International Journal of Current Engineering and Technology
E-ISSN 2277 – 4106, P-ISSN 2347 – 5161

Available at http://inpressco.com/category/ijcet

Research Article

Model-Based Design of Piezoelectric Patches used to Repair Damaged Beams under Static Load

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Received 15 Dec 2017, Accepted 15 Feb 2018, Available online 22 Feb 2018, **Vol.8, No.1 (Jan/Feb 2018)**

Abstract

Static loads exposing to mechanical components can cause cracks, which are lead to form stress concentration regions causing the failure of structure. Generally, from 80% to 90% of structure failure is due to initiation of the cracks. Therefore, it is necessary to repair the crack and reduce its effect on the structure where the effect of the crack is modelled as an additional flexibility to the structure. In the last few years, piezoelectric materials have been considered as one of the most favourable repairing techniques. The piezoelectric material converts the applied voltage on it to a bending moment to counter the bending moment caused by the external load on the beam at the crack location. In this study, the design of the piezoelectric materials used to repair effect of crack on the mechanical behaviour of beam subjected to static loads is analytically achieved. This design includes calculating of desired dimensions of the material with the required voltage applied on it. The additional flexibility is expressed in term of a proposed unitless factor which can be calculated depend on experimental work. The results show that increasing the patch thickness increases the beam resistance to crack and load effects, while increasing the length of the piezoelectric material reduces the magnitude of the voltage required to repair the cracked beam.

Keywords: *Piezoelectric materials, simply supported beam, additional flexibility*

1. Introduction

Using composite patches in repairing for effect of crack structures has been studied extensively where the composite patches have a good mechanical properties of high stiffness, high strength in addition to its lightweight. Although, these composite patches have proven effective but there are some of the limitations that exist in its application. Piezoelectric materials that have been extensively investigated can be used as an alternative method for repairing for effect of cracked components because it has feature of affecting the repair in an active method.

Most of researchers have been studied repairing of damaged beam that subjected to static and dynamic loads by using piezoelectric patches, focusing on damaged cantilever beam as structure where generally, from 80% to 90% of structure failure is due to initiate the cracks. Therefore, it is necessary to repair the crack and reduce its effect on the structure (Callister *et al*, 2011).

Agrawal and Treanor studied analytically and experimentally the best location and voltage of piezoelectric actuator for shape control using a model of intelligent structures to minimize the error percentages between the desired and achieved shape.

They concluded that using of four piezoelectric actuators and equally distributed along beam give the best results if it compared with using number of piezoelectric actuator less than number of above mentioned.

Wang *et al.* utilized the repair of a cracked cantilever beam under a dynamic load using the electro-mechanical properties of piezoelectric material. They mathematically showed the relationship between crack parameters, bending moment, voltages, and piezoelectric pitch lengths. They used ABAQUS to verify the proposed repair where they compared with the analytical results. They obtained parameter that represents value of G_0 where restore value of natural frequency of the damaged beam to its healthy state value.

Liu studied numerically repair process of damaged structure by multi-layer piezoelectric patches where it attached on the damaged cantilever beam. He noted that the best choosing for design of piezoelectric patches is increasing the number of layers, increasing the patch length and reducing the patch and also there is a negative effect for using high voltage on size of crack (enlarge the crack).

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Mineto *et al.* , used a cantilever beam as a model based on Euler-Bernoulli theory using some assumptions: the thickness of the piezoelectric patch layer is small compared to the length of the beam, the electric field between the upper surface and the lower surface of the piezoelectric layer is uniform to know the power generated in a cantilever beam where used a type of piezoelectric material (PZT 255) and prove that maximum force that occur at the end of the beam, thus the best position for the piezoelectric is at the end of the beam. They concluded a relationship between power and force location where they noted that the power is constant with increasing force location until 0.1 m from fixed area and gradually increased until the maximum value of power at end of beam.

