Optimization of Blades of Horizontal Wind Turbines by Choosing an Appropriate Airfoil and Computer Simulation

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Abstract

This study investigates the optimization methods for turbine blades with working speeds below 10 m/s. This type of optimization is based on analytic methods leading to optimal torsion angle and hypotenuse length for each section of the turbine blade to achieve the maximum power. To this end, we developed a code using MATLAB software and by changing the airfoil type and ambient conditions, the optimum geometry for the airfoil and maximum achievable power is calculated. We used the Schmitz and Betz method for optimization, and to calculate the power, we used the momentum theory of blade element. We executed this code for several different airfoils and finally calculated the optimal blade for intended conditions. In the end, we used the Fluent software to calculate the flow field around the optimal turbine blade and the results showed the consistency of the calculated power using numerical methods and the presented code as well as the past studies. Ultimately, we conducted an economic analysis for the selected optimal blade which showed the economic gain of the article at annual mean wind speed of 6 m/s.

Keywords: Wind turbine blade, airfoil, blade optimization, computer simulation

1. Introduction

Today, the increasing price of energy and environmental concerns has forced most of the countries to use new energies such as wind energy. Wind turbines convert the kinetic energy of the wind into mechanical energy and finally to electricity. The history of wind turbine goes back to 200 B.C. in Persia, but operational wind mills were developed in the seventh century in Iran which was called Sistan Wind Mill. Wind mill was effectively used in the United States farms in 1930 for production of electricity and water pumping. The first wind turbine connected to a network was installed by John Brown in 1951. The final capacity of wind power up to 2014 was approximately 369.6 gigawatts and it is expected that by 2019 it will reach 666.1 gigawatts.

There are several wind turbine designs. This turbine, based on direction of the axis of rotation, is classified into two horizontal and vertical groups. The vertical-axis turbine is placed very close to the ground with the advantage of placement of heavy equipment including gearbox and generator close to the surface, whilst the wind power at ground level is low and thus, less power will be produced. Horizontal-axis turbine is very similar to aircraft propeller. The airflow moves on the aerodynamic cross-section of the blades to generate the lift force which turns the rotor. The horizontal-axis turbine nacelle is a location for gearbox and generator. In this generator blades must be directly opposite the wind direction to achieve the maximum power. So, the horizontal-axis turbine should have a system to adjust itself against the wind which is called Yawing mechanism. Air flow on any surface creates two types of aerodynamic forces called drag and lift. The drag force in the direction of wind flow and lift force is perpendicular to the wind flow. The wind turbine, based on drag force, acts like a spread-out sail and the wind power moves the surface onward.

Using lift force generates more energy than the drag force but it needs an aerodynamic surface. Small-scale wind turbine generates ~10 kW power that is adequate for domestic needs. This energy can be used effectively because energy extracted from conventional sources can be reserved for long periods of time. To increase the efficiency of horizontal wind turbine, different parameters influencing the speed of rotation of the blade need to be optimized. In this article, we have used analytical methods and simulation programs to optimize turbine blades in order to increase the efficiency and power of the turbine with various airfoil designs.
2. Numerical calculation method

Equation (1) shows that power is a function of the density of the air, the encircled surface of the turbine and wind speed $V_1$ (Gundtoft, 2009).

$$c_P = \frac{P}{2 \rho A v_1^2} \quad (1)$$

The power extracted from the wind was considered on both sides of wind turbine and using Bernoulli’s equation on the upper and lower stream of the wind flow and by subtracting the equations from one another in the change of linear momentum $V_1$ to $V_3$, pressure change can be obtained from the following equation:

$$\Delta p = \rho \dot{\theta} \left( v_1 - v_3 \right) \quad (2)$$

As a result, the power produced by the wind turbine is equal to the kinetic energy of the air as the following:

$$P = \frac{1}{2} \rho \dot{\theta} \left( v_1^2 - v_3^2 \right) \quad (3)$$

The axial interference factor $a$ is a measure of air speed loss while passing over the wind turbine blade. The axial interference factor is defined as:

$$v = (1 - 2a)v_1$$

$$v_3 = (1 - 2a)v_1 \quad (4)$$

Considering the above definition, the power equation can be rewritten as:

$$P = 2 \rho a (1 - a)^2 v_1^3 A \quad (5)$$

Using the above equation, a more precise definition of the power extracted by wind turbine by taking into account the axial interference factor will be as follows:

$$C_p = 4a (1 - a)^2 \quad (6)$$

The theoretical value of maximum coefficient of performance is obtained by differentiation and setting equation (6) equal to zero to solve for $a$. Therefore, the $a$ value will be 1.3 and maximum coefficient of performance will be 16.27. This result is called Betz Limit which represents the maximum amount of theoretical power coefficient.

