

Research Article

Modified Selected Mapping Technique for Peak-to-Average Power Ratio Reduction in Orthogonal Frequency Division Multiplexing Systems

Dike U. Ike^{*,} Samuel N. John[#] and Charles Ndujiuba[#]

[#]Department of Electrical and Information Engineering, Covenant University, Ota, Nigeria

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Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is a type of multicarrier transmission technique with capability of supporting very high data rates. The high Peak-to-Average Power Ratio (PAPR), which leads to OFDM signal distortion and power inefficiency in the OFDM system transmitter radio frequency amplifier, shows a major limitation of OFDM. Several PAPR reduction techniques have been proposed and used, such as, tone injection, amplitude clipping, coding, tone injection, Partial Transmit Sequence (PTS), interleaving, and Selected Mapping (SLM). This paper introduced an effective SLM technique which gives a better OFDM PAPR reduction performance than conventional SLM technique. In achieving this, phase rotation of OFDM symbol is performed using a new set of phase rotation factors, and a Hann window function is applied to further reduce peaks in the OFDM signal. The results of computer simulations show that the PAPR reduction performance of the proposed technique is higher than conventional SLM by 0.2dB.

Keywords: OFDM, Multicarrier Transmission, Selected Mapping, Modified Selected Mapping, PAPR.

1. Introduction

Wireless communications continue to experience exponential growth in the wireless internet, cellular telephony and wireless home networking applications (Jordan, *et al*, 2002). The common feature of fourth generation wireless technologies is the convergence of multimedia services such as audio, video, image and data. There is an increased need for higher data rates. Presently, high data rate 4th Generation (4G) wireless networks can offer substantially higher bit rates of 300 Mbps and 75 Mbps in the downlink and uplink directions respectively.

While wireless communications offers many benefits, such as flexibility and mobility, it suffers from insufficient frequency spectrum utilization, inter-symbol interference (ISI), and frequency selective fading which is caused by multipath propagation. In order to support higher data rates, channel equalization of the conventional single carrier system can be too complex to implement. Thus, an effective modulation technique supporting high data rates with adequate robustness to radio channel impairments is required.

With the capability to suppress interference, and frequency selective fading, multi-carrier transmission

techniques are preferred to single carrier transmission techniques. Among the several multi-carrier transmission techniques available, Orthogonal Frequency Division Multiplexing (OFDM) is the most preferred and it makes use of parallel data streams. The onset of OFDM has raised the wireless standards to 100Mbps and higher and this has revolutionized the wireless world. The first multi-channel modulation systems transmitting binary coded data over Single Side Band (SSB) voice channel was implemented by (Doelz, *et al*, 1957). Multi-carrier modulation scheme was invented by (Chang, *et al*, 1968) who proposed the orthogonality concept. (Bingham, 1990) technically proved that multicarrier modulation provides greater resistance to noise and fades. (Cimini *et al*, 1985) introduced OFDM to the wireless community.

Although OFDM was introduced several decades ago, it has not been recognized as the optimum technique for high speed wireless data communication until recent years. Advancements in Digital Signal Processing and Very Large Scale Integration technologies have helped to overcome the initial obstacles to the implementation of OFDM. Presently, OFDM is widely adopted in various communication standards like Digital Audio Broadcasting, Digital Video Broadcasting-Terrestrial (DVB-T), IEEE 802.11 family of local area networks (IEEE, 1999), High Performance Radio Local Area Networks (HIPERLAN/2) (ETSI,

*Corresponding author's ORCID ID: 0000-0001-5766-6119

1997), Broadband Radio Access Networks (BRAN) (ETSI, 1998), Asymmetric Digital Subscriber Line [Nee, et al, 2000], 3GPP Long Term Evolution (LTE) 4G broadband system, and so on.

