

Research Article

Numerical Methods for Investigation the Two-Phase Flow-Description of the Model

Hameed Khalaf Saleh*

Engineering Techniques of Refrigeration and Air Conditioning Departments, Middle Technical University, Baghdad, Iraq

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Abstract

In the current work is making a solution of two phase jets using model of finite difference method. It is given in detailed the solution of the system of equations for solving a problem of two phase jets.

Keywords: Equations, finite difference method, two-phase flow

1. Introduction

Numerical modeling of turbulent non-isothermal flows is made on the basis of equation for movement of vertical non-isothermal jet. To close the system of equations is necessary to choose an appropriate model of turbulence.

In the work of (Elgobashi *et al*, 1982), (Elgobashi *et al*, 1983) to close the system of equations for the motion it is used a model with two equations (for the transfer of turbulent kinetic energy of the gaseous medium K_g and the rate of dissipation ε). In the work (Antonov *et al*, 1995), (Lien *et al*, 1996) and (Nam *et al*, 1991) is made a modification of the existing model of turbulence ($K-\varepsilon$), by introducing a new equation in terms of turbulent energy of the phase of impurities (K_p).

This pattern of turbulence can be conditionally write as $K_g - \varepsilon - K_p$. Thus as it mentioned has been introduced dividing the turbulent energy of the two phases, which corresponds more accurately the accepted two-fluid model of the flow.

2. Mathematical Model

In the above-mentioned work, processed equations are as follows:

Equation for the transport of turbulent kinetic energy of the carrier phase:

$$(y^j \rho_g U_g) \cdot \frac{\partial K_g}{\partial x} + (y^j \rho_g V_g) \cdot \frac{\partial K_g}{\partial y} = \frac{\partial}{\partial y} \left[\frac{y^j \rho_g \nu_{tK}}{\sigma_k} \cdot \frac{\partial K_g}{\partial y} \right] + y^j \rho_g \nu_{tK} \cdot \left[\frac{\partial U_g}{\partial y} \right]^2 - (1)$$

$$- y^j \rho_g (\varepsilon + \varepsilon_p)$$

Equation for the transport of turbulent kinetic energy to the phase of impurities

$$(y^j \rho_p U_p) \cdot \frac{\partial K_p}{\partial x} + (y^j \rho_p V_p) \cdot \frac{\partial K_p}{\partial y} = \frac{\partial}{\partial y} \left[\frac{y^j \rho_p \nu_{tK}}{\sigma_k} \cdot \frac{\partial K_p}{\partial y} \right] + y^j \rho_p \nu_{tK} \cdot \left[\frac{\partial U_p}{\partial y} \right]^2 - (2)$$

$$- y^j \rho_p \varepsilon_p^*$$

Equation for the velocity of dissipation

$$(y^j \rho_g U_g) \cdot \frac{\partial \varepsilon}{\partial x} + (y^j \rho_g V_g) \cdot \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y} \left[\frac{y^j \rho_g \nu_{t\varepsilon}}{\sigma_\varepsilon} \cdot \frac{\partial \varepsilon}{\partial y} \right] - (3)$$

$$- y^j \rho_g \Phi_p + C_{\varepsilon 1} y^j \rho_g \frac{\varepsilon}{K_g} \left[\nu_{tK} \cdot \left(\frac{\partial U_g}{\partial y} \right)^2 + G \right] - y^j \rho_g \cdot \frac{\varepsilon^2}{K_g} \cdot (C_{\varepsilon 2} - C_{\varepsilon 3} \cdot \chi)$$

