} = \int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_k dz = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_k dz \tag{4a}$$

Where σ_x , σ_y and σ_{xy} are normal and shear stress

$$\begin{Bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{Bmatrix} = \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xy}^{0}\\ \varepsilon_{xy}^{0}\\ \gamma_{xy}^{0}\end{Bmatrix} + \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}^{1}\\ \varepsilon_{yy}^{1}\\ \gamma_{xy}^{1}\end{Bmatrix} \tag{4b}$$

$$\begin{Bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix} = \int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_k z\,dz = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_k z\,dz \tag{5a}$$

$$\begin{Bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix} = \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xy}^{0}\\ \varepsilon_{xy}^{0}\\ \gamma_{xy}^{0}\end{Bmatrix} + \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}^{1}\\ \varepsilon_{yy}^{1}\\ \gamma_{xy}^{1}\end{Bmatrix} \tag{5b}$$

The extensional stiffness *is* A_{ij}, the coupling stiffness *is* B_{ij}, and the bending stiffness *is* D_{ij}.

$$A_{ij}=\sum_{k=1}^{N}(\bar{Q}_{ij})_k(z_{k+1} - z_k) \tag{6a}$$
$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N}(\bar{Q}_{ij})_k\,({z^2}_{k+1} - {z^2}_k) \tag{6b}$$
$$D_{ij}=\frac{1}{3}\sum_{k=1}^{N}(\bar{Q}_{ij})_k({z^3}_{k+1} - {z^3}_k) \tag{6c}$$

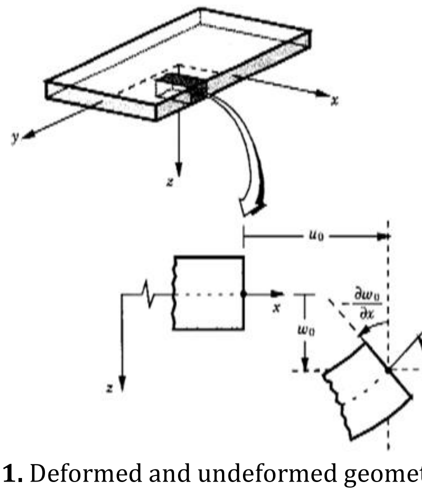

Fig.1. Deformed and undeformed geometries of an edge of a plate under the Kirchhoff assumptions

3. Experimental part

The current paper includes the experimental works below:

a. manufacturing of mold for composite materials for a zig-zag model laminated plates

The steps following are how to manufacture the mold for composite material zig-zag model laminated plates with different volume fraction:

1-Initially cutting the rectangular wood mold dimensions (60*50*10) cm^3.
2-Plan the wood and put white dots in the form of continuous lines and put spaces between the white dots about (1) cm between each point in dimensions (30*25*0.6) cm^3 rectangular shape.
3-Fix at each white point one nails to become the white point like zig-zag then put wax on photo sheet to prevent the adhesion of polyester.
4-Wrap the fibers around the nail and we do the same at each point in longitudinal direct in 90° for first layer,then repeat the same process in transverse direction in angle 0°for second layer, and repeat warp fibers in longitudinal direction in 90°in third layer to getting on three layers, Then we fix the fiber in the end with adhesive tape, as show in **figure (2)**.
5-Place the weight over the cover to ensure air bubbles will move out and the laminate will take smooth surface shape.
6-The laminate will have left for 1 days in room temperature, and then will extracted from wood mold after removing the cover.
7-For desired dimensions the laminate by(CNC) machine or by rotary cutter to obtain smooth surface with edges and reduced the thickness to (6mm). cutting composite materials plate to rectangular samples has dimensions (19*165) mm^2 using in tensile test.

Fig.2 Manufacturing of composite materials plate for a zig-zag model

b. clamped-clamped edges manufacturing

In the beginning, a solid cylinder of aluminum with a length of (40 cm) and a diameter of (13 cm), divided it into two parts, made the edge of the installation. The cylinder was rotated from the bottom almost to the jaw. The upper and lower jaw diameter (10 cm) The jaws and the position of (4) places for the screws. Afterward we turn the upper part of the cylinder on both sides and use the milling process to open a stream of (12 mm) to fit the thickness of the plate that will be used in the process of spawning and confirmed by (3) screws. as shown in **figure (3)**.

