

Research Article

Method to Reduce the Complexity of Centre of Gravity Method used for Defuzzification

Nikhil Binoy C^{†*}

[†]Department of Instrumentation and Control Engineering, N.S.S. College of Engineering, Palakkad, India

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Abstract

Fuzzy logic is accepted by scientific fields due to its simplicity. Defuzzification is one part which is used to convert the fuzzy value given by the inference system to crisp value. Several methods are used for defuzzification. Centre of gravity method is one method used for defuzzification due to its continuity. Several algorithms are available to solve defuzzification using the centre of gravity method. This paper suggests a method to reduce the complexity of centre of gravity method through partition where each partition has a maximum of four possibilities.

Keywords: Defuzzification; centre of gravity.

1. Introduction

Fuzzy logic is used to handle variables with values ranging from 0 to 1 where the Boolean logic is used to handle variables with two values 0 and 1. Fuzzy systems are used to process continuous variables by using fuzzy logic.

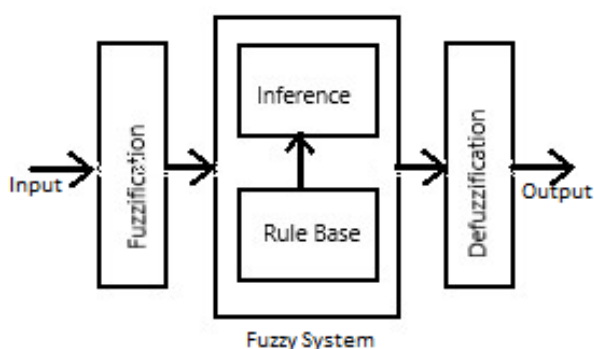


Fig.1 Block Diagram of Fuzzy System

Fig.1 shows the schematic diagram of fuzzy systems. Main parts of fuzzy system are (1) fuzzification, (2) inference, (3) rule base, and (4) defuzzification. Fuzzification is used to convert the value of input variable(s) into fuzzy value. Inference mechanism is used to find the output from input(s) using fuzzy logic based on the rules written in rule base. Defuzzification is the process of converting the output linguistic variable into numeric values.

Classical methods like centre of gravity method and centre of sum method are widely used because they can easily be implemented. Also, these methods have scientific reasoning with reasonable accuracy [Farzad Tahiri *et al*, 2014; H. R. Mahdiani *et al*, 2013]. Continuity of defuzzification is important. The change in output should be small with respect to changes in the membership value of output fuzzy set. Therefore the centre of gravity method and the centre of area method are the best choices for fuzzy controllers. For N quantizing samples, centre of gravity method and centre of area method respectively, have 3N-1 and 4N number of operations, quite small compared to other methods, and these methods are fast [Dragan Z *et al*, 2002].

Numerical accuracy is increased and cost implementation using hardware and software is decreased by using TMA, TWTMA and RWTMA [H. R. Mahdiani *et al*, 2013]. Intersection of convex membership functions like triangle or trapezoidal functions are convex functions. For these functions, arithmetic implication is possible instead of logic operations [J. Dombi *et al*, 2017].

The centre of gravity method calculates the centre of gravity of area of membership function of all active output fuzzy sets and then the numerical value of the output variable. Centre of gravity method is given in equation (1).

$$y_c = \frac{\int y \mu_B(y) dy}{\int \mu_B(y) dy} \quad (1)$$

where y_c : numerical value of output; y : output linguistic variable; and $\mu_B(y)$: membership value of union of fuzzy sets

*Corresponding author's ORCID ID: 0000-0002-5311-4330

2. Scope of this paper

Programming defuzzification is easy by using Matlab due to the availability of fuzzy tool box. But those who are designing fuzzy systems using microcontrollers, it is very difficult to write an algorithm for centre of gravity method. For real-time operations or discrete control operations, speed of algorithm is important. If the used algorithm is taking more time for calculation, chance of deadline miss is there or relative stability may be decreased if it is a part of real-time system or control system respectively.

The aim of this paper is to design an algorithm which is able to reduce the complexity and can increase the speed of computation of centre of gravity method.

3. Proposed method

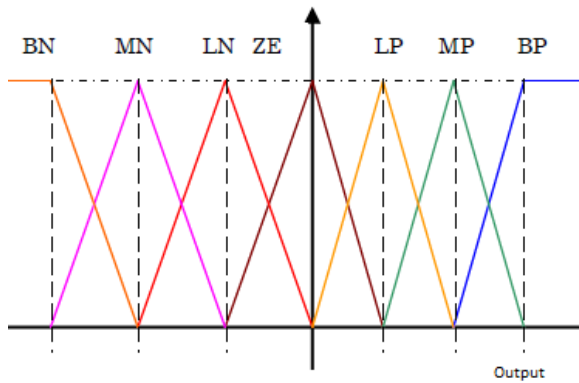


Fig.2 Partition of Union of Output Fuzzy Sets

Complexity of centre of gravity method is reduced through partition as shown in Fig.2.