Al-Ashtari studied analytically and numerically repair of cracked beam (cantilever) by using piezoelectric patches. He used perturbation method for deriving the solution in addition to a novel analytical model to design piezoelectric patches used to repair. He showed that there is a relationship between the thickness of piezoelectric patches and thickness of cracked beam, electro-mechanical properties of the piezoelectric patches material, applied load and crack location. He resulted that increasing of the patch thickness improved the beam resistance to crack and load effect.

This study focused analytically using of a notched simple supported beam model where it has manufactured required rigs for fixing the specimens that has investigated under effect of static loads at different boundary condition.

2. Model of Simply Supported Beam

An Aluminum simply supported beam that as shown in figure (1) has a total length (l_b), thickness (h) and unit width. A compressive load (W) is applied to this beam and it lies at distance (l_f) away from one end of the beam. This beam is selected to study the parameters, which influence the repair of the damaged beam by using piezoelectric patches. The same procedure that has followed to obtain the deflection of the healthy beam used in damaged and repaired beam. Three cases of the simply supported beam (healthy, damaged and repaired beam) used to obtain the value of deflection for the damaged beam to a value equaled to the value of deflection for the healthy beam by using the piezoelectric patch.

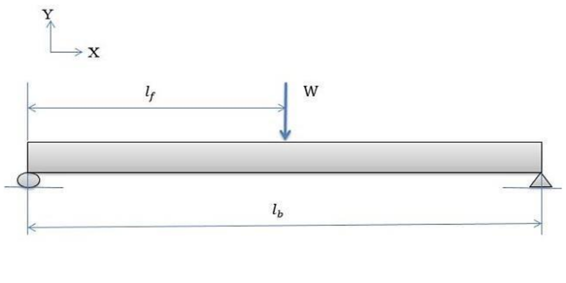

Fig.1: Simply supported beam

2.1 Healthy simply supported beam

The healthy beam that beam does not include any damage is as shown in figure 1. A static force (W) is applied at $x = l_f$ where the maximum deflection occurs. The x-axis denotes the coordinate along the length of the beam with its origin at the left end of the beam. The positive direction of the deflection for this beam (y) is defined downward. A static force W is applied at $x = l_f$. The slope and deflection can determine based on the Euler– Bernoulli equation that can be expressed as (Thomson, 1996): -

$$M = Y_b I_b \frac{d^2y}{dx^2} \tag{1}$$

By substituting the value of externally applied moment in equation (1), it becomes the following:

$$Y_b I_b \frac{d^2 y_h}{dx^2} = \frac{W}{2}.x - W\left(x - \frac{l_b}{2}\right) \tag{2}$$

Where:

y_h,($\frac{dy_h}{dx} = \emptyset_h$) are the deflection and the slope of the healthy beam respectively.

The slope and deflection of healthy beam becomes as the following:

$$\emptyset_h = \frac{W}{2Y_bI_b}\left(\frac{x^2}{2} - \frac{{l_b}^2}{16} - \left(x - \frac{l_b}{2}\right)^2\right) \quad 0 \le x \le l_b \tag{3}$$

$$y_h = \frac{W}{2Y_bI_b}\left(\frac{x^3}{6} - \frac{{l_b}^2}{8}x - \frac{1}{3}\left(x - \frac{l_b}{2}\right)^3\right) \quad 0 \le x \le l_b \tag{4}$$

Where: -

y: The deflection of the beam in (mm)

Y_b : The modulus of elasticity in (MPa).

I_b : Moment of inertia in mm$^{4.}$

M : The external applied moment in (N.m)

x: The distance between the compressive force and the end of beam in (mm)

It is known that the maximum deflection occurs at the midpoint of the beam while the maximum slopes occur at both of end for support in simply supported as shown in the following equations:

$$y_h(\max) = \frac{W{l_b}^3}{48Y_bI_b} \qquad at \quad x = \frac{l_b}{2} \tag{5}$$

$$\emptyset_h(\max) = \frac{W{l_b}^2}{16Y_bI_b} \qquad at \quad x = 0 \quad or \quad x = l_b \tag{6}$$

2.2 Damaged simply supported beam

The beam that contains a damage can be called damaged beam. The beam, which has a vertical notch located at l_d from the left end of the beam, is as shown in figure **(2).** The beam is isotropic with Young's modulus Y_b) and mass density (ρ).

