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Research Article

An Extended Result on Fixed Point Theorem in ε-Chainable Fuzzy Metric Space

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Abstract

In this paper we establish fixed point theorem for six weakly compatible mappings in a complete $\varepsilon -$ chainable fuzzy metric space by depreciating the condition of continuity of any mappings. Our results extend and generalize several known results of fixed point theory in different spaces.

Keywords: *Fuzzy Metric Space, $\varepsilon -$ Chainable Fuzzy Metric Space, Weakly Compatible Mappings,Common Fixed Point.*

1. Introduction

The foundations of fuzzy set theory and fuzzy mathematics were laid down by Zadeh (1965) by the introduction of the notion of fuzzy sets. The theory of fuzzy sets has vast applications in applied sciences and engineering such as neural network theory, stability theory, mathematical programming, genetics, nervous systems, image processing, control theory etc. to name a few. The theory of fixed points is one of the basic tools to handle the physical formulations. This has led to the development and fuzzyfication of several concepts of analysis and topology. Kramosil and Michalek (1975) introduced the concept of a fuzzy metric space by generalizing the concept of a probabilistic metric space to the fuzzy situation. The concept of Kramosil and Michalek of a fuzzy metric space was later modified by George and Veeramani (1994). Grabeic (1988) following the concept of Kramosil and Michalek (1975) obtained the fuzzy version of Banach's fixed point theorem. Using the notion of weak commuting property, Sessa (1982) improved commutative conditions in fixed point theorems. Jungck (1986 &1998) introduced the concept of compatibility and proved common fixed point theorem for set valued functions without continuity of mappings in metric spaces. Jungck and Rhoades (2006) introduced the concept of weakly compatible maps which was the generalization of the concept of compatible maps. The notion of compatible mappings in fuzzy metric spaces was introduced by Cho (1997). Vasuki (1999) introduced the concept of R – weakly commuting map and proved a fixed point theorem for fuzzy metric space using this concept. Singh and Chauhan (2000) introduced the concept of compatibility in fuzzy metric spaces. Singh and Jain (2005) studied the notions of semi compatibility and weak compatibility of maps in fuzzy metric spaces. Sharma and Deshpande (2009) established some results on common fixed point theorems for finite number of discontinuous, non-compatible mappings on non-complete fuzzy metric spaces. Furthermore, Sharma and Deshpande (2010) extended their own work by proving some common fixed point theorems for finite number of discontinuous, non-compatible mappings on non-complete intuitionistic fuzzy metric spaces. In this paper we establish fixed point theorems for six weakly compatible mappings in a complete $\varepsilon -$ chainable fuzzy metric space by depreciating the condition of continuity. Our results extend and generalize several known results of fixed point theory in different spaces.

2. Preliminaries

Definition 2.1.: A 3 – tuple $(X, \mathcal{M}, *)$ is called a $\mathcal{M} -$ fuzzy metric space if X is an arbitrary (non - empty) set, $*$ is a continuous $t - norm$, and $\mathcal{M}$ is a fuzzy set on $X^2 \times (0,\infty)$, satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$,
(i) $\mathcal{M}(x, y, t) = 0$,
(ii) $\mathcal{M}(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
(iii) $\mathcal{M}(x, y, t) = \mathcal{M}(y, x, t)$,

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(iv) $\mathcal{M}(x,y,t) * \mathcal{M}(y,z,s) \leq \mathcal{M}(x,z,t+s)$,
(v) $\mathcal{M}(x,y,.) : [0,1] \to [0,1]$ is left continuous.

Example 2.1.: Let (X,d) be a metric space. Define $a * b = ab$, or

$a * b = \min(a,b)$, and for all x, y and $t > 0$,

$$\mathcal{M}(x,y,t) = \frac{t}{t+d(x,y)}$$

then $(X,\mathcal{M},*)$ is a fuzzy metric space. We call this fuzzy metric $\mathcal{M}$ induced by the metric d, the standard fuzzy metric.