In these model equations has added one additional member $\frac{\chi_0}{1 + \chi_0} K_p$ (Abramovich *et al*, 1984), which

reflects this influence by the concentration of impurities on the turbulence of the flow. The equation for the transport of turbulent kinetic energy of the carrier medium for non-isothermal turbulent jets has the form

$$(y^j \rho_g U_g) \cdot \frac{\partial K_g}{\partial x} + (y^j \rho_g V_g) \cdot \frac{\partial K_g}{\partial y} = \frac{\partial}{\partial y} \left[\frac{y^j \rho_g \nu_{tK}}{\sigma_k} \cdot \frac{\partial \left(K_g + \frac{\chi_0}{1 + \chi_0} K_p \right)}{\partial y} \right] + (4)$$

$$+ y^j \rho_g \nu_{tK} \cdot \left[\frac{\partial U_g}{\partial y} \right]^2 - y^j \rho_g (\varepsilon + \varepsilon_p)$$

The equation applies to stationary axis jet stream at $j=1$ and flat jet stream at $j=0$.

In tree parametric models of turbulence for non-isothermal turbulent jets it is also include the equation for the velocity of dissipation of turbulent energy types.

*Corresponding author: **Hameed Khalaf Saleh**

$$(y^j \rho_g U_g) \frac{\partial \varepsilon}{\partial x} + (y^j \rho_g V_g) \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y} \left[\frac{y^j \rho_g \nu_{rg}}{\sigma_\varepsilon} \frac{\partial \left(\varepsilon + \frac{\chi_o}{1 + \chi_o} K_p K_g \right)}{\partial y} \right] - \quad (5)$$

$$- y^j \rho_g \Phi_p + C_{\varepsilon 1} y^j \rho_g \frac{\varepsilon}{K_g} \left[\nu_{rg} \left(\frac{\partial U_g}{\partial y} \right)^2 + G \right] - y^j \rho_g \frac{\varepsilon^2}{K_g} (C_{\varepsilon 2} - C_{\varepsilon 3} \chi)$$

The dissipation is determined by the relationship

$$\varepsilon = C_D \cdot K_g^{0.5} \cdot L_t \quad (6)$$

The additional dissipative term is determined by the expression

$$\varepsilon_p = \frac{1}{\rho_g} \sum_i \overline{F_i V'_{gi}} \quad (7)$$

Through this term is reported the impact that the presence of particulate matter have on the turbulent energy of the gas phase. In (7) is taking into account the pulsation of the forces F_i governing the movement of the two-phase jet. At free jet streams, which are characterized by strong velocity gradient cannot ignore the force of Saffman - f_s . At non-isothermal jet jets to additional dissipative member it is necessary to add terms, taking into account the forces f_T and f_G in which the additional dissipative member ε_p is obtained

$$\varepsilon_p = \varepsilon_{pA} + \varepsilon_{pM} + \varepsilon_{pS} + \varepsilon_{pT} + \varepsilon_{pG} \quad (8)$$

The additional dissipative member in the equation for the velocity of dissipation of turbulent energy arises from the presence of the forces of interfacial interaction. Through it is expresses the immediate impact of the second phase (particle of impurity) on the velocity of dissipation of pulsation energy. The additional dissipative member is determined by the expression

$$\Phi_p = 2 \nu \frac{1}{\rho_g} \sum_i \sum_k \left(\frac{\partial F'_i}{\partial x_i} \right) \left(\frac{\partial V'_{gi}}{\partial x_k} \right) \quad (9)$$

In the most common case Φ_p is seen as composed of individual components corresponding to the respective forces that cause them

$$\Phi_p = \Phi_{pA} + \Phi_{pM} + \Phi_{pS} + \Phi_{pT} + \Phi_{pG} \quad (10)$$

Additional dissipative members which are obtained ε_p , ε_p^* , Φ_p enable to solve the equations for turbulent (pulsed) energy of the gas phase and impurities and speed the dissipation of turbulent energy.