The extracted power from air is the result of torque and angular velocity of wind turbine. In accordance with the angular momentum conservation, the wind turbine shaft torque can be only generated by rotation of the downstream Wake flow against the direction of rotation of the rotor. By calculating the Wake generated flow in the opposite direction, the following equation shows the relative tangential velocity of the spinning blade. The term $\frac{1}{2} \Delta u$ is used to calculate the additional tangential wind speed that the blade receives because of the mean Wake speed of contour rotation.

$$u = \omega + \frac{1}{2} \Delta u \quad (7)$$

That extra tangential speed that the blade receives because of Wake is defined as a function of tangential the axial interference factor of $a'$.

$$u = \omega \left( 1 + a' \right) \quad (8)$$

It is clear that increasing the tangential speed is an effect of $\frac{1}{2} \Delta u$ and the axial interference factor causes an axial speed reduction that reduces the relative speed angle. The following equation defines the change in relative wind speed $\Delta \omega$ by taking into account the initial relative wind speed $\omega_1$ and change in the angle of the relative wind speed $\varphi$.

$$\Delta \omega = 2\omega_1 \sin \left( \varphi_1 - \varphi \right) \quad (9)$$

In order to expand the data for given angles of attack before the occurrence of the first stall (Viterna, 1982) Viterna has provided a simple procedure to link the coefficient of lift before the stall and drag to the overall geometry of the blade. Viterna's equations for lift and drag coefficient are as follows:

$$c_l = A_1 \sin (2a) + \frac{A_2 \cos (a)}{\sin (2a)} \quad (10)$$

$$c_d = B_1 \sin 2(a) + B_2 \cos a$$

$$c_d = B_1 \sin 2(a) + B_2 \cos a$$

$$A_1 = \frac{B_1}{2}$$

$$A_2 = (c_{L MAX} \sin (a_2) \cos (a_2) \sin (a_2)) $$

$$B_1 = c_{D MAX}$$

$$B_2 = c_{D MAX} \sin 2(a_2)$$

$$C_{D MAX} = 1.11 + 0.18 AR$$

These equations are used to calculate the coefficient of lift and drag with angles of attack between 20 to 90 degrees. For angles less than 20 degrees, a polynomial equation is fitted to the experimental data curves (Abbot, 1959) and the continuous iterated solving in BEM calculation in the following can determine the values without interpolation.

The tested airfoils in this turbine blade are NACA 23012, NACA 4412, NACA 4418 and NACA 23015. NACA 4412 airfoil was used in older wind turbines such as Wind Cruiser turbine with Crafts Kills Enterprises.

The basis of the mean wind speed at ground level is approximately 5 m/s at an altitude of 11.5 meters which corresponds to a blade with a radius of 2.5 meters. The wind speed and the radius of blade were chosen for the presentation of results.
The conditions governing the testing of blades in Qblade software:

\[ \text{Ro}_{\text{air}} = 1.225; \quad \text{density of air} \]
\[ \text{omega} = 15; \quad \text{rad/sec blade rotational speed} \]
\[ U = 5; \quad (\text{m/s}) \text{ wind speed} \]
\[ B = 3; \quad \text{number of blades} \]
\[ R = 2.5; \quad (\text{m}) \text{ Radius of blade} \]
\[ \text{rh}(1) = 0.3750; \quad (\text{m}) \text{ Radius of Hub} \]
\[ \text{dr} = 0.10;\% \quad (\text{dr}/R) \]

3. Results obtained from testing the blades with Qblade software

3.1 NACA 23012 airfoil

Based on NACA 23012 airfoil conditions presented in fig. 1, the optimal shape that shows the length of hypotenuse and angle of torsion in different cross-sections using Betz and Schmidt’s theories was obtained as fig. 2. In the following figures, the differences between these two methods is significant. By using the optimum conditions obtained from the Schmidt’s theory as the initial conditions of the BEM theory, the power and propulsion force per each cross-section are calculated and the results are in accordance with fig. 3.

![Fig.1 NACA23012 airfoil profile](image)

Total power: 750.48
Total thrust: 256.635

![Fig.2 Length of hypotenuse and torsion angle per each cross-section with NACA 23012 airfoil](image)

![Fig.3 Power and propulsion force per each cross-section with NACA 23012 airfoil](image)

To check the impact of the angular speed of the rotor on power and propulsion force, we kept other factors fixed and changed the angular velocity according to five different numbers as shown in fig. 4 and fig. 5.

