International Journal of Current Engineering and Technology
E-ISSN 2277 – 4106, P-ISSN 2347 – 5161

Available at http://inpressco.com/category/ijcet

Research Article

Estimation of State Variables of Active Suspension System using Kalman Filter

Shital M. Pawar†* and A.A. Panchwadkar†

†Mechanical Engineering Department, Pimpri-Chinchwad College of Engineering, Pune, Maharashtra, India

Accepted 22 April 2017, Available online 25 April 2017, **Vol.7, No.2 (April 2017)**

Abstract

Suspension systems are an integral part of the new age vehicles. These suspension systems contribute in supporting vehicle weight, and also increase vehicle stability and cut off the driver from road roughness. Active suspension systems use an actuator which is governed by the control strategy using an ECU. Most of the literature focuses on feedback control of active suspension systems. This study aims to make use of an algorithm which predicts the states of the suspension systems in response to the road input using Kalman filter and control the suspension travel between the sprung and unsprung masses of the suspension. The Kalman filter is used as the observer which will observe the system states and predict the next states of the plant model. A linear Quadratic Regulator (LQR) and a Linear Quadratic Gaussian (LQG) control strategy is used to control the required force to minimize the suspension travel. The suspension system model is prepared in Matlab™/Simulink and simulated. Comparison of the estimation errors in open loop (passive suspension), the LQR control and LQG control using Kalman filter is made to study the effectiveness of the new control strategy.

Keywords: *Active suspension system, suspension travel, Kalman filter, Linear Quadratic Regulator, Linear Quadratic Gaussian, estimation errors, control strategy.*

1. Introduction

Now a days automotive control systems have a number of applications such as space vehicle system, missile guidance system, robotic systems, manufacturing, aerospace and automobile industry etc. In twenty first century, suspension systems of automobile play a vital role in increasing ride comfort, safety of the driver, road holding capability and vehicle stability. For these reasons, many researchers are focusing on development of an active suspension system. Suspension system can be classified as Passive, Semi active, and Active. Passive suspension system is inexpensive but its spring and damping characteristics are constant. And it is difficult to decide the values of these parameters because one has to understand the tradeoff between vehicle stability and ride comfort. When the damping coefficient is less, then passengers feel more comfortable but in that case stability is reduced greatly and vice versa

In semi active suspension systems one can change the damping characteristics. It is less expensive than active suspension. The dampers and springs are replaced by an actuator in active suspension system. In active suspension systems, the most important and critical part is to develop a control strategy which achieves the desired performance characteristics such as ride comfort, suspension travel etc. Road roughness is the most influential input parameter which limits the control strategy. A poorly developed control strategy can result in discomfort to the user and can also affect the fuel economy of the vehicle.

Currently global positioning system (GPS) technology in smart phones, time of flight cameras, etc. are used to detect the road roughness in advance which can help the control strategy to act accordingly but all these technologies are very expensive because it requires high resolution cameras and expensive sensors. So, there is a need to develop new control strategy in which one can predict the state variables of the system and can take necessary action by controller to reduce the vertical acceleration that is felt by the passenger. Kalman filter is used to estimate the vertical velocity of the vehicle chassis and relative velocity between chassis and wheel for semi-active suspension system (Olof Lindgarde, 2002). In this study, a linear observer (Kalman filter) is used which observes and estimates the internal states of the plant model. The control strategy uses these estimates and modulates the control force to be applied between the sprung and unsprung masses of the suspension system using a Linear Quadratic regulator/Linear Quadratic Gaussian control.

*Corresponding author: **Shital M. Pawar**

3. Mathematical modeling for suspension system

Fig 1. describes the scheme of a 2DOF quarter car active suspension model. In this actuator force is given by F_c and equation of motion for the sprung and unsprung mass can be written as follows (Shi-Yuan Han, *et al*, 2014):