3. Equation Describe Two-Phase Vertical Non-Isothermal Flow in Dimensionless Type

$$\frac{\partial (\overline{y^j \cdot \rho_g \cdot U_g})}{\partial x} + \frac{\partial (\overline{y^j \cdot \rho_g \cdot V_g})}{\partial y} = 0 \quad (11)$$

$$\frac{\partial (\overline{y^j \cdot \rho_p \cdot U_p})}{\partial x} + \frac{\partial (\overline{y^j \cdot \rho_p \cdot V_p})}{\partial y} = 0 \quad (12)$$

$$\left(\overline{y^j \cdot U_p} \right) \frac{\partial \overline{\rho_p}}{\partial x} + \left(\overline{y^j \cdot V_p} \right) \frac{\partial \overline{\rho_p}}{\partial y} = \frac{\partial}{\partial y} \left(\overline{y^j \cdot \frac{\nu_{pp} \partial \overline{\rho_p}}{Sc_p \partial y}} \right) \quad (13)$$

$$\left(\overline{y^j \cdot \rho_g \cdot U_g} \right) \frac{\partial \overline{U_g}}{\partial x} + \left(\overline{y^j \cdot \rho_g \cdot V_g} \right) \frac{\partial \overline{U_g}}{\partial y} = \frac{\partial}{\partial y} \left(\overline{y^j \cdot \rho_g \cdot \nu_{rg} \frac{\partial \overline{U_g}}{\partial y}} \right) - \overline{F_x \cdot y^j} - (\overline{y_0} + \overline{y^j}) (\overline{\rho_g} - \overline{\rho_s}) \pi g \overline{y^j} \quad (14)$$

$$\left(\overline{y^j \cdot \rho_p \cdot U_p} \right) \frac{\partial \overline{U_p}}{\partial x} + \left(\overline{y^j \cdot \rho_p \cdot V_p} \right) \frac{\partial \overline{U_p}}{\partial y} = \frac{\partial}{\partial y} \left(\overline{y^j \cdot \rho_p \cdot \nu_{pp} \frac{\partial \overline{U_p}}{\partial y}} \right) + \left(\overline{y^j \cdot \frac{\nu_{pp} \partial \overline{\rho_p}}{Sc_p \partial y}} \right) \frac{\partial \overline{U_p}}{\partial y} + \overline{F_x \cdot y^j} \pm g \overline{m_p} \quad (15)$$

$$\left(\overline{y^j \cdot \rho_g \cdot U_g \cdot C_{pg}} \right) \frac{\partial \overline{T_p}}{\partial x} + \left(\overline{y^j \cdot \rho_g \cdot V_g \cdot C_{pg}} \right) \frac{\partial \overline{T_p}}{\partial y} = \overline{F_x \cdot y^j} (\overline{U_g} - \overline{U_p}) + \frac{\partial}{\partial y} \left(\overline{y^j \cdot \rho_g \cdot C_{pg} \frac{\nu_{rg}}{Pr_g} \frac{\partial \overline{T_p}}{\partial y}} \right) + \overline{F_x \cdot y^j} (\overline{V_g} - \overline{V_p}) - \overline{y^j \sum_i F_i V'_{pi}} - \overline{Q \cdot y^j} \quad (16)$$

$$\left(\overline{y^j \cdot \rho_p \cdot U_p \cdot C_{pp}} \right) \frac{\partial \overline{T_p}}{\partial x} + \left(\overline{y^j \cdot \rho_p \cdot V_p \cdot C_{pp}} \right) \frac{\partial \overline{T_p}}{\partial y} = \quad (17)$$

$$\frac{\partial}{\partial y} \left(\overline{y^j \cdot \rho_p \cdot C_{pp} \frac{\nu_{pp}}{Pr_p} \frac{\partial \overline{T_p}}{\partial y}} \right) + \left(\overline{y^j \cdot C_{pp} \frac{\nu_{pp}}{Sc_p} \frac{\partial \overline{\rho_p}}{\partial y}} \right) \frac{\partial \overline{T_p}}{\partial y} + \overline{Q \cdot y^j}$$

$$\left(\overline{y^j \cdot \rho_g \cdot U_g} \right) \frac{\partial \overline{K_g}}{\partial x} + \left(\overline{y^j \cdot \rho_g \cdot V_g} \right) \frac{\partial \overline{K_g}}{\partial y} = \quad (18)$$