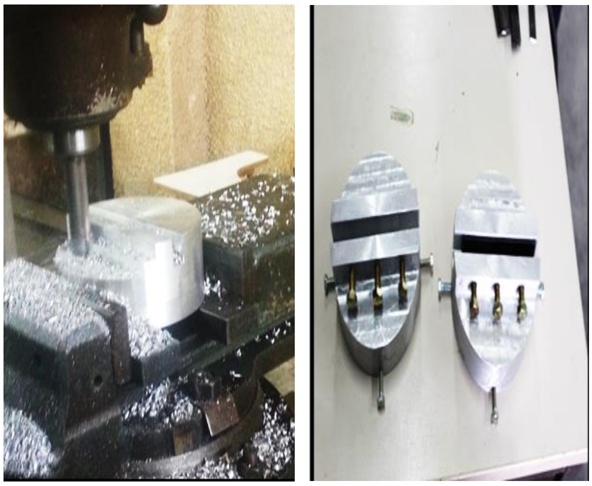

Fig.3 Jaws clamped manufacturing

c. Poisson's ratio calculation

Then we stick two strain gages on specimen of composite material put one of strain gage in the place in specimen which expect happen fracture in tensile test in longitudinal direction to extract and transverse direction then pin it on the sample. We connect the ends of the strain meter to the ends of strain gage and measure the amount of strain in longitudinal direction (ϵ_y) and transverse direction (ϵ_x). See **figure (4)**.

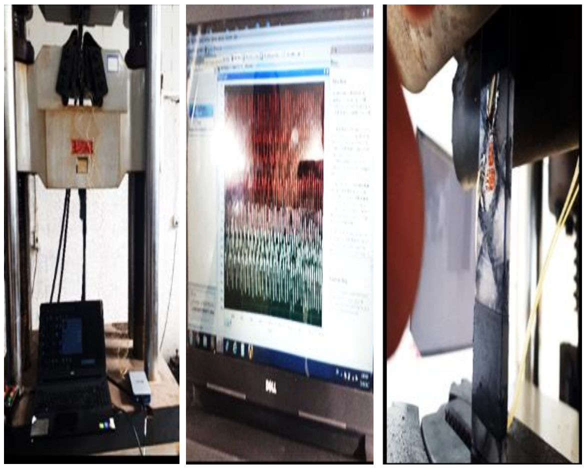

Fig.4 Poisson's ratio measuring device

d. shear test for zig-zag composite materials specimens

The shear stress of zig-zag composite materials plates test is to measure the separation of the layers in the beginning with a sheet of 18 mm thickness. After that, the samples are reduced in the form of beams. Then process the specimen to form a (10 mm) solid shaft with a length of 10 cm according to ASTM D7078 / D7078M - 12. , as shown in **figure (5)**, then we take a sample and insert it into the shearing test machine to extract the force and compare it with the critical buckling load that mean delaminating not happen. If the critical buckling force is less than the force of shear, this means that the separation of layers occurs this mean the manufacturing is good.

Fig.5 Shear stress specimens and device test

e. tensile test for zig-zag composite materials specimens

The tensile experimental test of zig-zag composite materials specimens includes the evaluation of the modulus of elasticity for composite material for three types the first glass fiber and polyester resin in directions (0°, 90°). Five samples are used for each test. Every sample is divided according to the dimensions according to ASTM (D3039/D03039M), as shown in **figure (6-a)**. The device is calibrated by the Central Organization for Standardization and Quality Control so the error rate can be considered very few.

Glass fiber-polyester specimens

Carbon fiber - polyester specimens

Hybrid-polyester specimens

Fig.6-a Tensile test specimens for three types of zig-zag composite materials

The elastic modulus of the composite material can be calculated using the load-extension curve obtained through the experimental test as well as measuring the stress of yield. Using 5 specimens for every type of composite materials model zig-zag and take readings for the five samples. see **figure (6-b)**.

Fig.6-b Tensile test machine

f. buckling test for a zig-zag composite materials laminated plates

The direction of buckling load on the plate samples is vertical, utilizing tensile test programming machine of (100kN) capability, and has dimensions, thickness 6mm and width 80 mm, with different length (80,120,160,200, and 240) mm. The specimen, it is clamped-clamped from two ends while is stay free-free at the other two ends. The buckling load at specimen is slowly (0.5mm/min). See **figure (7)**.

Fig.7 Buckling stress test

iv. Numerical part

This part deals with the finite element modeling of composite materials laminated plate for a zig-zag model, built with ANSYS software version 15.0. ANSYS element type called (shell 181). The mesh element is contain from four nodes with six degrees of freedom at each node **[9, 10]**. The modeling accuracy of composite plates is governed by the first order shear deformation theory. See **figure (8)**.