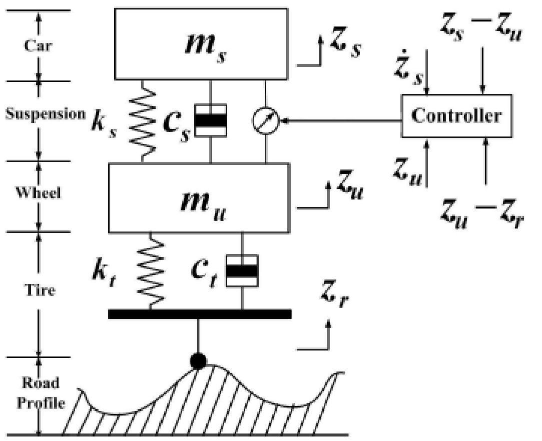

Fig.1 A Typical 2DOF Quarter Car Active Suspension System Model

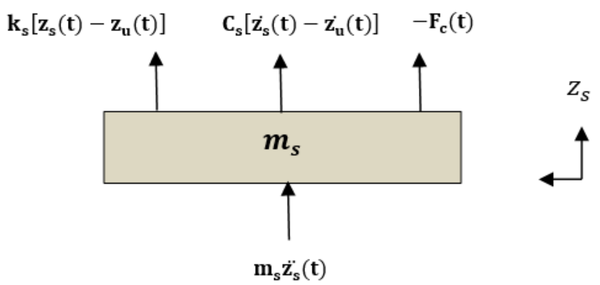

Fig.2 Typical Free Body Diagram of Sprung Mass

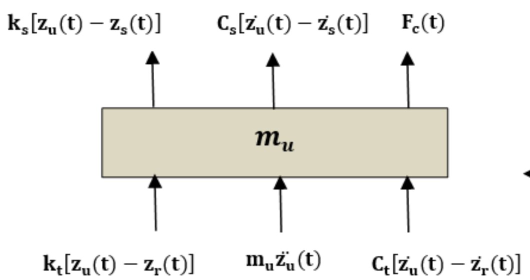

Fig.3 Typical Free Body Diagram of Unsprung Mass

$$F_c(t) = m_s\ddot{z}_s(t) + C_s[\dot{z}_s(t) - \dot{z}_u(t)] + k_s[z_s(t) - z_u(t)]$$

$$-F_c(t) = m_u\ddot{z}_u(t) + C_s[\dot{z}_u(t) - \dot{z}_s(t)] + k_s[z_u(t) - z_s(t)] + k_t[z_u(t) - z_r(t)] + C_t[\dot{z}_u(t) - \dot{z}_r(t)] \quad (1)$$

Where m_s is the sprung mass; m_u is the unsprung mass; C_s and k_s are damping and stiffness coefficient of passive suspension system; k_t and C_t are compressibility and damping coefficient of pneumatic tire, z_s (t) and z_u (t) are the displacements of sprung and unsprung masses respectively wherez_r is the road roughness inputs.

The purpose of active suspension system is to minimize following parameters:

Ride Comfort: is related to vehicle body motion sensed by the passengers i.e. sprung mass acceleration $[\ddot{z}_s(t)]$.
Suspension Travel: refers to relative displacement between the vehicle body and the tire i.e. relative displacement between sprung and unsprung mass $[z_s(t) - z_u(t)]$.

Road handling is associated with contact forces between the road surface and the vehicle tires and this depends on the tire deflection which is given by relative displacement between unsprung mass and road$[z_u(t) - z_r(t)]$.

State space representation for active suspension system can be given by following equations:

$$\dot{x} = Ax + Bu \quad 2(a)$$
$$y = Cx + Du \quad 2(b)$$

Where, x is the state vector,u and y are input and output vector respectively, A is the state matrix, B is called the input matrix, C is the output matrix, D is the direct transmission matrix.

The four system states variables can be written as:
$x_1(t) = z_s(t) - z_u(t)$
$x_2(t) = \dot{z}_s(t)$
$x_3(t) = z_u(t) - z_r(t)$
$x_4(t) = \dot{z}_u(t)$
Where,
$x_1(t)$ =Suspension deflection
$x_2(t)$ = Speed of sprung mass
$x_3(t)$ = Tire deflection
$x_4(t)$ =Speed of unsprung mass

$$\text{So } x = \begin{bmatrix} z_s(t) - z_u(t) \\ \dot{z}_s(t) \\ z_u(t) - z_r(t) \\ \dot{z}_u(t) \end{bmatrix}$$

Two inputs and two outputs also can be given by:

$$u(t) = \begin{bmatrix} \dot{z}_r(t) \\ F_c \end{bmatrix} \qquad y = \begin{bmatrix} z_s(t) - z_u(t) \\ \ddot{z}_s(t) \end{bmatrix}$$

Matrices A, B, C, D can be derived by equations of motion from equation 1:

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -k_s/m_s & -C_s/m_s & 0 & C_s/m_s \\ 0 & 0 & 0 & 1 \\ k_s/m_u & C_s/m_u & -k_t/m_u & -(C_s + C_t)/m_u \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 1/m_s \\ -1 & 0 \\ C_t/m_u & -1/m_u \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -k_s/m_s & -C_s/m_s & 0 & C_s/m_s \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 1/m_s \end{bmatrix}$$

Table 1 Vehicle Parameter of Active Suspension System

System Parameters	Value	Unit
Sprung mass (m_s)	2.45	Kg
Unsprung mass (m_u)	1	Kg
Stiffness (k_s)	900	N/m
Compressibility of the pneumatic tire (k_t)	1250	N/m
Damping of the active suspension system (C_s)	7.5	N-s/m
Damping of the pneumatic tire (C_t)	5	N-s/m

Substituting the table 1 values in matrix equations, A, B, C, and D matrices in a continuous time state space representation of the suspension system will become as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -367.347 & -3.0612 & 0 & 3.0612 \\ 0 & 0 & 0 & 1 \\ 900 & 7.5 & -1250 & -12.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0.4081 \\ -1 & 0 \\ 5 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -367.347 & 3.0612 & 0 & 3.0612 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0.4081 \end{bmatrix}$$

These continuous time state space matrices need to be converted to discrete time state space representation using the impulse time of the system.

4. Linear quadratic regulator

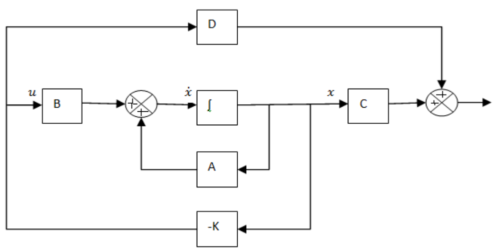

Fig.4 Closed loop Control system of LQR Controller

In LQR controller, the main aim is to find the feedback control gain for state space system by minimizing the cost function which is given by:

$$J = \int_0^{\infty} x(t)'\,Qx(t) + RF_c(t)^2 dt \qquad (3)$$

The system matrices A and B which are the state transition matrix and control matrix are needed along with the Q and R weighting matrices in order to obtain the gain matrix K which will be multiplied to the plant states and fed back to the plant as input.

The solution to the optimization problem in equation (3) is

$$\mathrm{F_c} = -\mathrm{Kx}$$

The state space equation becomes

$$\dot{x} = Ax + Bu$$
$$\therefore \dot{x} = Ax + B(-Kx)$$
$$\therefore \dot{x} = (A - BK)x \qquad (4)$$

5. Kalman Filter

In real world sensors are expensive and also impractical to measure all states of system so there is need to design estimator so here well-known Kalman Filter is designed with LQR. Kalman Filter is an iterative method used to estimate state variables when there are uncertainties in the measurements which are difficult to measure. It consists of two steps, prediction step and correction step. In this we estimate current state at time k from previous state at time k-1 and corrects the predicted state using the Kalman gain.

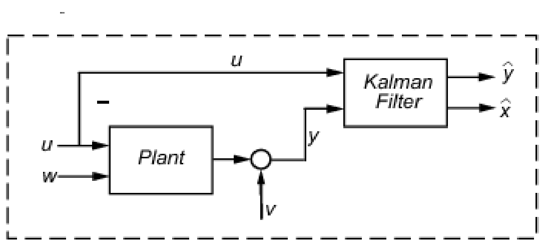

Fig.5 Kalman Filter

In fig. 5. U is known inputs, w is white process noise , and v is measurement noise, $\hat{x}$ and $\hat{y}$ are estimated state vector and measurement vector.