Therefore the equation of the centroid method given in equation (1) can be written as

$$y_c = \frac{\int y \mu_B dy}{\int \mu_B dy} = \frac{\int_1 y \mu_B dy + \int_2 y \mu_B dy + \dots}{\int_1 \mu_B dy + \int_2 \mu_B dy + \dots} \tag{2}$$

There are four possibilities for the union of fuzzy sets in a particular section shown in Fig.2 and are explained below.

2.1 Possibility 1

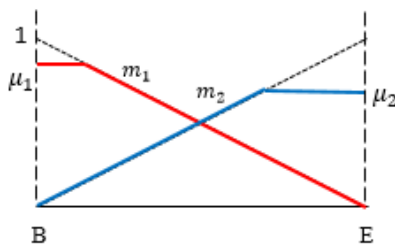


Fig.3 Possibility 1

In fig. 3, B and E are the starting and ending values of linguistic variable y of one partition, μ_1 and μ_2 are the membership values of saturation part of left and right fuzzy sets respectively, and $m_1 = \frac{-1}{E-B}$ and $m_2 = \frac{1}{E-B}$ are the slopes of non-saturation part of left and right fuzzy sets respectively. In this possibility, both μ_1 and μ_2 are greater than or equal to 0.5.

Expression for $\int_j y \mu_B dy$ and $\int_j \mu_B dy$ are given in equation (3) and equation (4) respectively.

$$\begin{aligned} \int_j y \mu_B dy &= \left(\frac{\mu_1}{2}\right) \left[\left(\frac{\mu_1 - C_1}{m_1}\right)^2 - B^2 \right] \\ &+ \left[\frac{m_1}{3} \left(\frac{B+E}{2}\right)^3 + \frac{C_1}{2} \left(\frac{B+E}{2}\right)^2 \right] \\ &- \left[\frac{m_1}{3} \left(\frac{\mu_1 - C_1}{m_1}\right)^3 + \frac{C_1}{2} \left(\frac{\mu_1 - C_1}{m_1}\right)^2 \right] \\ &+ \left[\frac{m_2}{3} \left(\frac{\mu_2 - C_2}{m_2}\right)^3 + \frac{C_2}{2} \left(\frac{\mu_2 - C_2}{m_2}\right)^2 \right] \\ &- \left[\frac{m_2}{3} \left(\frac{B+E}{2}\right)^3 + \frac{C_2}{2} \left(\frac{B+E}{2}\right)^2 \right] \\ &- \left(\frac{\mu_2}{2}\right) \left[\left(\frac{\mu_2 - C_2}{m_2}\right)^2 - E^2 \right] \end{aligned} \tag{3}$$

$$\begin{aligned} \int_j \mu_B dy &= \mu_1 \left[\left(\frac{\mu_1 - C_1}{m_1}\right) - B \right] \\ &+ \left[\frac{m_1}{2} \left(\frac{B+E}{2}\right)^2 + C_1 \left(\frac{B+E}{2}\right) \right] \\ &- \left[\frac{m_1}{2} \left(\frac{\mu_1 - C_1}{m_1}\right)^2 + C_1 \left(\frac{\mu_1 - C_1}{m_1}\right) \right] \\ &+ \left[\frac{m_2}{2} \left(\frac{\mu_2 - C_2}{m_2}\right)^2 + C_2 \left(\frac{\mu_2 - C_2}{m_2}\right) \right] \\ &- \left[\frac{m_2}{2} \left(\frac{B+E}{2}\right)^2 + C_2 \left(\frac{B+E}{2}\right) \right] \\ &- \mu_2 \left[\left(\frac{\mu_2 - C_2}{m_2}\right) - E \right] \end{aligned} \tag{4}$$

$$C_2 = 1 - \frac{E}{E-B} = 1 - C_1 \tag{5}$$

2.2 Possibility 2

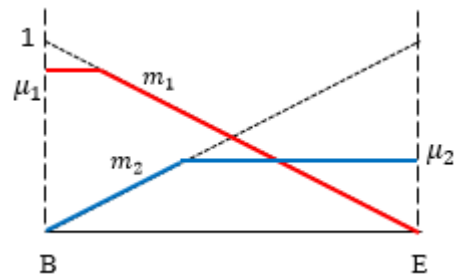


Fig.4 Possibility 2

In this possibility μ_2 is less than both μ_1 and 0.5. Expression for $\int_j y \mu_B dy$ and $\int_j \mu_B dy$ are given in equation (6) and equation (7) respectively.