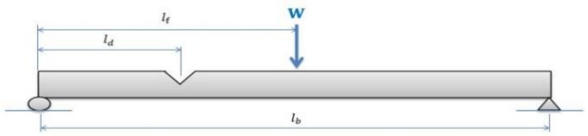

Fig.2: Simply supported beam includes damage

Fig.3: A. Dimensions of notch   B. Slope of the beam at damaged location

At the same procedure that has followed, to obtain the deflection of the healthy beam used in the damaged beam. The deflection of the beam can be found by using Euler– Bernoulli equation taking into consideration the location of force and crack on the beam as shown in the following equations (Wang *et a*l, 2002).

$$y_{d1} = \frac{l_b - l_f}{6 l_b Y_b I_b} W x^3 + ax + b \qquad 0 \le x \le l_d \tag{7}$$

$$y_{d2} = \frac{l_b - l_f}{6 l_b Y_b I_b} W x^3 + cx + d \qquad l_d \le x \le l_f \tag{8}$$

$$y_{d3} = \frac{l_f}{6 l_b Y_b I_b} W (l_b - x)^3 + ex + f \quad l_f \le x \le l_b \tag{9}$$

The constants a, b, c, d, e and f that appeared in equations (7), (8) and (9) can be calculated by applying the following boundary conditions:

$$\begin{array}{lll} at & x = 0 & y_{d1} = 0 \\ at & x = l_b & y_{d3} = 0 \\ at & x = l_d & y_{d1} = y_{d2} \\ at & x = l_f & y_{d2} = y_{d3} \\ at & x = l_f & y'_{d2} = y'_{d3} \\ at & x = l_d & y'_{d2} - y'_{d1} = \mu y'_{d2} \end{array}$$

Where y'_d is the first derivative of the deflection with respect to $x-$axis.Thus, the value of constants a, b, c, d, e and f are as followings:

$$a = A \times \left[-3 \times \mu \times {l_d}^2 \times B\right] - [c(\mu - 1)]$$

$$b = 0$$

$$c = \frac{\left[A \times [(l_f \times B^3) - ({l_f}^3 \times B)]\right] + \left[[A \times l_d] \times [3 \times \mu \times {l_d}^2 \times B]\right] - \left[[A \times B] \times \left[\left(3 \times l_f \times (B^2)\right) + (3 \times {l_f}^2 \times B)\right]\right]}{[l_f + B - (\mu \times l_d)]}$$

$$d = A \times \left[[l_f \times B^3] - [{l_f}^3 \times B]\right] + \left[[A \times B] \times [-3 \times l_f \times B^2] - [-3 \times B \times {l_f}^2]\right] - c[l_f + B]$$

$$e = c - \left[A \times \left[[-3 \times l_f \times B^2] - [-3 \times B \times {l_f}^2]\right]\right]$$

$$f = -e l_b$$

Where:

$$A = \frac{W}{6 l_b Y_b I_b}$$

$$B = l_b - l_f$$

By substituting, the value of the constants a, b, c, d, e and f in equation (7), (8) and (9) then deflections can be calculated.

The notch has a depth (t_d) and width (b_d) as shown in figure (3A) where this notch causes discontinuous of the slope of the beam as shown in figure (3B).

From figure (3B), the effect of the damage can be represented by a break in the slope at the damage location (Krawczuk *et al*, 1995). The parameter μ represents the additional flexibility of the beam due to the damage where this parameter can be found by Matlab programming (Try and errors) and according to the results of maximum deflection for healthy and damaged beams ($l_b/2$, $l_b/4$, $l_b/8$) that obtained from experimental work. Additional flexibility depends on the mechanical properties of beam and dimensions and location of notch.