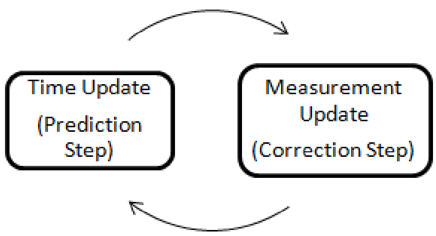

Fig.6 Discrete Kalman filter cycle

The Equations for time and measurement updates are (G. Bishop, *et al*, 2001):

Discrete Kalman filter time Update Equations:

$$x_k^- = Ax_{k-1} + Bu_k \quad (5)$$
$$P_k^- = AP_{k-1}A^T + Q \quad (6)$$

Discrete Kalman filter measurement updates equations:

$$K_k = P_kH^T(HP_kH^T + R)^{-1} \quad (7)$$
$$x_k = x_k^- + K_k(z_k - Hx_k^-) \quad (8)$$
$$P_k = (I - K_kH)P_k^- \quad (9)$$

In the above equation x_k^- is the predicted state vector containing interested state variables, x_{k-1}, previous state vector, u_k is the input vector, A is the state transition matrix while B is control input matrix, Q is process noise covariance matrix, R is the measurement noise covariance matrix, K_k is the Kalman Gain which minimizes the posteriori error covariance, P_k and I is identity matrix.

6. Simulation results

In order to provide road profile input to the system an impulse of 0.1m is given to the system to simulate a sudden bump on the road. This kind of input also makes it possible to study the time taken by the system to stabilize or converge. This road input is given to passive suspension system. It is expected that the sprung and unsprung masses deflect and settle after some time. In passive suspension system, this value is about 5~6 sec (fig. 8). Four system states: suspension travel, sprung mass velocity, tire deflection and unsprung mass velocity are plotted. Both the sprung mass and unsprung mass deflect for about 6 sec. The maximum value of suspension travel which is the output is 6mm.

A linear quadratic regulator is used for controlling the suspension system. Weighting matrices Q and R are obtained and the K gain to minimize the cost function is

$$Q = \begin{bmatrix} 100000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0. & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rlqr =0.01

K= [2387.9 75.7 750.5 -29.5]

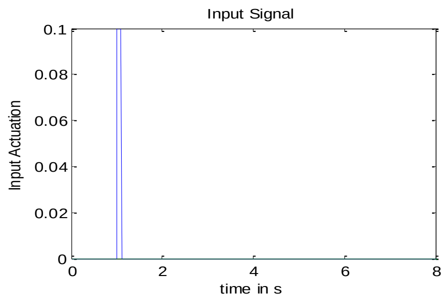

Fig.7 Impulse input signal given to system

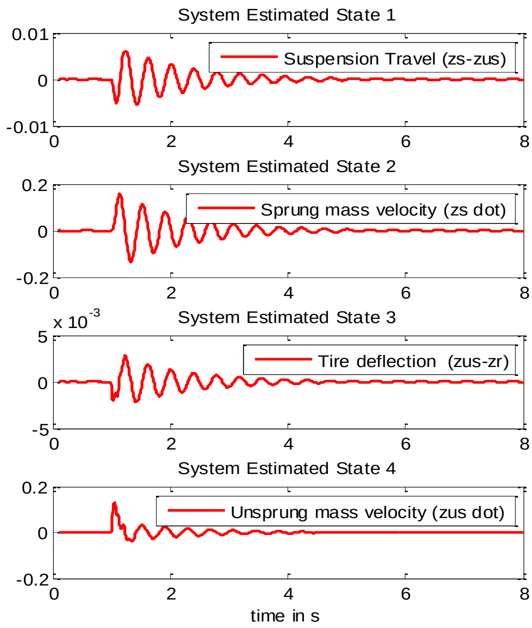

Fig.8 Passive suspension system internal states in response to impulse input

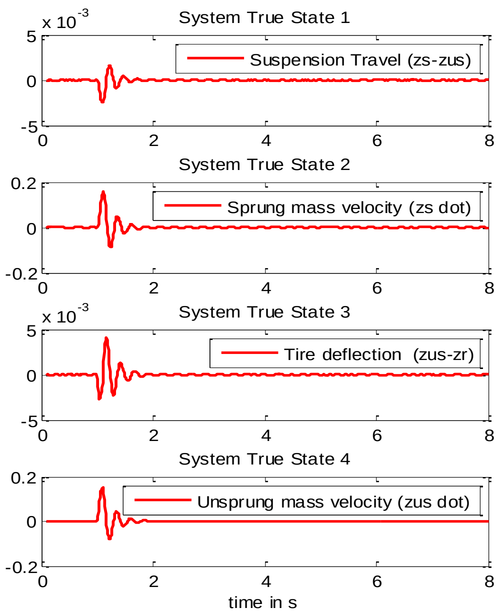

Fig.9 Active suspension system internal states in response to impulse input

This K matrix is multiplied by the system states and fed back to the control force input port. This linear quadratic regulator makes the system stabilize within 2 sec. The maximum value of suspension travel is reduced to 1.6 mm. The system states for active suspension system are shown in Fig (9). It is clear that the suspension travel has decreased considerably reducing the motion between the chassis and the wheel.