Lemma 2.1.: $\mathcal{M}(x,y,.)$ is non-decreasing for all $x, y \in X$.

Proof: Suppose $\mathcal{M}(x,y,t) > M(x,y,s)$ for some

$0 < t < s$. Then $\mathcal{M}(x,y,t) * \mathcal{M}(y,y,s-t) \leq \mathcal{M}(x,y,s) < M(x,y,t)$.

Since $\mathcal{M}(y,y,s-t) = 1$,therefore,$\mathcal{M}(x,y,t) \leq \mathcal{M}(x,y,s) < M(x,y,t)$, which is a contradiction. Thus, $\mathcal{M}(x,y,.)$ is non-decreasing for all $x, y \in X$.

Definition 2.2.: Let $(X,\mathcal{M},*)$ be a fuzzy metric space:

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$, if $\lim_{n\to\infty} \mathcal{M}(x_n,x,t) = 1$, for all $t > 0$.
(ii) A sequence $\{x_n\}$ in X is called a Cauchy sequence if

$$\lim_{n\to\infty} \mathcal{M}(x_{n+p},x_n,t) = 1, \text{ for all } t > 0 \text{ and } p > 0.$$

(iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Remark 2.1.: Since $*$ is continuous, if follows from the condition (iv) of Definition 2.1 that the limit of the sequence in fuzzy metric space is uniquely determined.

Let $(X,\mathcal{M},*)$ be a fuzzy metric space with the following condition :

$\lim_{n\to\infty} \mathcal{M}(x,y,t) = 1$ for all $x, y \in X$and $t > 0$

Lemma 2.2.: If for all $x, y \in X$, $t > 0$ and $0 < k < 1$, $\mathcal{M}(x,y,kt) \geq \mathcal{M}(x,y,t)$, then $x = y$.

Proof: Suppose that there exists $0 < k < 1$ such that $\mathcal{M}(x,y,kt) \geq \mathcal{M}(x,y,t)$ for all $x, y \in X$ and $t > 0$. Then, $\mathcal{M}(x,y,t) \geq \mathcal{M}\left(x,y,\frac{t}{k}\right)$, and so $\mathcal{M}(x,y,t) \geq \mathcal{M}\left(x,y,\frac{t}{k^n}\right)$ for positive integer n. Taking limit as $n \to \infty$, $\mathcal{M}(x,y,t) \geq 1$ and hence $x = y$.

Lemma 2.3.: Let$(X,\mathcal{M},*)$ be a fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in (0,1)$such that

$$\mathcal{M}(y_{n+2},y_{n+1},kt) \geq \mathcal{M}(y_{n+1},y_n,t),$$

for all $t > 0$ and $n = 1,2,\ldots$, then $\{y_n\}$ is a Cauchy sequence in X.

Definition 2.3.: Let A and B be mappings from a fuzzy metric space $(X,\mathcal{M},*)$ into itself. The mappings A and B are said to be compatible if

$\lim_{n\to\infty} \mathcal{M}(ABx_n,BAx_n,t) = 1$, for all $t > 0$,

Whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Definition 2.4.: Two self mappings A and B of a fuzzy metric space $(X,\mathcal{M},*)$ are said to be weakly compatible if $ABu = BAu$ whenever $Au = Bu$ for some

$u \in X$. If the self mappings A and B of a fuzzy metric space $(X,\mathcal{M},*)$ are compatible, then they are weakly compatible, but the converse is not necessarily true.

Example 2.2.: Let $X = [0,4]$ and $a * b = \min\{a,b\}$. Let $\mathcal{M}$ be the standard fuzzy metric induced by d, where $d(x,y) = |x-y|$ for $x, y \in X$. Define two self mappings A and B of the fuzzy metric space $(X,\mathcal{M},*)$ by:

$$Ax = \begin{cases} 4-x, & 0 \leq x \leq 2 \\ 4, & 2 \leq x \leq 4 \end{cases}$$

$$Bx = \begin{cases} x, & 0 \leq x \leq 2 \\ 4, & 2 \leq x \leq 4 \end{cases}$$

Let $\{x_n\} = \{1 - (^1/_n)\}$. Then it can be easily proved that the self mappings A and B are weakly compatible but they are not compatible.