$$= \frac{\partial}{\partial y} \left(\overline{y^j \cdot \rho_g \frac{\nu_{rg} \partial \left(\overline{K_g} + \frac{\chi_o}{1 + \chi_o} \overline{K_p} \right)}{\sigma_k \partial y}} \right) + y^j \cdot \rho_g \cdot \nu_{rg} \left(\frac{\partial \overline{U_g}}{\partial y} \right)^2 - y^j \cdot \rho_g \cdot (\overline{\varepsilon_g} + \overline{\varepsilon_p})$$

$$\left(\overline{y^j \cdot \rho_p \cdot U_p} \right) \frac{\partial \overline{K_p}}{\partial x} + \left(\overline{y^j \cdot \rho_p \cdot V_p} \right) \frac{\partial \overline{K_p}}{\partial y} = \frac{\partial}{\partial y} \left(\overline{y^j \cdot \rho_p \frac{\nu_{pp} \partial \left(\overline{K_p} + \frac{1}{1 + \chi_o} \overline{K_g} \right)}{\sigma_k \partial y}} \right) + y^j \cdot \rho_p \cdot \nu_{pp} \left(\frac{\partial \overline{U_p}}{\partial y} \right)^2 - y^j \cdot \rho_p \cdot \overline{\varepsilon_p} \quad (19)$$

$$\left(\overline{y^j \cdot \rho_g \cdot U_g} \right) \frac{\partial \overline{\varepsilon_g}}{\partial x} + \left(\overline{y^j \cdot \rho_g \cdot V_g} \right) \frac{\partial \overline{\varepsilon_g}}{\partial y} = \frac{\partial}{\partial y} \left(\overline{y^j \cdot \rho_g \frac{\nu_{rg} \partial \left(\overline{\varepsilon_g} + \frac{\chi_o}{1 + \chi_o} \overline{K_p} \overline{K_g} \right)}{\sigma_\varepsilon \partial y}} \right) - y^j \cdot \rho_g \cdot \overline{\Phi_p} + \quad (20)$$

$$+ C_{\varepsilon 1} \overline{y^j \cdot \rho_g \frac{\varepsilon_g}{K_g}} \left[\nu_{rg} \left(\frac{\partial \overline{U_g}}{\partial y} \right)^2 - C_{\varepsilon 1} \frac{\varepsilon_g}{K_p} \nu_{pp} \left(\frac{\partial \overline{U_p}}{\partial y} \right)^2 + G \right] - y^j \cdot \rho_g \frac{\varepsilon_g^2}{K_g} (C_{\varepsilon 2} - \chi C_{\varepsilon 3})$$

$$\overline{P} = \overline{\rho_g \cdot T_g} \quad (21)$$

To closed the system (11) ÷ (21) is necessary to add dependencies for determining the coefficients of turbulent viscosity ν_{rg} and ν_{pp}

$$\overline{\nu_{rg}} = C_{\mu} \frac{\overline{K_g^2}}{\varepsilon}; \overline{\nu_{pp}} = C_{\mu} \frac{\overline{K_p^2}}{\varepsilon} \quad (22)$$

and

$$\overline{\varepsilon} = C_D \frac{\sqrt{\overline{K_g^3}}}{L_t} \quad (23)$$

The dimensionless is made with initial values of the relevant parameters

$$\overline{x} = \frac{x}{y_0}; \overline{y} = \frac{y}{y_0}; \overline{U_g} = \frac{U_g}{U_{g0}}; \overline{U_p} = \frac{U_p}{U_{p0}}; \overline{V_g} = \frac{V_g}{U_{g0}}; \overline{V_p} = \frac{V_p}{U_{p0}}; \overline{\rho_g} = \frac{\rho_g}{\rho_{g0}}; \overline{\rho_p} = \frac{\rho_p}{\rho_{p0}}; \quad (24)$$

