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Research Article

New Simplified Formula for Crack Growth Direction under Multiaxial Cyclic Loads

Fathi Alshamma[#] and Shwan Abdulmuhsin Zainalaabdeen[^*]

[#]College of Engineering/University of Baghdad, Baghdad, Iraq
[^]State Company for Oil Projects/Ministry of Oil, Baghdad, Iraq

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Abstract

In the present research, new simple formula has been proposed for mixed modes I/II conditions due to multiaxial cyclic loads which is suitable only to predict crack growth direction with initial transverse crack, so the bonus of this study represented by the facilities of simple implementation to find out (θ_C) and compare the results with old approach that implemented with complex SIFs for modes I/II to be substituted in Erdogan-Sih law which was established to predict crack growth direction, hence, the case study has been proposed for thin copper pipe 12200 under specified stress ratio (R=-1) and the test has been conducted by using new test rig for this purpose, adding to this that the results have been verified via comparison between old and new procedure as well as the convergent between experiment and available formulas with good agreement.

Keywords: *Multiaxial cyclic loads, crack growth direction, new formula*

Nomenclature

σ = Applied direct stress (Mpa)
τ= Applied shear stress (Mpa)
r_o=Outside radius of the specimen (m)
G=Shear modulus (Gpa)
$\emptyset$=Angle of twist (rad.)
I=Second moment of area (m^4)
M=Bending moment (N.m)
F_{app}=Applied force (N)
$K_I$and K_{II}=stress intensity factors (SIFs) for mode I and mode II respectively.
$\propto$=90^0 (for transverse crack with respect to axial axis).
$\rho^2 = \frac{\sqrt{12(1-\upsilon^2)a_i^2}}{8rt}$ (non dimensional factor).
a_i = initial crack length (m)
υ=0.33 (Poisson's ratio)
r=mean radius(m)
θ_C= crack path direction with respect to crack line (circumferential line).
S=stress ratio ($^{\tau}/_{\sigma}$)

1. Introduction

Stress intensity factors (K_I and K_{II}) used in engineering solid mechanics to predict stress state at crack tip as a consequence to remote applied stresses conditions and the magnitude of these SIFs depends on specimen configuration, initial crack length, location of crack and applied remote stresses (P.R. Gawande and A. Bharule, 2014). There are different structures like piping system in practical implementation that are subjected to multiaxial loads which cause a crack propagation in particular direction that leads to final fracture, therefore, case study proposed to investigate path direction of a crack growth for long initial crack that introduced transversely with respect to pipe axial axis;

It is worth mentioning that the dimension of specimens were selected from real service conditions to be applicable and suitable with previous well known formulas. Special stress intensity factors (SIFs) have been used in this work that based on Ph.D. thesis (J. J. F. Bonnen, 1998), that depended basically on the particular study that had been developed (H.V. Lakshminarayana and M.V.V.Murthy 1976) for long pipe and short crack. Noting that state of art concept has been achieved in this work for long cracks in the specimens (which established already for short crack), adding to this that one of the most famous formula (F.Erdogan and G.C.Sih,1963) concerning crack direction that had been established for brittle plate were adopted in this work for ductile pipe and simplified new innovative formula developed which depends on the ratio of shear stress to direct stress $^{\tau}/_{\sigma}$. (F.Erdogan and G.C.Sih, 1963) concluded that the crack extended in plate in the direction which is perpendicular on the maximum tension which was

*Corresponding author **Shwan Abdulmuhsin Zainalaabdeen** is working as Senior Engineer **and Dr. Fathi Alshamma** is working as Assistant Professor

considered as satisfactory form for brittle material under plane loading and transverse shear. (H.A.Richard *et al*, 2005) studied a mixed mode crack growth, and new empirical model had been proposed to guess crack path direction (P. Arora *et al*, 2011) studied solid and tubular specimens of SA333Gr.6 material under pure axial, pure shear, axial-torsion. (M. Blazic *et al*, 2014) considered crack growth path for plate specimen with two holes under mixed mode conditions with studying of residual life for the two dimensional structural elements. (V. chaves *et al*, 2015) investigated the crack growth direction in AISI 304L stainless steel under axial – torsional loads. New test rig have been designed and manufactured to be suitable for multiaxial cyclic loads, where the data monitored for each period as well as special graphs have been drawn to illustrate crack behavior, the motivation of this research is to proof the durability of the rig by estimation the direction of crack path to be compared with previous well known laws concerning this issue and demonstrated the compliance between real service and previous theories, as well as proposing new formulas to find out crack growth direction.

Copper pipes 12200 (specimens) with the same dimensions in real practice were considered in this work. There is lack of information regarding multiaxial cyclic load effect (cyclic bending-cyclic torsion) on this type of copper pipe that adopted with the proposed rig. Based on Figure (1), the crack grew in counterclockwise direction and θ_C were estimated with respect to initial crack line.