Definition 2.5.: A finite sequence $x = x_0, x_1, \cdots, x_n = y$ in a fuzzy metric space $(X,\mathcal{M},*)$ is called $\varepsilon -$ chain from x to y if there exists $\varepsilon > 0$ such that $\mathcal{M}(x_i,x_{i-1},t) > 1 - \varepsilon$ for all $t > 0$ and $i = 1,2,\cdots,n$.

A fuzzy metric space $(X,\mathcal{M},*)$ is called $\varepsilon -$ chainable if there exists an $\varepsilon -$ chain from x to y, for any $x, y \in X$.

3. The Main Result

Theorem 3.1: Let $(X,\mathcal{M},*)$ be a complete $\varepsilon -$ chainable fuzzy metric space and let A, B, S, T, P and Q be the self mappings of X, satisfying the following conditions:

(1) $A(X) \subset ST(X)$ and $B(X) \subset PQ(X)$;
(2) The pair (A, PQ) and (B, ST) are weakly compatible;
(3) There exists a constant $k \in (0,1)$, such that for every $x, y \in X$ and $t > 0$,

$$\mathcal{M}(Ax,By,kt) \geq \left\{ \mathcal{M}(PQx,STy,t) * \mathcal{M}(Ax,PQx,t) * \mathcal{M}(By,STy,t) * \mathcal{M}(Ax,STy,t) * \mathcal{M}(By,PQx,t) * \frac{\mathcal{M}(By,PQx,t)}{\mathcal{M}(PQx,STy,t) * \mathcal{M}(By,PQx,t)} \right\}.$$

Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: We can find a Cauchy sequence $\{y_n\}$ in X such that

$y_{2n-1} = STx_{2n-1} = Ax_{2n-2}$ and $y_{2n} = PQx_{2n} = Bx_{2n-1}$ for $n = 1,2,3,\cdots$ From completeness, $y_n \to z$ for some $z \in X$, and so $\{Ax_{2n-2}\}, \{PQx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{STx_{2n-1}\}$ also converge to z. Similarly as Cho S.H. and Jung J.H. (2006) we can show that $\{x_n\}$ is a Cauchy sequence in X. Since X is complete, hence there exists $z \in X$ such that $\{x_n\}$ converge to z. Hence there exists $u, v \in X$ such that $PQu = z$ and $STv = z$ respectively. By (3), we have

$$\mathcal{M}(Au, y_{2n}, kt) = \mathcal{M}(Au, Bx_{2n-1}, kt) \geq \Big\{\mathcal{M}(PQu, STx_{2n-1}, t) * \mathcal{M}(Au, PQu, t) * \mathcal{M}(Bx_{2n-1}, STx_{2n-1}, t) * \mathcal{M}(Au, STx_{2n-1}, t) * \mathcal{M}(Bx_{2n-1}, PQu, t) * \frac{\mathcal{M}(Bx_{2n-1}, PQu, t)}{\mathcal{M}(PQu, STx_{2n-1}, t) * \mathcal{M}(Bx_{2n-1}, PQu, t)}\Big\}.$$

Taking the limit as $n \to \infty$,

$$\mathcal{M}(Au, z, kt) \geq \Big\{\mathcal{M}(z, z, t) * \mathcal{M}(Au, z, t) * \mathcal{M}(z, z, t) * \mathcal{M}(Au, z, t) * \mathcal{M}(z, z, t) * \frac{\mathcal{M}(z, z, t)}{\mathcal{M}(z, z, t) * \mathcal{M}(z, z, t)}\Big\}.$$

$$\mathcal{M}(Au, z, kt) \geq \Big\{1 * \mathcal{M}(Au, z, t) * 1 * \mathcal{M}(Au, z, t) * 1 * \frac{1}{1*1}\Big\}.$$