2.3. Effect of Piezoelectric Patches on Beam

Piezoelectric materials convert mechanical energy into electrical energy (direct piezoelectric effect) and electrical energy to mechanical energy (indirect piezoelectric effect) (Vijaya, 2012). The charge, which generated by the piezoelectric layer can be calculated from below equation (Lee *et al*, 2004):

$$Q = -e_{31} \int_0^{l_b} \left(\frac{h+\delta}{2}\right) y''_d \, dx \tag{10}$$

Where:
$\boldsymbol{Q}$: The charge generated by the piezoelectric layer.
$\boldsymbol{e_{31}}$: The piezoelectric constant.
$\boldsymbol{\delta}$: Thickness of piezoelectric patches.
The sensor output voltage is given by:

$$V_s = \frac{Q}{C_v} = -e_{31} \left(\frac{h+\delta}{2 C_v}\right) \int_0^{l_b} y''_d \, dx \tag{11}$$

Where $\boldsymbol{C_v}$ is the electric capacitance

The voltage applied to the piezoelectric actuator can be written as:

$$V_a = \mathrm{g}\, V_s = -s \frac{e_{31}(h+\delta)}{2} \int_0^{l_b} y''_d \, dx \tag{12}$$

Where g is the control gain factor; $s = \frac{\mathrm{g}}{C_v}$

The axial stress along the piezoelectric by the applied voltage (Sun *et al*, 1995), as

$$\sigma_x = e_{31} \frac{V_a}{\delta} = -s \frac{e_{31}^2 (h+\delta)}{2\delta} \int_0^{l_b} y''_d \, dx \tag{13}$$

This will affect a bending moment on the beam given by:

$$M_a = \sigma_x \delta \left(\frac{h+\delta}{2}\right) = -\frac{s e_{31}^2}{4} (h+\delta)^2 \int_0^{l_b} y''_d \, dx = -G y'_d \big|_0^{l_b} \tag{14}$$

Where $G = \frac{se_{31}^2}{4}(h+\delta)^2$ and it represents repair moment coefficient

The voltage that applied to the piezoelectric patch is obtained using equation (12) and can be expressed as

$$V_a = -\frac{2(l_b - l_f)Wl_d}{l_b e_{31}(h+\delta)} = -\frac{2M_{Fc}}{e_{31}(h+\delta)} \quad (15)$$

Where:
M_{Fc} : Moment at the damage location.
The applied voltage is depended on not only h, δ and e_{31} but also depends on the external moment and hence W, l_d, i_f and l_b

2.4. Repair of Damaged Beam Using Piezoelectric Patches

The Piezoelectric Patches used to repair the effect of the notch in the damaged beam as shown in figure (4) where the principle of repairing depends on generating of opposite moment for the resulted moment of applied external force that means the deflection of the damaged beam will be as close as possible of the value of deflection for the healthy thus, the damaged beam acts as if the notch does not exist.

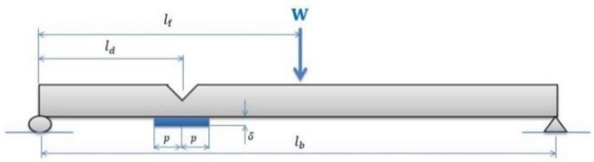

Fig. 4: Repaired beam with piezoelectric patch

Piezoelectric material that used to repair damaged beam has a total length ($2p$), thickness (δ), width (w) in addition to Young modulus (Y_p).

The required voltage that operates the piezoelectric can be calculated from equations of piezoelectric. This patch fixed at the lower part of the beam as shown in figure (4).