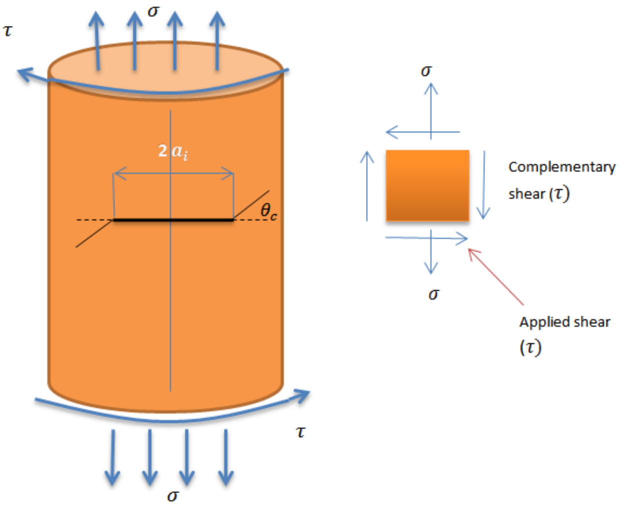

Fig.1 Illustrates crack growth direction due to multiaxial cyclic loads

2. Experiment Procedure and Data Monitoring

Actual practice condition has been demonstrated with its simulation in Figure (2) for the new rig via exhibition of the important portion, so the reciprocated shaft acting on the action arm to twist and bend the specimen where the oscillation was equaled to around 1830 r.p.m. , noting that the process was governed by control system.

Firstly, cracks have been introduced in pipes in such away the crack was through toward inside portion, see Figure (3) . Secondly the specimen settled in the rig and the limitation pads located in such away the displacement governed to be checked and measured again during the test by mechanical and digital caliper, subsequently, these displacements substituted in the following formulas (D. Rozumek *et al*, 2008). To find out shear stress τ and direct stress σ:

$$\tau = r_o \frac{G\emptyset}{L} \tag{2.1}$$

$$F_{app} = \frac{3EIx}{l^3} \tag{2.2}$$

$$\sigma = \frac{Mr_o}{I} = \frac{(wl)r_o}{I} \tag{2.3}$$

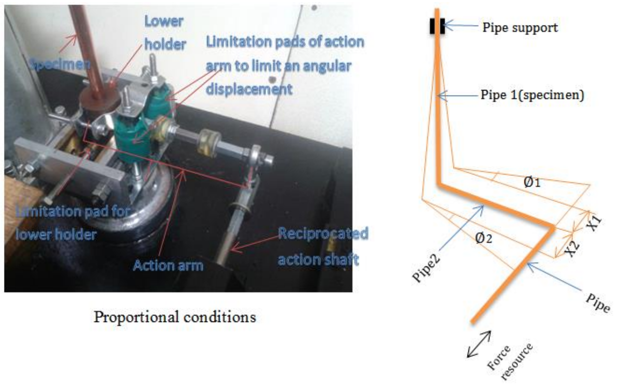

Fig.2 Multiaxial cyclic loads

Table1. Mechanical and chemical properties (copper pipe)

Mechanical properties				
Yield point stress (Mpa) , $\sigma_{y.p.}$	Ultimate stress (Mpa) , $\sigma_{ult.}$		modulus of rigidity (Gpa) , G	modulus of elasticity (Gpa) , E
240	269.5		43.583	115.93
Chemical properties				
Zn%	Pb%	P%	Fe%	Al%
0.0062	0.0205	0.0376	0.0127	0.0221
S%	Ni%	Bi%	Sb%	Cu%
0.0116	0.0052	0.0147	0.114	≈99

An incremental in crack lengths recorded at each period to be discussed by figures, where the rig was turned off each period, according to crack propagation to be monitored easily. If total crack length (a) reaches to around 8 mm after growth at each side (i.e. 2a≈16 that equaled to around 40% of specimen circumference) the displacement will be changed suddenly and Mode III appeared, so any readings of crack increment out this range have not been considered and the test should be stopped where the fracture will be occur after very few seconds and the number of cycles (N) should be recorded to be discussed via figures. For considered specimen, gauge length was equal to (180 mm), the wall thickness equaled to 0.7 mm and outside diameter equaled to 12.6 mm. The angular displacement $\emptyset_1$ and $\emptyset_2$ was equal to (0.0367 rad.), i.e. shear stress (τ) equal to55.92 Mpa , and x_1, x_2 equal to 3.25 mm, so σ = 219.77 Mpa, adding to this that initial surface crack length ($2a_i$) was equal to 6.94mm, so the crack grew after test commencement in the left hand side (LHS) and right hand side (RHS) in counterclockwise direction with respect to crack tip, so the most dangerous crack growth was considered. See Figure 4.

Fig.3 Initial crack in pipes (before test commencement)

Fig.4 Illustrates a sample of crack path due to multiaxial load

Noting that the crack non uniformly grew at both sides and this phenomena had been observed also by (Y. Hos, and M. Vormwald, 2015). As well as the change of growth is a property of the material microstructure and this change might be observed in the same specimen at both sides as stated also by reference (L. Damkilde, 2015).