which gives $\mathcal{M}(Au, z, kt) \geq \mathcal{M}(Au, z, t)$.
Therefore by the Lemma 2.2, we have $Au = z$. Since $PQu = z$,
thus $Au = PQu = z$, that is u is a coincidence point of A and PQ.
Similar to (3), we have

$$\mathcal{M}(y_{2n-1}, Bv, kt) = \mathcal{M}(Ax_{2n-2}, Bv, kt) \geq \Big\{\mathcal{M}(PQx_{2n-2}, STv, t) * \mathcal{M}(Ax_{2n-2}, PQx_{2n-2}, t) * \mathcal{M}(Bv, STv, t) * \mathcal{M}(Ax_{2n-2}, STv, t) * \mathcal{M}(Bv, PQx_{2n-2}, t) * \frac{\mathcal{M}(Bv, PQx_{2n-2}, t)}{\mathcal{M}(PQx_{2n-2}, STv, t) * \mathcal{M}(Bv, PQx_{2n-2}, t)}\Big\}.$$

Taking the limit as $n \to \infty$,

$$\mathcal{M}(z, Bv, kt) \geq \Big\{\mathcal{M}(z, z, t) * \mathcal{M}(z, z, t) * \mathcal{M}(Bv, z, t) * \mathcal{M}(z, z, t) * \mathcal{M}(Bv, z, t) * \frac{\mathcal{M}(Bv, z, t)}{\mathcal{M}(z, z, t) * \mathcal{M}(Bv, z, t)}\Big\}.$$

$$\mathcal{M}(z, Bv, kt) \geq \Big\{1 * 1 * \mathcal{M}(Bv, z, t) * 1 * \mathcal{M}(Bv, z, t) * \frac{\mathcal{M}(Bv, z, t)}{1 * \mathcal{M}(Bv, z, t)}\Big\}.$$

which gives $\mathcal{M}(z, Bv, kt) \geq \mathcal{M}(Bv, z, t)$.
Therefore by the Lemma 2.2, we have $Bv = z$. Since $STv = z$,
thus $Bv = STv = z$, that is u is a coincidence point of B and ST.
Since the pair $\{A, PQ\}$ is the weakly compatible therefore A and PQ commute at their coincidence point that is $A(PQu) = PQ(Au)$ or $Az = PQz$.
Similarly the pair $\{B, ST\}$ is the weakly compatible therefore B and ST commute at their coincidence point that is $B(STv) = ST(Bv)$ or $Bz = STz$.
Now we prove that $Az = z$, By (3), we have

$$\mathcal{M}(Az, Bx_{2n-1}, kt) \geq \Big\{\mathcal{M}(PQz, STx_{2n-1}, t) * \mathcal{M}(Az, PQz, t) * \mathcal{M}(Bx_{2n-1}, STx_{2n-1}, t) * \mathcal{M}(Az, STx_{2n-1}, t) * \mathcal{M}(Bx_{2n-1}, PQz, t) * \frac{\mathcal{M}(Bx_{2n-1}, PQz, t)}{\mathcal{M}(PQz, STx_{2n-1}, t) * \mathcal{M}(Bx_{2n-1}, PQz, t)}\Big\}.$$

Taking the limit as $n \to \infty$, we have

$$\mathcal{M}(Az, z, kt) \geq \Big\{\mathcal{M}(PQz, z, t) * \mathcal{M}(Az, PQz, t) * \mathcal{M}(z, z, t) * \mathcal{M}(Az, z, t) * \mathcal{M}(z, PQz, t) * \frac{\mathcal{M}(z, PQz, t)}{\mathcal{M}(PQz, z, t) * \mathcal{M}(z, PQz, t)}\Big\}.$$