$$\overline{T_g} = \frac{T_g R}{U_{g0}^2}; \overline{T_p} = \frac{T_p R}{U_{p0}^2}; \overline{K_g} = \frac{K_g}{U_{g0}^2}; \overline{K_p} = \frac{K_p}{U_{p0}^2}; \overline{\varepsilon} = \frac{\varepsilon y_0}{U_{g0}^3}; \overline{C_{pg}} = \frac{C_{pg}}{R}; \overline{C_{pp}} = \frac{C_{pp}}{R}; \overline{\nu_{rg}} = \frac{\nu_{rg}}{y_0 U_{g0}};$$

$$\overline{\nu_{pp}} = \frac{\nu_{pp}}{y_0 U_{p0}}; \overline{F} = \frac{F y_0}{\rho_{g0} U_{g0}^2}; \overline{Q} = \frac{Q y_0}{\rho_{g0} U_{g0}^3}; \overline{\Phi_p} = \frac{\Phi_p y_0^2}{U_{g0}^4}; \overline{g} = \frac{U_{g0}}{l}; \overline{\rho_s} = \frac{\rho_s}{\rho_{g0}}$$

4. Discretization of Numerical Model

The solution of axis non-isothermal turbulent flow with different density is carried out based on a system of (11) ÷ (21) at j=1 (for axis jet).

Equation in Finite differences

The system of differential equation for two-phase vertical non-isothermal axis flow can be introduce with characteristic equation of the type

$$A \cdot \frac{\partial \bar{Z}}{\partial x} + B \cdot \frac{\partial \bar{Z}}{\partial y} = C \cdot \frac{\partial^2 \bar{Z}}{\partial y^2} + D \tag{25}$$

Where Z, A, B, C, D are the variables are adjusted and the dimensions, whose meanings are given in Table 1.

The parameters of the flow, shown in Table. 1 ($Z \equiv \rho_p \div \varepsilon$) is resolved by the same scheme in finite differences. This allows the differential operators in the characteristic equation be replaced by the same difference schemes as follows

$$\frac{\partial \bar{Z}}{\partial x} = \frac{\bar{Z}_{i,j} - \bar{Z}_{i-2,j}}{2K}$$

$$\frac{\partial \bar{Z}}{\partial y} = \frac{\bar{Z}_{i-1,j+1} - \bar{Z}_{i-1,j-1}}{2H} \tag{26}$$

$$\frac{\partial^2 \bar{Z}}{\partial y^2} = \frac{1}{H^2} \left[\bar{Z}_{i-1,j+1} - \frac{1}{2}(\bar{Z}_{i,j} + \bar{Z}_{i-2,j}) - \frac{1}{2}(\bar{Z}_{i,j} + \bar{Z}_{i-2,j} - \bar{Z}_{i-1,j-1}) \right]$$

This equation can be solved or each Z.