![Fig.4 The effect of angular velocity of the rotor on power with NACA 23012 airfoil](image)

![Fig.5 Evaluation of the impact of angular velocity of the rotor on the thrust force with NACA 23012 airfoil](image)

To check the influence of the blade diameter on power and propulsion force, we kept other parameters fixed and changed the diameter according to four different numbers as shown in fig. 6 and fig. 7. Other parameters are considered as follows.
Fig. 6 The impact of the rotor blade diameter on power with NACA 23012 airfoil

Fig. 7 Evaluation of the impact of rotor blade diameter on thrust force with NACA 23012 airfoil

3.2 NACA 4418 airfoil

Fig. 8 NACA 4418 airfoil profile

Based on NACA 4418 airfoil conditions presented in fig. 8, the optimal shape including the length of hypotenuse and angle of torsion in different cross-sections using Betz and Schmidt’s theories was obtained as fig. 9. In the following figures, the differences between these two methods is significant. By using the optimum conditions obtained from the Schmidt’s theory as the initial conditions of the BEM theory, power and propulsion force per each cross-section were calculated and the results are in accordance with fig. 10.

Total power: 757.6674
Total thrust: 258.7440

Fig. 9 The length of hypotenuse and angle of torsion per each cross-section with NACA 4418 airfoil

(a)

(b)

Fig. 10 Power and propulsion force per each cross-section with NACA 4418 airfoil

To check the impact of the angular speed of the rotor on power and propulsion force, we kept other factors fixed and changed the angular velocity according to five different numbers as shown in fig. 11 and fig. 12.

Fig. 11 The effect of angular velocity of the rotor on power with NACA 4418 airfoil

(a)
Fig. 12 Evaluation of the impact of angular velocity of the rotor on the thrust force with NACA 4418 airfoil

In accordance with the diagram, with an increase in angular velocity of the rotor the overall power has dropped but the propulsion force has increased. Also, the lower the angular velocity, the higher the difference between the two methods of Betz and Schmidt’s as shown in fig. 13 and fig. 14.

Fig. 13 The difference between the two methods of Betz and Schmidt’s with NACA 4418 airfoil at 30 rad/s speed

Fig. 14 The difference between the two methods of Betz and Schmidt’s with NACA 4418 airfoil at 10 rad/s speed

3.3 NACA 23015 airfoil

Fig. 15 NACA 23015 airfoil profile

Based on NACA 23015 airfoil conditions presented in fig. 15, the optimal shape including the length of hypotenuse and angle of torsion in different cross-sections using Betz and Schmidt’s theories was obtained as fig. 16 and the differences between these two methods is significant. By using the optimum conditions obtained from the Schmidt’s theory as the initial conditions of the BEM theory, power and propulsion force per each cross-section were calculated and the results are in accordance with fig. 17.

Total power: 741.6298
Total thrust: 259.6492

Fig. 16 The length of hypotenuse and angle of torsion per each cross-section with NACA 23015 airfoil

(a)

(b)

Fig. 17 Power and propulsion force per each cross-section with NACA 23015 airfoil

(a)

(b)
To check the impact of the angular speed of the rotor on power and propulsion force, we kept other factors fixed and changed the angular velocity according to five different numbers as shown in fig. 18 and fig. 19.

![Fig.18](image1.png)
**Fig.18** The effect of angular velocity of the rotor on power with NACA 23015 airfoil

![Fig.19](image2.png)
**Fig.19** Evaluation of the impact of angular velocity of the rotor on the thrust force with NACA 23015 airfoil

The diagram shows that with an increase in angular velocity of the rotor the overall power has dropped but the propulsion force has increased.

### 3.4 NACA 4412 airfoil

![Fig.20](image3.png)
**Fig.20** NACA 4412 airfoil profile

Based on NACA 4412 airfoil conditions presented in fig. 20, the optimal shape including the length of hypotenuse and angle of torsion in different cross-sections using Betz and Schmidt’s theories was obtained as fig. 21. In the following figures, the differences between these two methods is significant. By using the optimum conditions obtained from the Schmidt’s theory as the initial conditions of the BEM theory, power and propulsion force per each cross-section were calculated and the results are in accordance with fig. 22.

![Fig.21](image4.png)
**Fig.21** The length of hypotenuse and angle of torsion per each cross-section with NACA 4412 airfoil

![Fig.22](image5.png)
**Fig. 22** Power and propulsion force per each cross-section with NACA 4412 airfoil
To check the impact of the angular speed of the rotor on power and propulsion force, we kept other factors fixed and changed the angular velocity according to five different numbers as shown in fig. 23 and fig. 24.