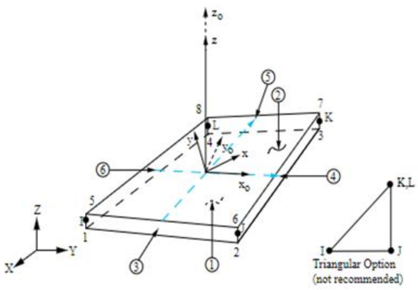

Fig.8 Shell 181 geometry (ANSYS 2012; Madenci E *et al*, 2006)

5. Results and discussion

a. experimental results and discussion

a.1 Poisson's ratio results and discussion

Dividing ϵ_y/ϵ_x to find Poisson's ratio results for three types of zig-zag composite materials laminated plates for E-glass, carbon and hybrid fibers with polyester. These results are listed in **table (1)**.

Table 1. Mechanical properties for three types of composite materials for a zig-zag model

Property	Symbol	Unit	E-glass-polyester	Carbon-polyester	Hybrid-polyester
Modulus of elasticity	E	GPa	1.522	2.0265	1.44
Poisson's ratio	ν	---	0.34	0.38	0.36
Density	ρ	kg/m³	1609	1510	1470

a.2 tensile test results and discussion

The machine of tensile test is used to calculate the modulus of elasticity for three types of zig-zag composite materials laminated plates for E-glass, carbon and hybrid fibers with polyester. See **table (1)**. Machine tensile test characterized by a maximum load (0-100 kN) with feed rate (0.005-200 mm/min). The results in this table show the modulus of elasticity value of carbon fiber with polyester is greater than that values of E-glass fiber and hybrid with polyester.

a.3 densities result and discussion

To calculate the densities and volume fractions of the three types of zig-zag composite materials laminated plates, the dial Phial and sensitive balance tools are used. All densities and volume fraction results are shown in **tables (1) and (2)** respectively.

Table 2. Volume fraction for three types of composite materials laminated plates

E-glass-polyester	Carbon-polyester	Hybrid-polyester
12.01%	13.08%	12.77%

a.4 shear test results and discussion

Table (3) shown the shear stress results for three types of zig-zag composite materials laminated plates for e-glass, carbon and hybrid fibers with polyester. These results show the shear stress value of carbon fiber with polyester is greater than that values of E-glass fiber and hybrid with polyester.

Table 3. Shear and ultimate buckling stresses values

Type of composite materials	Shear stress (τ) MPa	Ultimate buckling stress (σ_{buck}) MPa	Delamination
E-glass-polyester	19.653	18.54	Not happen
Carbon-polyester	25.74	21.354	Not happen
Hybrid-polyester	18.86	16.58	Not happen

a.5 buckling test results and discussion

The results of buckling test for clamped-clamped three types of zig-zag composite materials laminated plates are shown in **table (3).** The results in this table show the ultimate buckling stress value of carbon fiber with polyester is greater than that values of E-glass fiber and hybrid with polyester. The results of experimental critical buckling load for three types of composite materials laminated plates model zig-zag with different dimensions are shown in **table (4)**.

From the values of **table (3)**, because of the values of maximum shear stress are greater than that the values of ultimate buckling stress for three types of zig-zag composite materials laminated plates, that mean the layers delaminating for three types of zig-zag composite materials laminated plates is not happen.

Table 4 Experimental critical buckling load for three types of composite materialslaminated plates for a zig-zag model

No.	Dimensions of plate (mm²)	E-glass/polyester Pcr (kN)	Carbon/polyester Pcr (kN)	Hybrid/polyester Pcr (kN)
1	80*80 a/b = 1)(	8.9	10.25	7.96
2	120*80 (a/b = 1.5)	3.3	5.2	3.101
3	160*80 (a/b = 2)	2.06	2.25	2.1

b. numerical results and discussion

The dimensions of the composite materials laminated plates model zig-zag were used in this work are mentioned in **figures (9, 10, and 11)** and **table (5)** with critical buckling load numerically results by using ANSYS program15.0.