Fig. (10) and fig. (11) show the differences in suspension travel of passive and active suspension systems. Not much changes in values of sprung mass velocity, tire deflection and unsprung mass velocity are seen because the LQR controller invests more effort in reducing suspension travel as is evident from the Qlqr matrix.

A Kalman filter is used to estimate the internal states of the system. The weighting matrices Qk and R are obtained after a few iterations. The process noise covariance of the system for all the states is assumed to be 0.0001 times a zero-mean random variable and the measurement noise covariance matrix is set as 0. 01 $\times 10^{-6} m^2$.

The initial states of the Kalman filter are assumed to be all zeros and the initial error covariance matrix is set to all zeros. The system outputs and the input to the system (road input and control force input) are fed to the Kalman filter. Kalman filter estimates the internal states and the outputs of the system. Table (2) shows the mean squared error between actual and estimated states.

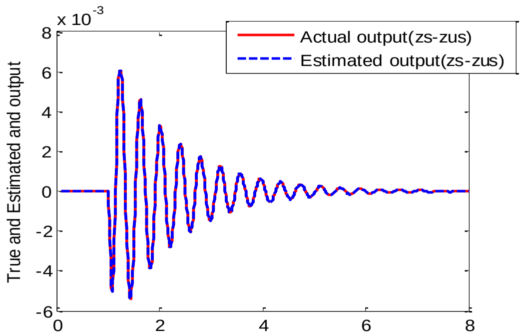

Fig.10 Impulse Passive suspension system outputs for impulse input simulated as road roughness signal

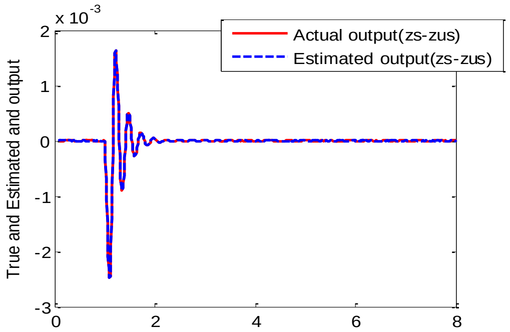

Fig.11 Impulse Active suspension system outputs for impulse input simulated as road roughness signal

Fig. (12) shows the estimation done by Kalman filter for a passive suspension system. The red signal is the true value of the signal and the blue signal is the Kalman estimated signal. It can be seen that Kalman filter correctly estimates the next state and Table (2) shows that there is negligible error in estimation of the Kalman filter. The assumption is that all the internal states of the system are measurable and hence the use of Kalman filter. The measured output of the system is given as input to the Kalman filter along with the inputs of the system. The Kalman filter block estimates the internal states and the current system output using the weights of the Q, and R matrices.

Linear quadratic regulator and linear quadratic Gaussian (LQG) strategy is tested in Matlab™/Simulink software for both passive and active suspension systems.

Table 2 Experimental procedure parameters

	Suspension Travel	Sprung mass velocity	Tire deflection	Unsprung mass velocity
Passive Suspension System	2.14e-8	4.80e-7	9.43e-9	2.75e-07
LQR	1.58e-8	3.42e-7	7.22e-9	2.62e-7
LQG	1.59e-8	3.37e-7	7.58e-9	2.75e-7

Fig. (11) shows the true and estimated system states of passive suspension system. The signal in red is the actual signal and the signal in blue is the Kalman estimated signal. Fig. (12) shows true and estimated signals of the LQR regulator strategy. Fig. (14) shows the correctly estimated internal states used in the LQG strategy of the system.