![Fig. 23](image) Evaluation of the effect of angular velocity of the rotor on power with NACA 4412 airfoil

In accordance with the diagram, with an increase in angular velocity of the rotor the overall power has dropped but the propulsion force has increased.

4. Choosing the optimal airfoil

By comparison of the calculated power for the two airfoils, we can see that NACA airfoil provides a better performance. So, this is the perfect choice for an airfoil. In the following, for a more detailed review of NACA 4412 airfoil we studied the impact of wind flow and the impact of change in the radius. To check the impact of the wind flow we used a blade with a radius of 2.5 meters and to check the impact of the radius we used a flow at a speed of 5 and 6 m/s which are shown in figures 25 to 27.

![Fig. 24](image) Evaluation of the impact of angular velocity of the rotor on the thrust force with NACA 4412 airfoil

According to the obtained information we can conclude that the most power for the above turbine blade was obtained at a wind speed of 6 m/s.

4.1 The results of numerical simulation for a 3-blade turbine with NACA 4412

After the convergence, the solution for the torque and power production in x, y, z coordinates was reported as follows. The general torque generated by the turbine blade is as follows:

\[ M = 90.55i + 0.1461j + 0.06k \]

Considering that the axis of rotation was x, the values were obtained as indicated above. To calculate power we will have:

\[ P = T \cdot \omega = 90.55 \cdot 15 = 1358 \text{ watt} \]

We observe that the results have a small difference with the results calculated by BEM method that calculated the power as 1352.4 watts in the same conditions. After making sure of the results, in the following, we study the speed contour around the blade. According to fig. 28, after the turbine, the wind up to about three times the diameter of the turbine behind the turbine the flow is still under the influence of the presence of turbine.
Fig.28 The intensity of the turbulence around the wind turbine

4.2 Choosing the optimal airfoil

By comparison of the calculated power for the four airfoils, we can see that NACA 4412 airfoil provides a better performance. So, this is the perfect choice for an airfoil. In the following, for a more detailed review of NACA 4412 airfoil, we studied the impact of wind flow and the impact of change in the radius.

To check the impact of the wind flow we used a blade with a radius of 2.5 meters and to check the impact of the radius we used a flow at a speed of 5 and 6 m/s which are shown in figures 29 to 31.

Fig. 29 The power diagram in terms of wind speed for a wind turbine with NACA 4412 blade and a radius of 2.5 m

Fig. 30 The power diagram in terms of the radius for a wind turbine with NACA 4412 blade and a wind speed 5 m/s

Fig.31 The power diagram in terms of the radius for a wind turbine with NACA 4412 blade and a wind speed 6 m/s

5. Economic analysis of the project

5.1 Variables affecting the cost of installing a wind turbine

Since the useful life of wind turbine is 25 years (Taylor, 2015), calculation is carried out for a period of 25 years with 20% rate of inflation and 22% rate of interest. The cost of installation and maintenance and repair of the wind turbine installed on land will be as follows:

Table 1 Variables affecting the cost of installing a wind turbine

<table>
<thead>
<tr>
<th>Cost variable</th>
<th>Cost of wind turbine installed on land in the year 2014 (USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean installation cost per kW</td>
<td>1785</td>
</tr>
<tr>
<td>Mean weight to generate one kWh</td>
<td>0.090</td>
</tr>
<tr>
<td>Cost of repair and maintenance to generate one kWh</td>
<td>02025.0</td>
</tr>
</tbody>
</table>

Since based on the statistics of meteorological organization, in a 10-year period, the highest amount of mean wind speed in the country was recorded in Manjil station with 6.25 m/s and next to that, Zabol station with 6 m/s; the amount of annual electrical energy produced in kWh is based on the hypothetical mean wind speed of 6 m/s and the calculations are performed based on it. Also, for a wind speed of 6 m/s for optimal blade with NACA 4412 airfoil according to fig. 32, we clearly see that the higher radius of blade can increase the generated power.