a/b = 1 a/b = 1.5 a/b = 2

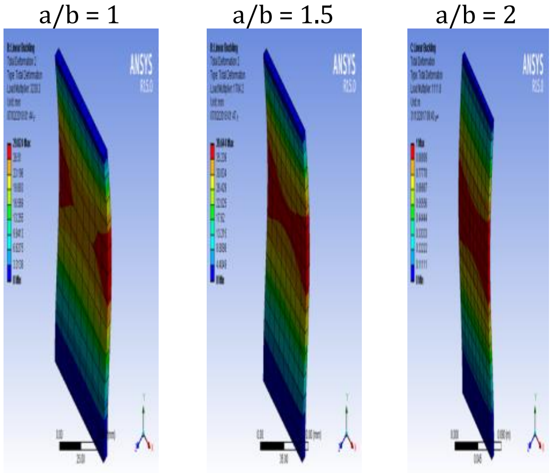

Fig.9 Numerical critical buckling analysis for E-glass/polyester laminated plate with different aspect ratios

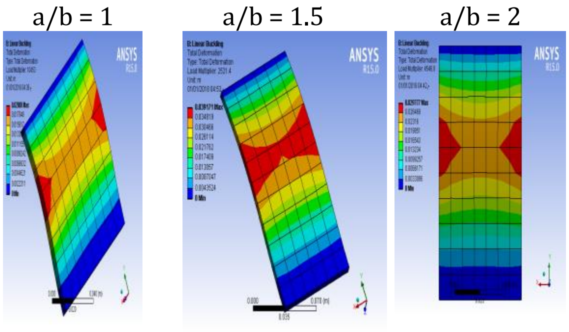

Fig.10 Numerical critical buckling analysis for carbon fiber/polyester laminated plate with different aspect ratios.

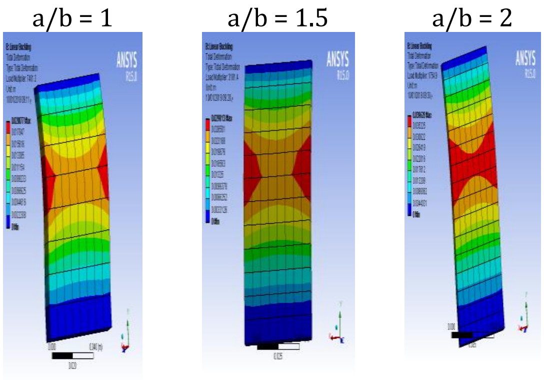

Fig..11 Numerical critical buckling analysis for hybrid fibers / polyester laminated plate with different aspect ratios.

In order to compare between the experimental and numerical results, a good agreement between these results of three types of zig-zag composite materials laminated plates for E-glass, carbon and hybrid fibers with polyester from **tables (4 and 5)**, the maximum absolute percentage errors are 17.26%, 20.35% and 19.72% for E-glass, carbon and hybrid fibers with polyester respectively.

Table 5. Numerical critical buckling load for three types of composite materials laminated plates a zig-zag model

No.	Dimensions of plate(mm²)	E-glass/polyester Pcr (kN)	Carbon/polyester Pcr (kN)	Hybrid/polyester Pcr (kN)
1	80*80 (a/b = 1)	7.502	10.453	7.401
2	120*80 (a/b = 1.5)	3.23	4.543	3.181
3	160*80 (a/b = 2)	1.784	2.521	1.754

Conclusion

The critical buckling load increasing with decrease of deflection for three types.

The layers delaminating for three types of composite materials laminated plates for zig-zag model is not happened because the maximum shear stress results for these composite are greater than the ultimate buckling stress results. The results showed the critical buckling load is decrease when the aspect ratio is increase.

A good agreement between the results of three types of zig-zag composite materials laminated plates for E-glass, carbon and hybrid fibers with polyester, the maximum absolute percentage errors are 17.26%, 20.35% and 19.72% for E-glass, carbon and hybrid fibers with polyester.

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Nomenclature

Latin Symbols

Symbol	Description	Units
A	Extensional stiffness	N/mm
a	Plate length	mm
B	Coupling stiffness	N/mm
b	Plate width	mm
D	Bending stiffness	N/mm
E	Modulus of elasticity	kN/mm²
h	Plate thickness	mm
M	Moment	N.mm
N	Force	N
P_{cr}	Critical buckling load	kN
u, v, and w	Displacements in x, y, and w respectively	mm

Greek Symbols

Symbol	Description	Units
ρ	Material density	(kg/m^3)
τ	Shear stress	MN/m^2
ν	Poisson's ratio	dimensionless
σ_{buck}	Ultimate buckling stress	MN/m^2
ε	Strain	mm/mm
γ	Shear strain	radian