When compared with single carrier modulation, OFDM has many advantages, such as: immunity to inter-carrier interference, resilient to inter-symbol interference, robustness to channel fading, resistance to multipath, much lower complexity, and so on (Nee, et al, 2000). However, OFDM has some limitations to its implementation in practical telecommunications systems. The major drawback is that OFDM signal has high peak to average power ratio (PAPR). The high PAPR of the OFDM signal can cause inter-modulation noise and out-of-band radiation due to power amplifier non-linearity. A lot of research has been done on PAPR reduction with many techniques proposed such as clipping, coding, interleaving, windowing, Partial Transmit Sequence (PTS), Selective Mapping and so on. These techniques are mainly categorized into signal scrambling, signal distortion techniques, and coding techniques. Signal scrambling techniques scramble each OFDM symbol with different scrambling sequences (Eetvelt, et al, 1996), (Muller, et al, 1997).

The scrambled sequence with the lowest PAPR is selected and transmitted. Though scrambling techniques are very efficient in reducing PAPR, they still have their limitations. The scrambling techniques has to calculate PAPR at the transmitter and the information of the chosen scrambling sequence needs to be known at the receiver for the sequence to be descrambled. They also suffer from high computational complexity and a slight loss of data transmission rate. Selected Mapping (SLM) (Muller, et al, 1997), (Bauml, et al, 1996) and Partial Transmit Sequence (PTS) (Muller, et al, 1997), (Muller et al, 1997) are the two most important scrambling techniques. Signal distortion techniques distort the peak valued portion of OFDM signals using different techniques for PAPR reduction (Fuji, et al, 2002), (May, et al, 1998). The easiest way to reduce the PAPR is clipping which is a signal distortion technique that clip the transmit signal power below a threshold level. However, clipping will cause in-band- distortion and out-of-band radiation. The main idea of coding techniques is to select some code-words with low PAPR (Jones, et al, 1994), (Tarokh, et al, 2000), (Golay, et al, 1961). Among them, Golay complementary sequences derived from Reed-Muller codes are very good code-words that also has error correcting capability. Although coding is a good technique to solve the PAPR problem, it is hard to find enough code-words with small PAPR for OFDM systems with large number of subcarriers. In our proposed scheme, we modify

2. System Model

2.1 OFDM System

In an OFDM system as shown in Figure 1, the collection of data symbols $X_n, n=0,1,2,\dots,N-1$, as a vector $X=[X_0, X_1,$

$X_2,\dots, X_{N-1}]^T$ is called an OFDM data block. An OFDM signal is formed with each symbol modulating a set of subcarriers $f_k, k = 0, 1, 2,\dots, N-1$. These subcarriers are orthogonal with each other. The complex baseband OFDM signal is given by:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}, 0 \leq t < T \tag{1}$$

Where X_k is the symbol carried by the k^{th} subcarrier, $f_k = kf$ with $f=1/T$ and T is the duration of an OFDM data block. The signal that passes through the power amplifier is in the time domain, and is usually higher than the discrete time estimate, thus, it is necessary to oversample the signal by a factor of at least 4 to allow for accurate peak detection (Jung Chen, et al, 2010). For an oversampling factor of L , the input signal to the IFFT is extended by including $N(L-1)$ zeros at the centre of the signal (Siddiq, et al, 2012).

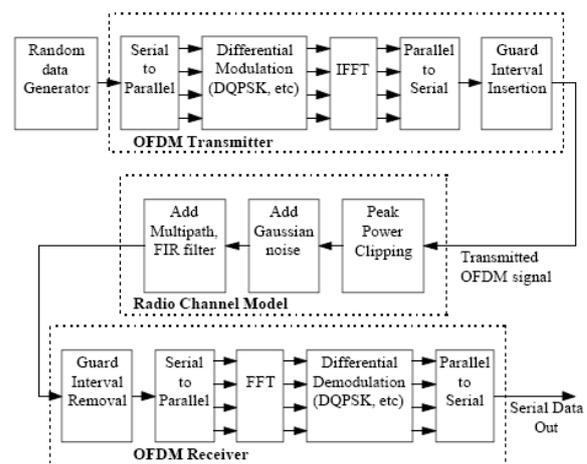


Fig. 1 Block Diagram of OFDM System

The method used to generate an OFDM symbol is as follows:

- First, there is zero padding of the N input complex symbols to get N_s symbols which are then used to compute the IFFT. The IFFT output is the basic OFDM symbol.
- A guard time interval is chosen based on the maximum possible delay spread of the channel. The number of samples that correspond to this guard time must be taken from the beginning of the OFDM symbol and placed at the end of the symbol. Also, the exact same number of samples must be taken from the end of the OFDM symbol and placed at the beginning.
- The OFDM symbol is then multiplied with the Raised Cosine Window to take out the power of the out of band sub-carriers.
- The windowed OFDM symbol is then added to the output of the previous OFDM symbol with a delay of T_s , so that there is an overlap region of βT_s between each symbol (Chandran, 2001).
- At the receiver end, the transmitted signal is received and the process is somewhat opposite of

the transmitter process. OFDM signal plot is shown in Figure 2.

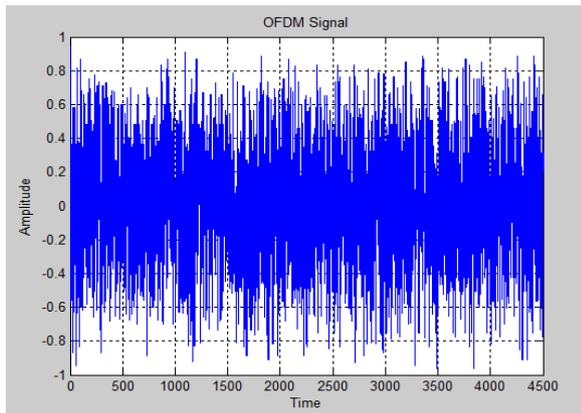


Fig. 2: OFDM Signal Plot

2.2 The PAPR Problem in OFDM

A major problem with OFDM transmission is that it exhibits a high Peak to Average Power Ratio (PAPR). The OFDM signal is a sum of N complex variables, each of which can be considered as a complex modulated subcarrier at different frequencies. When these subcarriers are added up coherently, the peak power of an OFDM signal will be far greater than the average power, leading to a large PAPR. The worst case scenario occurs when the N signals are added when they all have the same phase, their combination will have a peak power that is N times the average power. Fig. 3 shows a typical OFDM signal PAPR plot. If a nonlinear power amplifier is used to boost OFDM signals the large peak power brings about its nonlinearity, which creates out of band signal emission and in-band distortion and significantly degrades OFDM system performance (Pervez, et al, 2012).

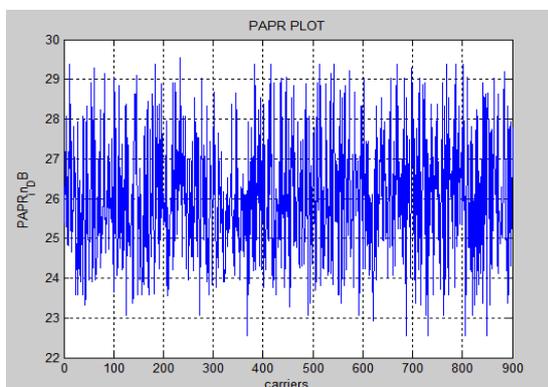


Fig. 3 Typical OFDM Signal PAPR Plot

RF linear power amplifiers are used in the transmitter, so the operating point must be in the linear region. Due to the high PAPR the operating point moves to the saturation region hence clipping of signal peaks takes place which generates in-band and out-of-band distortion. In other to keep the operating point in the

linear region the dynamic range of the RF power amplifier should be increased which leads to high power consumption and reduces efficiency. Hence a trade-off exists between non-linearity and efficiency [Han, et al, 2005]. Also, the analog-to-digital (A/D) and digital-to-analog (D/A) converters must also have a wide range to avoid clipping. Thus, the high PAPR increases the complexity and cost of implementation of OFDM system. As a result of the degrading effect of high PAPR to system performance, as researchers our objective should be to reduce this high PAPR.