Fig.32 The effect of changing the blade radius on the amount of generated power at a wind speed of 6 m/s

5.2 Calculating the effective values in the economic estimation of the article

Since the generated power for a wind speed of 6 m/s for the optimum blade with the selected NACA 4412 airfoil with 2.5 m radius is 1.3524 kW, after carrying out an economic analysis based on this blade radius we will have:

Capital Cost:

\[
\text{Capital Cost} = \frac{1.3524 \text{kW} \times 1785}{\text{kW}} = $2414.034
\]
Annual energy production:

\[ 1.3524kW \times 365 \frac{day}{year} \times \frac{24 \text{ hour}}{1 \text{ day}} = 11847.024kW/h \]

Also, since the tariff of guaranteed electricity power purchase for wind power plants with a capacity of 1 MW and less (specific to electricity subscribers and limited to capacity of branches) for each kWh is 593 Tomans (http://www.suna.org.ir/), and assuming 1 USD = 3500 Tomans, we will have:

\[ 11847.024 \frac{kWh}{year} \times 0.1694 \frac{\$}{kWh} = 2006.886 \frac{\$}{year} \]

Also, the cost of repair and maintenance according to the wind turbine power generation is calculated as follows:

\[ 11847.024 \frac{kWh}{year} \times 0.02025 \frac{\$}{kWh} = 239.902 \frac{\$}{year} \]

Therefore, the Annual Net Real Savings of the article can be calculated as follows:

\[ 2006.886 \frac{\$}{year} - 239.902 \frac{\$}{year} = 1766.984 \frac{\$}{year} \]

Simple Payback Period:

\[ \text{Simple Payback Period} = \frac{2414.034\$}{1766.984\$} = 1.366 \text{ year} \approx 16.39 \text{ months} \]

Using the above calculation data, the diagram of cash flow of the wind turbine installation article will be as shown in fig. 32:

![Cash flow diagram of the wind turbine installation article for a period of 25 years](image)

**Fig.32** Cash flow diagram of the wind turbine installation of the article for a period of 25 years

In the following, we use the equations below to calculate the inflation index and interest index (Blank, 2008)

\[ IF = (1 + \frac{IR}{100})^{-n} \]

\[ DF = (1 + \frac{IR}{100})^{-n} \]

Where, is the rate of inflation and interest rate. Also, the real interest rate is equal to the difference between the rate of inflation and interest rate (Blank, 2008).

### 5.3 Results of the economic analysis

Calculation of NPV and IRR for the installation of a wind turbine for a 25-year period is shown in table 2:

**Table 2** Cash flow and calculation of NPV and IRR for the installation of a wind turbine for a 25-year period

<table>
<thead>
<tr>
<th>Year</th>
<th>Capital Cost</th>
<th>Annual Net Cost Saving</th>
<th>NPV</th>
<th>IRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2414.034$</td>
<td>1766.984$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2414.034$</td>
<td>1766.984$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2414.034$</td>
<td>1766.984$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2414.034$</td>
<td>1766.984$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2414.034$</td>
<td>1766.984$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is clear that according to table 2:

NPV: $32083.60121

IRR: 73%

### Conclusion

According to the results obtained from the comparison between the shape of blade airfoils we observed that blades with NACA 4412 airfoil have the optimum power generation. The recommendation to continue the work is that the higher the number of the selected airfoils for the analysis, the better will be the results. Also, economic analysis showed that the investment profit is highly dependent on wind speed and stability of the current. Therefore, design of turbine for lower split speeds will increase the profit and reliability of capital investment on the article. This requires a continuous analysis of feasibility of acquisition of the energy at lower wind speeds design based on it. Also, studying the impact of the number of blades and their weight on the power generated by the turbine can be extremely helpful to choose a more efficient design.

### Abbreviations

- A: the axial interference factor
- A': tangential interference index
- A: surface area swept by the turbine blade (m²)
- B: number of blades
- \( C_l \): coefficient of lift
- \( C_p \): power coefficient
- \( C_f \): coefficient of axial forces
\( C_x \) coefficient of tangential forces
\( C \) length of the blade hypotenuse (m)
\( P \) Power (W)
\( r \) Annular cross-section radius of the blade (m)
\( T_h \) Axial force on the rotor, thrust (N)
\( T \) torque (N.m)
\( U \) tangential force on the rotor (N)
\( u \) axial wind speed in rotor plane (m/s)
\( V \) upstream wind speed of the rotor (m/s)
\( V_1 \) downstream wind speed of the rotor (m/s)
\( V_2 \) blade tip speed (m/s)
\( V_{tip} \) relative wind speed (m/s)
\( W \) relative tip speed
\( \alpha \) angle of attack (deg)
\( \beta \) angle of the blade pitch relative to rotor plane (deg)
\( \Phi \) angle of the relative wind relative to rotor plane (deg)
\( \mu \) Dynamic viscosity (kgm\(^{-1}\)s\(^{-1}\))
\( \rho \) air density (kg/m\(^3\))
\( \omega \) rotor angular velocity (1/s)

**References**

Gundtoft, Soren (2009), University of Aarhus. Wind Turbines. Copyright


http://www.suna.org.ir/