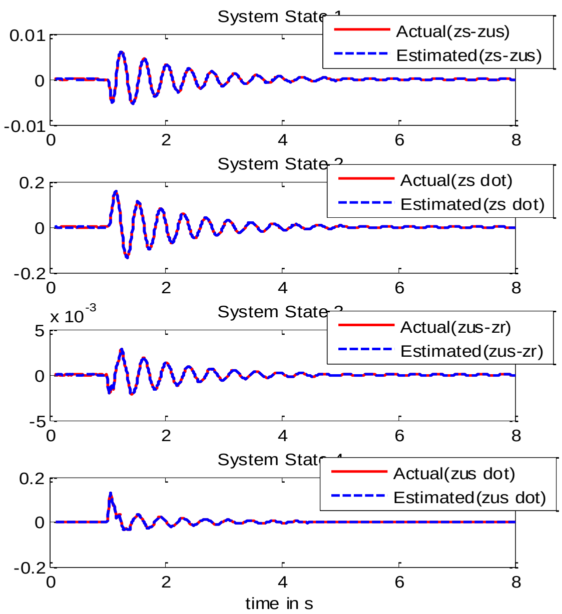

Fig.12 Impulse Comparison of actual and estimated states for given impulse input for passive suspension system

It is easily observable that there is negligible error in internal states of LQR and LQG controller strategy.

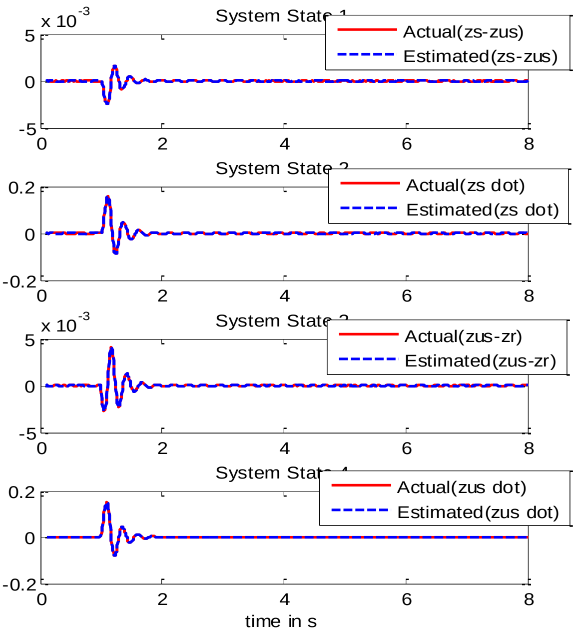

Fig.13 Closed loop system estimated state output compared with actual state for given impulse input for LQR

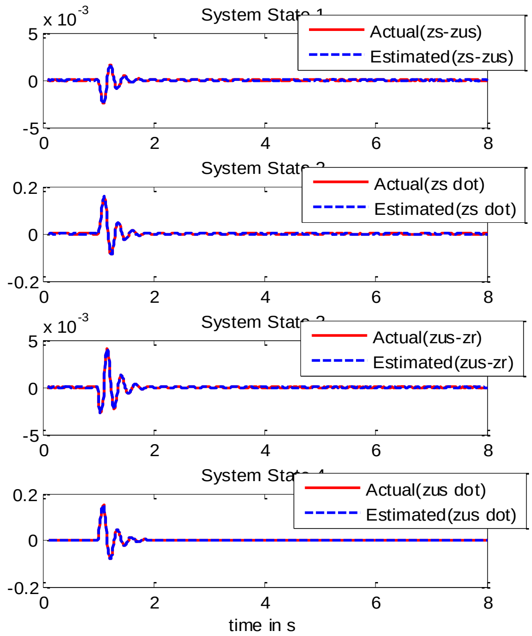

Fig.13 Closed loop system estimated state output compared with actual state for given impulse input for LQG

Conclusions

Active suspension systems greatly reduce the suspension travel and increase ride comfort of the passenger. Linear Quadratic Regulator and Linear Quadratic Gaussian is used to control the active suspension. The states are multiplied by the LQR gain matrix K and are penalized to minimize the cost function. The active suspension thus adjusts the travel between sprung and unsprung mass and at the same time increase the comfort of the passenger.

A linear quadratic Gaussian controller is also designed further by using the Kalman filter estimated states along with the linear quadratic control. The estimated states can be multiplied by the K matrix and a linear quadratic Gaussian controller is thus obtained. To achieve this, the Kalman Filter needs to be capable to estimate the internal states of the system accurately. In order to achieve this, Q and R covariance matrices also need to be accurately tuned. Thus, the performance of active suspension system can be improved with state estimation.

The control strategy can be further optimized to improve the LQG controller performance to effectively damp the road transmission from reaching the passenger. The LQG performance needs to be evaluated in presence of varying amounts of noise in the system and measurements and if the LQG is susceptible to noise then a robust control strategy needs to be designed further.

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