3. Analytical Part

The experiments have been analyzed to confirm the validity of the rig and its results, moreover, crack trajectory has been investigated by using maximum tangential stress theory that proposed by (F.Erdogan and G.C.Sih,1963). Hence specified SIFs have been used for (mode I) and (mode II) that proposed by (J. J. F. Bonnen, 1998), so these SIFs substituted in Erdogan-Sih formula and as indicated hereunder.

3.1 Old Procedure (Model)

Old procedure was adopted previously to find out θ_C, as following

$$K_I = \sigma\sqrt{\pi a_i}\left[sin^2 \propto + \frac{\pi\rho^2}{32}(3 - 2cos2 \propto - cos4 \propto)\right]$$
$$+\tau\sqrt{\pi a_i}\left[sin2 \propto + \frac{\pi\rho^2}{32}(9sin2 \propto + 2sin4 \propto)\right] \quad (3.1)$$
$$K_{II} = \sigma\sqrt{\pi a_i}\left[\frac{1}{2}sin2 \propto + \frac{\pi\rho^2}{32}sin4 \propto\right]$$
$$+\tau\sqrt{\pi a_i}\left[cos2 \propto - \frac{\pi\rho^2}{16}(1 - 2cos2 \propto - cos4 \propto)\right] \quad (3.2)$$

Maximum tangential stress formula (MTS) for Erdogan-Sih is

$$\theta_C = \pm arccos\left(\frac{3K_{II}^2 + K_I\sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2}\right) \quad (3.3)$$

It is evident that above mentioned formulas depend on Mode I and Mode II and insensitive to Mode III, furthermore, material geometry and SIFs must be known to find out θ_C .

3.2 New Procedure (Model)

New simple procedure developed and adopted, as indicated hereunder:

$$\theta_C = \pm arccos\left(\frac{3S^2 + \sqrt{1 + 8S^2}}{1 + 9S^2}\right) \quad (3.4)$$

It is evident also that new formula depends only on the ratio ($^{\tau}/_{\sigma}$) and there is no need to the geometry and stress intensity factors SIFs to find out θ_C .

Noting that the sign of θ_C is positive (counterclockwise) if K_{II} negative and vice versa (F.Erdogan and G.C.Sih,1963), in other word crack grew in the direction of applied shear stress and as exhibited in Figure (1). Adding to this that $K_{min} = 0$ in the case of R≤0 as recommended by reference ASTM E647.

4. Results and Discussion

Based on the observed reading of crack growth direction and increasing in crack length that received from experiments by considering most dangerous state which was at right hand side where the crack grew in counterclockwise at θ_C=21^0 whereas θ_C that estimated from analytical part was equal to 25.68^0 namely equation (3.3) or (3.4). By comparing between analytical and experimental work regardingθ_c the error was equal to around 18% which represent good agreement. In the other hand, by comparing between old and new procedures the error was equal to zero % which is represent excellent agreement.

By drawing ((a)m) against ((N)cycles) the slope increased by increasing (N) and this is normal phenomena that observed and discussed in previous researches. It is worth emphasizing that new procedure was applicable on any suggested case study for transverse crack that depend on the ratio ($^{\tau}/_{\sigma}$) only with easiest implementation.

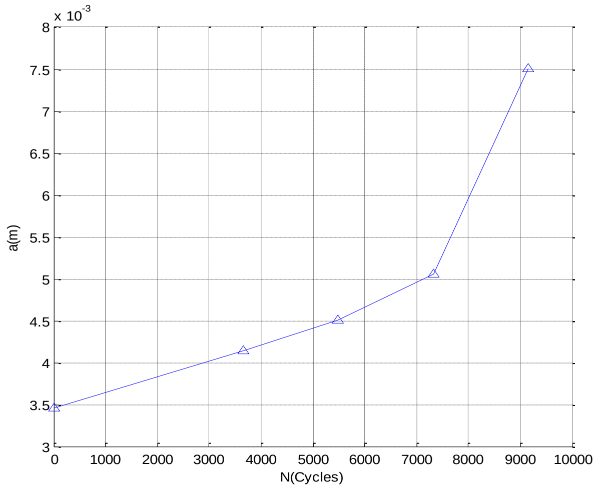

Fig.5 Crack growth according to cycles

Conclusions

1) It is evident that Erdogan-Sih established for brittle plate under mixed mode conditions and in this study used for very ductile pipe with good agreement. Moreover, SIFs that had been developed for initial small crack length were used in this work for initial long crack length ($2a_i$=17% of specimen circumference) with good agreement also.
2) It is evident also that new formula is much easier than old procedure in terms of implementation and depends only on the ratio ($^{\tau}/_{\sigma}$) and there are no needs to the geometry and stress intensity factors SIFs to find out θ_C , so the results shown excellent agreement.
3) According to above mentioned results the durability of the new test rig confirmed to test this type of pipes under real service conditions, and actual practice is compatible with previous theories.

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