$$\mathcal{M}(Az, z, kt) \geq \Big\{\mathcal{M}(z, z, t) * \mathcal{M}(Az, z, t) * \mathcal{M}(z, z, t) * \mathcal{M}(Az, z, t) * \mathcal{M}(z, z, t) * \frac{\mathcal{M}(z, z, t)}{\mathcal{M}(z, z, t) * \mathcal{M}(z, z, t)}\Big\}.$$

$$\mathcal{M}(Az, z, kt) \geq \Big\{1 * \mathcal{M}(Az, z, t) * 1 * \mathcal{M}(Az, z, t) * 1 * \frac{1}{1*1}\Big\}.$$

$$\mathcal{M}(Az, z, kt) \geq \mathcal{M}(Az, z, t).$$

Therefore by Lemma 2.2, we have $Az = z$. Since $PQu = Az$,
thus $Az = PQz = z$. Similar to (3), we have

$$\mathcal{M}(Ax_{2n-2}, Bz, kt) \geq \Big\{\mathcal{M}(PQx_{2n-2}, STz, t) * \mathcal{M}(Ax_{2n-2}, PQx_{2n-2}, t) * \mathcal{M}(Bz, STz, t) * \mathcal{M}(Ax_{2n-2}, STz, t) * \mathcal{M}(Bz, PQx_{2n-2}, t) * \frac{\mathcal{M}(Bz, PQx_{2n-2}, t)}{\mathcal{M}(PQx_{2n-2}, STz, t) * \mathcal{M}(Bz, PQx_{2n-2}, t)}\Big\}.$$

Taking the limit as $n \to \infty$, we have

$$\mathcal{M}(z, Bz, kt) \geq \Big\{\mathcal{M}(z, z, t) * \mathcal{M}(z, z, t) * \mathcal{M}(Bz, z, t) * \mathcal{M}(z, z, t) * \mathcal{M}(Bz, z, t) * \frac{\mathcal{M}(Bz, z, t)}{\mathcal{M}(z, z, t) * \mathcal{M}(Bz, z, t)}\Big\}.$$

$$\mathcal{M}(z, Bz, kt) \geq \Big\{1 * 1 * \mathcal{M}(Bz, z, t) * 1 * \mathcal{M}(Bz, z, t) * \frac{\mathcal{M}(Bz, z, t)}{1 * \mathcal{M}(Bz, z, t)}\Big\}.$$

This gives $\mathcal{M}(z, Bz, kt) \geq \mathcal{M}(Bz, z, t)$.
Therefore by Lemma 2.2, we have $Bz = z$. Since $STz = Bz$,
thus $Bz = STz = z$.
For uniqueness, let w be another common fixed point of A, B, S, T, P and Q. By (3),
we have

$$\mathcal{M}(z, w, kt) = \mathcal{M}(Az, Bw, kt) \geq \Big\{\mathcal{M}(PQz, STw, t) * \mathcal{M}(Az, PQz, t) * \mathcal{M}(Bw, STw, t) * \mathcal{M}(Az, STw, t) * \mathcal{M}(Bw, PQz, t) * \frac{\mathcal{M}(Bw, PQz, t)}{\mathcal{M}(PQz, STw, t) * \mathcal{M}(Bw, PQz, t)}\Big\}.$$

$$\mathcal{M}(z, w, kt) \geq \Big\{\mathcal{M}(z, w, t) * \mathcal{M}(z, z, t) * \mathcal{M}(w, w, t) * \mathcal{M}(z, w, t) * \mathcal{M}(w, z, t) * \frac{\mathcal{M}(w, z, t)}{\mathcal{M}(z, w, t) * \mathcal{M}(w, z, t)}\Big\}.$$

$$\mathcal{M}(z, w, kt) \geq \mathcal{M}(z, w, t).$$

From Lemma 2.2, $z = w$.
Therefore z is common fixed point of A, B, S, T, P and Q.

Conclusions

In this paper we establish fixed point theorem for six weakly compatible mappings in a complete ε-chainable fuzzy metric space by depreciating the condition of continuity of any mappings. Our result is more gripping and useful for other researchers.

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