The determination of the deflection for the repaired beam based on Euler– Bernoulli equation as following:

$$Y_b I_b y''_{d1} = \frac{l_b - l_f}{l_b} Wx \qquad 0 \le x \le (l_d - p) \quad (16)$$

$$Y_b I_b y''_{d2} = \frac{l_b - l_f}{l_b} Wx + G(y'_4|_{l_d+p} - y'_1|_{l_d-p}) \qquad (l_d - p) \le x \le l_d \quad (17)$$

$$Y_b I_b y''_{d3} = \frac{l_b - l_f}{l_b} Wx + G(y'_4|_{l_d+p} - y'_1|_{l_d-p}) \qquad l_d \le x \le (l_d + p) \quad (18)$$

$$Y_b I_b y''_{d4} = \frac{l_b - l_f}{l_b} Wx \qquad (l_d + p) \le x \le l_d \quad (19)$$

$$Y_b I_b y''_{d5} = \frac{l_f}{l_b} W(l_b - x) \qquad l_f \le x \le l_b \quad (20)$$

The deflection is calculated by double integrating of the above-mentioned equations:

$$y_{d1} = \frac{l_b - l_f}{6 l_b Y_b I_b} Wx^3 + ax \quad 0 \le x \le (l_d - p) \quad (21)$$

$$y_{d2} = \frac{l_b - l_f}{6 l_b Y_b I_b} Wx^3 + \frac{G}{2Y_b I_b}\left\{ \frac{l_b - l_f}{2 l_b Y_b I_b} W(l_d + p)^2 + b - \frac{l_b - l_f}{2 l_b Y_b I_b} W(l_d - p)^2 - a \right\} x^2 + ex + f \quad (22)$$

$$y_{d3} = \frac{l_b - l_f}{6 l_b Y_b I_b} Wx^3 + \frac{G}{2Y_b I_b}\left\{ \frac{l_b - l_2}{2 l_b Y_b I_b} W(l_d + p)^2 + b - \frac{l_b - l_f}{2 l_b Y_b I_b} W(l_d - p)^2 - a \right\} x^2 + gx + h \quad (23)$$

$$y_{d4} = \frac{l_b - l_f}{6 l_b Y_b I_b} Wx^3 + bx + c \quad (l_d - p) \le x \le l_f \quad (24)$$

$$y_{d5} = \frac{l_f}{6 l_b Y_b I_b} W(l_b - x)^3 + d(x - l_b) \quad l_f \le x \le l_b \quad (25)$$

The constants a, b, c, d, e, f, g and h that appeared in equations (21), (22), (23), (24) and (25) can be calculated by applying the following boundary conditions:

at $x = l_d - p$ $\quad y_{d1} = y_{d2}$; $y'_{d1} = y'_{d2}$
at $x = l_d + p$ $\quad y_{d3} = y_{d4}$; $y'_{d3} = y'_{d4}$
at $x = l_f$ $\quad y_{d4} = y_{d5}$; $y'_{d4} = y'_{d5}$
at $x = l_d$ $\quad y_{d2} = y_{d3}$
at $x = l_d$ $\quad y'_{d3} - y'_{d2} = \mu y'_{d3}$

The value of constants a, b, c, d, e, f, g and h are as following:

$$a = \frac{3zw(l_d)^2 - m(s\mu l_d - sr - s^2 + \mu s^2)}{m(r - k - \mu l_d + \mu k) + 1} - \frac{b(m(\mu l_d - k + \mu k) - 1 - \mu)}{m(r - k - \mu l_d + \mu k) + 1}$$

$$b = \frac{\left[ms\left\{ \frac{k^2}{2} - \frac{3}{2}r^2 - l_d(r-k) \right\} - \frac{l_f w}{6 l_b Y_b I_b}(l_b - l_f)^3 + zw(l_f)^3 + (l_f - l_b)\left\{ \frac{l_f w}{2 l_b Y_b I_b}(l_b - l_f)^2 - 3zw(l_f)^2 \right\} \right]}{\left[\left\{ m\left(\frac{r^2}{2} - l_d k + \frac{k^2}{2} \right) + l_d - l_b \right\} - \left\{ \frac{(m(\mu l_d + k - \mu k) + \mu - 1)\left(m\left(\frac{r^2}{2} + l_d k - l_d r - \frac{k^2}{2} \right) - l_d \right)}{\{ m(r - k - \mu l_d + \mu k) + 1 \}} \right\} \right]}$$