Consider an OFDM signal consisting of N subcarriers. Let a block of N complex symbols $X=\{X_k, k=0,1,\dots,N-1\}$ is formed with each complex symbol modulating one set of subcarriers with frequencies $f_k = k\Delta f$, where $\Delta f=1/(NT)$ and T is the original symbol period. The complex baseband continuous time domain OFDM signal is given as:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}, 0 \leq t < NT \tag{2}$$

Peak to Average Power Ratio (PAPR) is the ratio between the peak instantaneous power and the average power of a signal. The PAPR can be expressed for the time domain OFDM signal $x(t)$ as:

$$PAPR = 10 \log_{10} \frac{\text{Max}|x(t)|^2}{E\{|x(t)|^2\}}, 0 \leq t \leq NT \tag{3}$$

Where $E\{\cdot\}$ represent the expectation operation. As more subcarriers are used for transmission, higher peak instantaneous powers may occur, thus PAPR increases proportionally with the number of subcarriers. Reducing $\text{Max}|x(t)|$ is the principle goal of PAPR reduction techniques. To accurately estimate the PAPR of continuous time OFDM signal, the OFDM signal samples are derived by L times oversampling. By sampling the OFDM signal $x(t)$ at frequency $f_s = L/T$, where L is the oversampling factor, the discrete-time OFDM signal is given as:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{kn}{NL}}, 0 \leq n < NL-1 \tag{4}$$

Equation 4 can be obtained through the use of an (NL) length IFFT operation. The new input vector X is extended from the original input vector by the use of zero-padding, which is achieved by inserting (L-1)N zeros in the middle of X. The PAPR calculated from the L-times oversampled continuous time domain OFDM signal is given by:

$$PAPR [x(n)] = 10 \log_{10} \frac{\text{Max}|x(n)|^2}{E\{|x(n)|^2\}}, 0 \leq n \leq NL-1 \tag{5}$$

From the central limit theorem, the real and imaginary part of the complex symbol X_k are Gaussian distributed for larger values of N. Therefore, the amplitude of OFDM signal has a Rayleigh distribution (Zhong, et al, 2006). Thus, the power of the signal has a central chi-square distribution with zero mean and two degree of

freedom. The cumulative distribution function (CDF) of the signal power is given by:

$$F(\mu) = 1 - e^{-\mu} \tag{6}$$

With the assumption of mutual un-correlation among signal samples, which is the case with Nyquist rate sampling, the probability that the PAPR value is lower than some threshold PAPR value is given as:

$$\Pr(\text{PAPR} \leq \mu) = F(\mu)^N = (1 - e^{-\mu})^N \tag{7}$$

The cumulative distribution function described above is a frequently used measure for PAPR reduction Performance. In this paper, same with other related works, the complementary cumulative distribution (CCDF) is used instead of the CDF. CCDF measures the probability that the PAPR of an OFDM block is higher than some threshold PAPR value. The CCDF of the PAPR of an OFDM block assuming Nyquist rate sampling is given as:

$$\begin{aligned} \Pr(\text{PAPR} > \mu) &= 1 - \Pr(\text{PAPR} \leq \mu) \\ &= 1 - (1 - e^{-\mu})^N \end{aligned} \tag{8}$$

The CCDF of conventional selected mapping scheme after U mapping of each data block with U mutually independent phase rotation factors is given by:

$$\Pr(\text{PAPR} > \mu) = (1 - (1 - e^{-\mu})^N)^U \tag{9}$$

Where U is the number of phase rotation factors

2.3 Conventional Selected Mapping (SLM) OFDM Scheme

The SLM technique was first proposed by Bauml *et al.* In SLM technique as shown in Figure 4, from a single complex symbol sequence X having a length of N, a number of sequences are generated that convey the same information using some phase rotation factors and the sequence with the lowest PAPR is transmitted. If the number of generated new sequence is U, then the sequences are derived from multiplying the incoming complex symbol sequence X by U different phase rotation factors. These phase rotation factors are given in vector form as $B^{(u)} = [b_{u,0}, b_{u,1} \dots b_{u,N-1}]^T$ for $u = 1, 2, \dots, U$. The U transformed data blocks X_u , for $u = 1, 2, \dots, U$, are generated by multiplying X and all U different phase rotation factors. Each element of phase rotation factor $B^{(u)}$ has unit magnitude to preserve the same power and the first phase rotation factor $B^{(1)}$ will be set as an all one vector of length N, this is to ensure that the original signal is included in the set of candidate signals. The phase rotation factors are appropriately selected such that multiplying a complex symbol by these factor results in rotation of the complex symbol to another point on the constellation diagram. Thus the phase factors are given as:

$$B_i^n = \exp j\alpha_n^i \text{ where } 0 \leq \alpha_n^i \leq 2\pi \tag{10}$$

After U parallel IFFTs, we get U different time domain candidate signals with different PAPR values. Among them, the signal with the lowest PAPR value is selected for transmission. The normalized time domain OFDM signal is given as:

$$X^u(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(n) \cdot b^u \cdot e^{j2\pi n \Delta f t} \tag{11}$$

Where $0 \leq t < NT$, $u = 1, 2, \dots, U$, $\Delta f = 1/T$ and T is the duration of data block.

The phase rotation factors are used as side information which is transmitted for signal recovery at the receiver. The efficiency of SLM approach depends on the amount of scrambling done by these rotation factors on the original complex symbol sequence and the length of U. As we increase the number of phase rotation factors, PAPR performance improves but at the expense of increase in system complexity. The complexity of a typical SLM method with no oversampling (i.e. L=1) in terms of complexity additions is given by (Ibrahim Hussain, 2013):

$$\text{Complexity} = UN \log_2 N \tag{12}$$

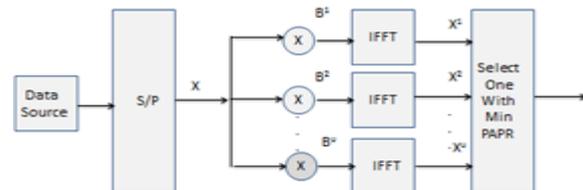


Fig. 4 Block Diagram of Conventional SLM

2.4 Proposed Method

A new scheme for PAPR reduction is proposed in this paper by using a new set of phase rotation factors for the complex symbol transformation, and using a Hann window function to further reduce the PAPR value of the OFDM signal that has been selected. The new phase sequence proposed is a series of the first six terms of the complex exponential j^n . That is:

$$B_n = \begin{cases} j^n & \text{for } n > 1 \\ 1 & \text{for } n = 1 \end{cases} \tag{13}$$

Hann window function is defined by:

$$w(n) = 0.5(1 - \cos(2\pi n/N)), 0 \leq n \leq N \tag{14}$$

The advantage of the Hann window is very low aliasing and the tradeoff is slightly decreased resolution. The peak windowing can be expressed as a multiplication of input signals with a scaling function at the peak level. It can be represented by:

$$x(n) = s(n) w(n) \tag{15}$$

Where $s(n)$ represents the scaling function that is used to lower the peak signal level. The scaling function can be expressed as a convolution between the window function $w(n)$ and weighting coefficient $c(n)$:

$$s(n) = 1 - \sum_{k=-\infty}^{\infty} c(k)w(n - k) \tag{16}$$

Peak windowing method can limit the instantaneous peak value to threshold value while maintaining its spectrum (Akhilesh Chandra, et al, 2012). The block diagram of the proposed method is shown in Figure 5. First the input complex symbols are multiplied by each of the proposed phase rotation factors to generate alternative input complex symbol sequences. Each of these alternative input complex symbol sequence is made the IFFT operation and the one with the lowest PAPR is selected for transmission. The Hann window function is then applied to further reduce the PAPR of the selected sequence.

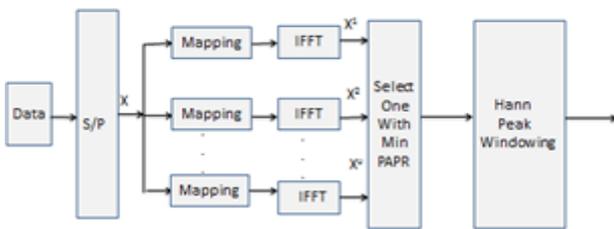


Fig. 5 Block Diagram of Proposed Method

3. Simulation Results

This section presents the results of simulation conducted to examine the PAPR performance of the proposed scheme. Table 1 summarizes the simulation parameters. These parameters correspond to the IEEE 802.11a standard.