$$\int_j y \mu_B dy = \left(\frac{\mu_1}{2}\right) \left[\left(\frac{\mu_1 - C_1}{m_1}\right)^2 - B^2 \right]$$

$$\begin{aligned}
 & + \left[\frac{m_1}{3} \left(\frac{\{1-\mu_2\}-C_2}{m_2} \right)^3 + \frac{C_1}{2} \left(\frac{\{1-\mu_2\}-C_2}{m_2} \right)^2 \right] \\
 & + \left[\frac{m_1}{3} \left(\frac{\mu_1-C_1}{m_1} \right)^3 + \frac{C_1}{2} \left(\frac{\mu_1-C_1}{m_1} \right)^2 \right] \\
 & - \left(\frac{\mu_2}{2} \right) \left[\left(\frac{\{1-\mu_2\}-C_2}{m_2} \right)^2 - E^2 \right]
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 \int_j \mu_B dy & = \mu_1 \left[\left(\frac{\mu_1-C_1}{m_1} \right) - B \right] \\
 & + \left[\frac{m_1}{2} \left(\frac{\{1-\mu_2\}-C_2}{m_2} \right)^2 + C_1 \left(\frac{\{1-\mu_2\}-C_2}{m_2} \right) \right] \\
 & + \left[\frac{m_1}{2} \left(\frac{\mu_1-C_1}{m_1} \right)^2 + C_1 \left(\frac{\mu_1-C_1}{m_1} \right) \right] \\
 & - \mu_2 \left[\left(\frac{\{1-\mu_2\}-C_2}{m_2} \right) - E \right]
 \end{aligned} \tag{7}$$

2.3 Possibility 3

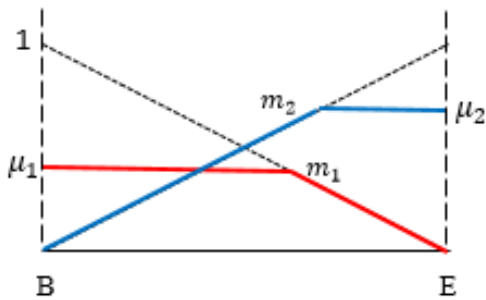


Fig.5 Possibility 3

In this possibility μ_1 is less than both μ_2 and 0.5. Expression for $\int_j y\mu_B dy$ and $\int_j \mu_B dy$ are given in equation (8) and equation (9) respectively.

$$\begin{aligned}
 \int_j y\mu_B dy & = \left(\frac{\mu_1}{2} \right) \left[\left(\frac{\{1-\mu_1\}-C_1}{m_1} \right)^2 - B^2 \right] \\
 & + \left[\frac{m_2}{3} \left(\frac{\mu_2-C_2}{m_2} \right)^3 + \frac{C_2}{2} \left(\frac{\mu_2-C_2}{m_2} \right)^2 \right] \\
 & - \left[\frac{m_2}{3} \left(\frac{\{1-\mu_1\}-C_1}{m_1} \right)^3 + \frac{C_2}{2} \left(\frac{\{1-\mu_1\}-C_1}{m_1} \right)^2 \right] \\
 & - \left(\frac{\mu_2}{2} \right) \left[\left(\frac{\mu_2-C_2}{m_2} \right)^2 - E^2 \right]
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 \int_j \mu_B dy & = \mu_1 \left[\left(\frac{\{1-\mu_1\}-C_1}{m_1} \right) - B \right] \\
 & + \left[\frac{m_2}{2} \left(\frac{\mu_2-C_2}{m_2} \right)^2 + C_2 \left(\frac{\mu_2-C_2}{m_2} \right) \right] \\
 & - \left[\frac{m_2}{2} \left(\frac{\{1-\mu_1\}-C_1}{m_1} \right)^2 + C_2 \left(\frac{\{1-\mu_1\}-C_1}{m_1} \right) \right] \\
 & - \mu_2 \left[\left(\frac{\mu_2-C_2}{m_2} \right) - E \right]
 \end{aligned} \tag{9}$$

2.4 Possibility 4

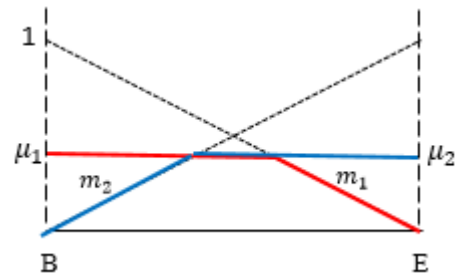


Fig.6 Possibility 4

In this possibility both μ_1 and μ_2 are equal and less than 0.5. Expression for $\int_j y\mu_B dy$ and $\int_j \mu_B dy$ are given in equation (10) and equation (11) respectively.

$$\int_j y\mu_B dy = \left(\frac{\mu_1}{2} \right) [E^2 - B^2] \tag{10}$$

$$\int_j \mu_B dy = \mu_1 [E - B] \tag{11}$$

Conclusions

A simplified method used to reduce the complexity of centre of gravity method used for defuzzification has been introduced. Compared to the MATLAB fuzzy tool box this method is not a good method. But for applications using microcontrollers, defuzzification can easily be implemented using this method. Speed of defuzzification is significantly increased by using this method.

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