$$c = (-l_b)b + \frac{l_f w}{6 l_b Y_b I_b}(l_b - l_f)^3 - zw(l_f)^3 + (l_f - l_b)\left\{ -\frac{l_f w}{2 l_b Y_b I_b}(l_b - l_f)^2 + 3zw(l_f)^2 \right\}$$

$$d = b - \frac{l_f w}{2 l_b Y_b I_b}(l_b - l_f)^2 + 3zw(l_f)^2$$

$$e = (mr + 1)a - mrs$$

$$f = \frac{mr^2}{2}(-a - b + 3s)$$

$$g = (mk)a + (1 - mk)b - mks$$

$$h = (l_d)mra + (l_d)a - (l_d)mrs - \frac{mr^2}{2}\left(a + b + \frac{3}{2}s\right) - (l_d)mka - (l_d)b + (l_d)mkb + (l_d)mks$$

Where:
$m = \frac{G}{Y_b I_b}$ $\qquad r = (l_d - p)$ $\qquad k = l_d + p$ $\qquad s = 3zw(k^2 - r^2)$ $\qquad z = \frac{l_b - l_f}{6 l_b Y_b I_b}$

By substituting, the value of the constant a, b, c, d, e, f, g and h in equation (21), (22), (23), (24) and (25) then deflections can be calculated.

3. Design of Piezoelectric Patches

In order to design the piezoelectric patches correctly and effectively, this is done by using a safety factor so that the design is at the maximum load applied to ensure the safety of patches and keep them from damage. The safety factor value is greater than one. The relationship between the safety factor and applied voltage can be express as the following:

$$V_{max} = -\frac{2(l_b - l_f)W(S.F)l_d}{l_b e_{31}(h+\delta)} \qquad (26)$$

$$V_{max} = E.\delta \qquad (27)$$

Where:
E : is the maximum electrical field on the piezoelectric patches

By equalling the equations (26) and (27), then the thickness of piezoelectric patch as follows:

$$\delta = \frac{1}{2}\left[\sqrt{h^2 - \frac{8(l_b - l_f)W(S.F)l_d}{E l_b e_{31}}} - h\right] \qquad (28)$$

Using equation (15) with the voltage factor has a value less than one.

$$\alpha_V = \frac{V_a}{V_{max}} \qquad (29)$$

Where:
α_V: The voltage factor that has a value from zero to one.

$$V_a = -\frac{2(l_b - l_f)W(\alpha_V)l_d}{l_b e_{31}(h+\delta)} \qquad (30)$$

In this paper, a theory of repair is obtained if the piezoelectric patches change the slope of the damaged beam and return to the original condition when there is no damage. This means that the slope of the repaired beam is equal to the slope of the healthy beam, thus it is $y'_{d2} = y'_h \quad at \quad x = l_d$

The slope $y'_{d2}\ at\ x = l_d$ can be found from equation (22) and substituting the value of constant previous-mentioned in repaired beam then the slope is as follows:

$$y'_{d2} = \frac{l_b - l_f}{2l_b Y_b I_b} Wx^2 + \frac{G}{Y_b I_b}\left\{\frac{l_b - l_f}{2l_b Y_b I_b} W(l_d + p)^2 + b - \frac{l_b - l_f}{2l_b Y_b I_b} W(l_d - p)^2 - a\right\} x + e \qquad (31)$$

The required length of the piezoelectric patch can be calculated from equation (31) and equation (4), by using the above-mentioned boundary condition then the length of piezoelectric patches:

$$2p = \frac{Y_b I_b}{G}\left[\frac{l_b\left\{\frac{l_d^2}{2} - \frac{l_b^2}{16}\left(l_d - \frac{l_b^2}{2}\right)\right\}}{2l_d^2(l_b - l_f)} - \left\{\frac{1}{2} - \frac{Y_b I_b l_b}{w l_d (l_b - l_f)}\left((b-a) - \frac{e}{l_d}\right)\right\}\right] \qquad (32)$$

4. Experimental work

This part includes determination of value additional flexibility, which appeared in the most of equations and to calculate this parameter, a rig has manufactured for achieving this purpose as shown in figure (5).