Table1. Simulation Parameters of OFDM

Parameter	Value
Modulation	QPSK
Number of data subcarriers	64
Number of pilot subcarriers	4
Number of unused subcarriers	12
Number of candidate symbols	16
Oversampling factor L	4
Channel	AWGN
Data Source	Random
Number of OFDM symbols	10000

Figure 6, 7, and 8 shows the CCDF PAPR performance plot for normal OFDM, conventional SLM and the proposed modified SLM (M-SLM) technique. The proposed method reduces PAPR more than conventional SLM by 0.2db.

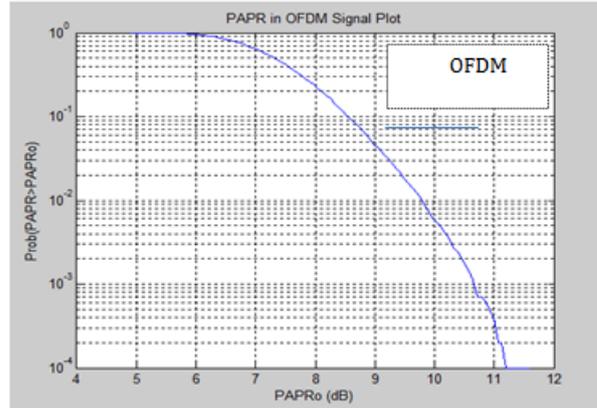


Fig. 6 CCDF Plot for OFDM with no PAPR Reduction

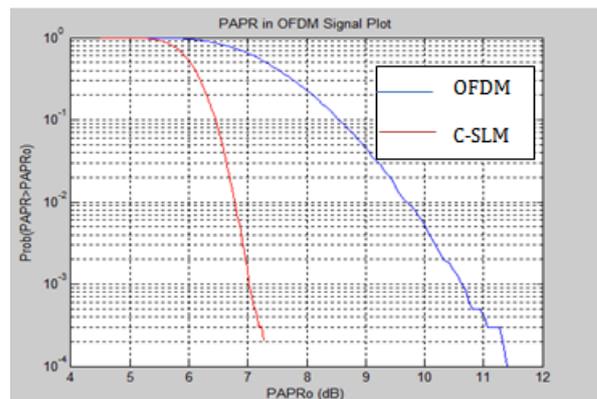


Fig. 7 CCDF Plot of OFDM with Conventional SLM

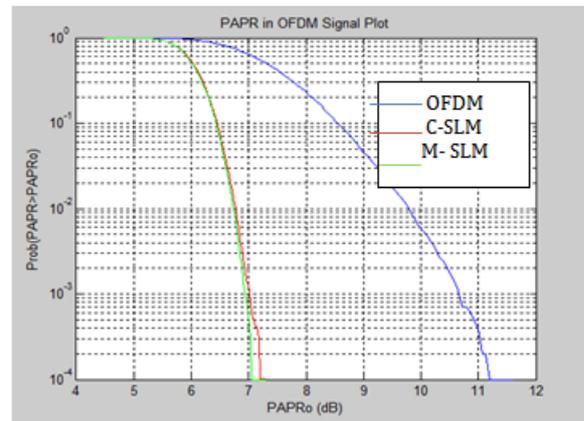


Fig. 8 CCDF Plot of OFDM, Conventional SLM and Proposed Modified SLM

Conclusion

A modified selected mapping technique is proposed in this paper to improve OFDM system performance by reducing PAPR. This technique makes use of new phase rotation factors combined with Hann peak windowing to further reduce PAPR in OFDM systems. The PAPR reduction performance was evaluated by computer simulations software, MATLAB. Results of computer simulations show that the PAPR reduction performance of the proposed technique is higher than conventional SLM by 0.2db.

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