Fig. 5: Required rig

Four samples of aluminium beam that has (600 mm) length, (10 mm) width and (6 mm) thickness are used for calculating the deflection for everyone. These four samples represent one for healthy beam and three for damaged beams but every beam has a notch at a certain location of the length of beam ($l_b/2$, $l_b/4$, $l_b/8$) as shown in figure (6). The width of the piezoelectric is considered equal to beam width.

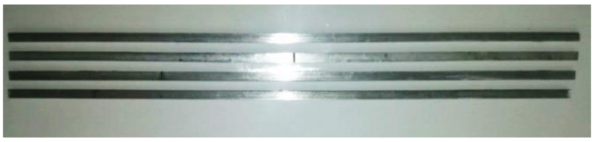

Fig.6: The used samples

One end of beam is fixed (U_y= 0, U_x =0, U_z =0) and the second the ((U_y= 0, U_x =0, U_z is free) where (10 N) compressive force subjected to at the midpoint for all of beams using dial gauge for measuring the maximum deflection that occurs because of applied force as shown in figure (7) as an important data input to get the additional flexibility.

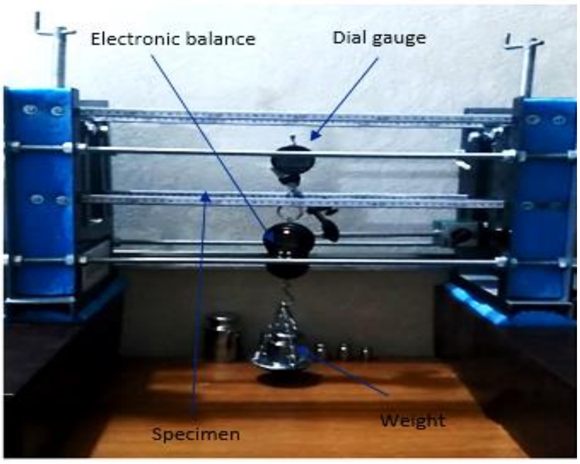

Fig.7: Procedure for measuring the maximum deflection using the rig

5. Results and Discussion

5.1 Results of experimental work

The results of additional flexibility that have obtained by experimental work can be shown in Table 1.

Table 1: Results of additional flexibility for different cases

Case	Location of crack	Additional flexibility
1	0.5 l_b	0.05
2	0.25 l_b	0.09
3	0.125 l_b	0.19

It is observed that the values of additional flexibility were close for the above-mentioned cases because of the low value for the applied force and moment of inertia. In general, the minimum value of additional flexibility occurred in case number one but in case number three, its value is the highest that means the damaged beam is close to healthy beam thus, the effect of crack is unnoticeable.

5.2 Results of analytical part

A piezoelectric layer that used in this study, bonded at the bottom of the beam and symmetrically attached according to the location of damage has piezoelectric constant (e_{31} = -9.29). The required length, thickness and voltage of piezoelectric material that have been obtained from equations (28) and (32) as follows:

2 p = 45 mm , δ = 0.9 mm and V_a = 46.8 volts

Figure (8) gives a plot between the deflection and length of the beam for healthy and damaged beams where it is noted that maximum value of deflection occurred when the location of crack is at the midpoint of the beam. The value of maximum deflection in case three was equal to its value in the healthy beam that means generally, in simply supported beam whenever the crack is close to the support then its the effect will be less.

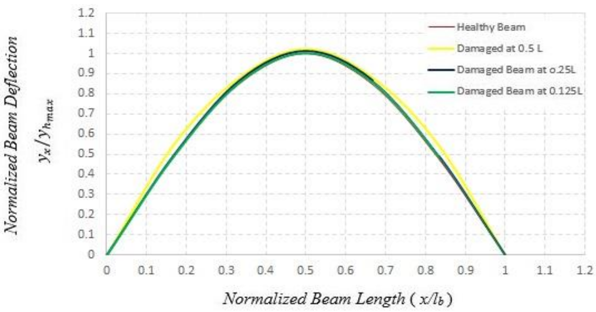

Fig.8: Deflection of the beam before repair plot

The influence of piezoelectric material on the damaged beam in three cases is as shown in figure (9). It is observed that there is very little variation in values of the deflections for three cases where all results of deflection that have obtained from analytical solution are close or equal to values of deflections for the healthy beam.

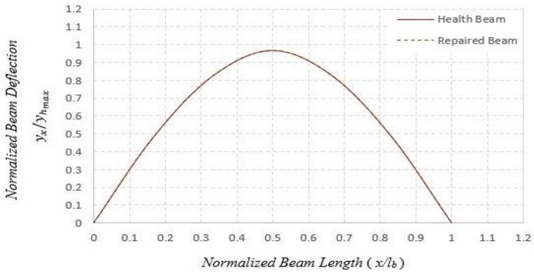

Fig. 9: Deflection of the damaged beam with piezoelectric patches plot

Through the results that have obtained from analytical solution, there is a relationship between the required voltage for piezoelectric material and the location of crack as shown in figure (10). It is found that the required voltage increases when the location of a crack approaches to the middle of the beam then its value decreases as the location of crack moves away from the middle of the beam. The cause is very clear because the required voltage directly proportional to the location of the crack as shown in equation (30).

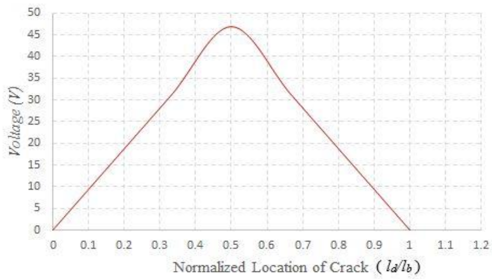

Fig.10: Voltage – Location of the damage beam plot

The relationship between the required voltage and the thickness of the piezoelectric material is an inverse relationship as shown in figure (11). The voltage decreases when the thickness of the piezoelectric material is increased. The increasing the value of thickness will increase the value of the denominator in equation (30) thus the required voltage will decrease.

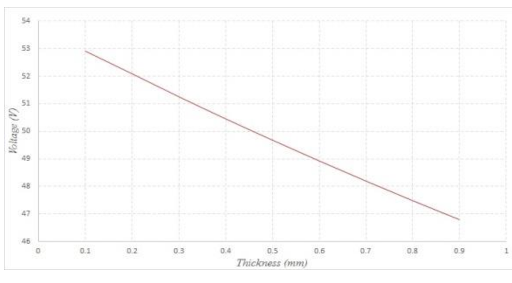

Fig.12: Voltage – Thickness of patch plot

Conclusion

This paper shows a method to repair damaged beam using piezoelectric materials analytically. The piezoelectric materials are used as actuators and moment treatment where they generated opposite moment, thus decreasing the high slope, which is increased due to damage. To clarify this concept, a simply supported beam is used to find a relationship between required voltage to operate the piezoelectric patches with some of parameters such as engineering properties of beam material and piezoelectric patch layers. The most important conclusions can be summarized as follows:

1) The piezoelectric patches technique is the best technique for repairing of damaged beam.
2) Increasing the patch thickness enhances the beam resistance to crack and load effect, while increasing the length of piezoelectric patch reduce the magnitude of the voltage required to repair the crack beam.
3) The required voltage increases when the location of a crack approaches to the middle of the beam.

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