Table 1 Meaning of Z, A, B, C, D

Z	A	B	C	D
$\bar{\chi}$	$\bar{y} \cdot \bar{U}_p$	$\bar{y} \cdot \bar{V}_p - \frac{\partial}{\partial y} \left(\bar{y} \cdot \frac{\bar{v}_{tp}}{Sc_t} \right)$	$\bar{y} \cdot \frac{\bar{v}_{tp}}{Sc_t}$	$-\bar{\chi} \cdot \frac{\partial}{\partial y} (\bar{y} \cdot \bar{V}_p) - \bar{y} \cdot \bar{\rho}_p \cdot \frac{\partial \bar{U}_p}{\partial x}$
\bar{U}_g	$\bar{y} \cdot \bar{\rho}_g \cdot \bar{U}_g$	$\bar{y} \cdot \bar{\rho}_g \cdot \bar{V}_g - \frac{\partial}{\partial y} (\bar{y} \cdot \bar{\rho}_g \cdot \bar{u}_{ig})$	$\bar{y} \cdot \bar{\rho}_g \cdot \bar{u}_{ig}$	$-\bar{y} \cdot \bar{F}_x - (\bar{y}_0 + \bar{y}^j) (\bar{\rho}_2 - \bar{\rho}_g) \pi g \bar{y}^j$
\bar{U}_p	$\bar{y} \cdot \bar{\rho}_p \cdot \bar{U}_p$	$\bar{y} \cdot \bar{\rho}_p \cdot \bar{V}_p - \frac{\partial}{\partial y} (\bar{y} \cdot \bar{\rho}_p \cdot \bar{u}_{tp}) -$ $-\bar{y} \cdot \frac{\bar{v}_{tp}}{Sc_t} \cdot \frac{\partial \bar{\rho}_p}{\partial y}$	$\bar{y} \cdot \bar{\rho}_p \cdot \bar{u}_{tp}$	$\bar{y} \cdot \bar{F}_x$
\bar{T}_g	$\frac{\bar{y} \cdot \bar{\rho}_g \cdot \bar{U}_g \cdot \bar{C}_{pg}}{R}$	$\frac{\bar{y} \cdot \bar{\rho}_g \cdot \bar{U}_g \cdot \bar{C}_{pg}}{R} - \frac{\partial}{\partial y} \left[\frac{\bar{y} \cdot \bar{\rho}_g \cdot \bar{u}_{ig}}{R \cdot Pr_t} \right]$	$\bar{y} \cdot \bar{\rho}_g \cdot \frac{\bar{C}_{pg} \cdot \bar{u}_{ig}}{R \cdot Pr_t}$	$-\bar{y} \cdot \bar{Q} + \bar{F}_x \cdot \bar{y} \cdot (\bar{U}_g - \bar{U}_p) +$ $+ \bar{F}_y \cdot \bar{y} \cdot (\bar{V}_g - \bar{V}_p)$
\bar{T}_p	$\frac{\bar{y} \cdot \bar{\rho}_p \cdot \bar{U}_p \cdot \bar{C}_{pp}}{R}$	$\frac{\bar{y} \cdot \bar{\rho}_p \cdot \bar{U}_p \cdot \bar{C}_{pp}}{R} - \frac{\partial}{\partial y} \left[\frac{\bar{y} \cdot \bar{\rho}_p \cdot \bar{u}_{tp}}{R \cdot Pr_t} \right] -$ $-\bar{y} \cdot \frac{\bar{C}_{pp} \cdot \bar{u}_{tp}}{R \cdot Sc_t} \cdot \frac{\partial \bar{\rho}_p}{\partial y}$	$\bar{y} \cdot \bar{\rho}_p \cdot \frac{\bar{C}_{pp} \cdot \bar{u}_{tp}}{R \cdot Pr_t}$	$\bar{y} \cdot \bar{Q}$
\bar{K}_g	$\bar{y} \cdot \bar{\rho}_g \cdot \bar{U}_g$	$\bar{y} \cdot \bar{\rho}_g \cdot \bar{V}_g - \frac{\partial}{\partial y} \left(\bar{y} \cdot \bar{\rho}_g \cdot \frac{\bar{u}_{ig}}{\sigma_k} \right)$	$\frac{\bar{u}_{ig} + \frac{\chi_0}{1 + \chi_0} \cdot \bar{v}_{tp}}{\bar{y} \cdot \bar{\rho}_g \cdot \sigma_k}$	$\bar{y} \cdot \bar{\rho}_g \cdot \bar{u}_{ig} \cdot \left(\frac{\partial \bar{U}_g}{\partial y} \right)^2 - \bar{y} \cdot \bar{\rho}_g \cdot (\varepsilon + \varepsilon_p)$
\bar{K}_p	$\bar{y} \cdot \bar{\rho}_p \cdot \bar{U}_p$	$\bar{y} \cdot \bar{\rho}_p \cdot \bar{V}_p - \frac{\partial}{\partial y} \left(\bar{y} \cdot \bar{\rho}_p \cdot \frac{\bar{u}_{tp}}{\sigma_k} \right)$	$\frac{\bar{v}_{tp} + \frac{1}{1 + \chi_0} \cdot \bar{u}_{ig}}{\bar{y} \cdot \bar{\rho}_p \cdot \sigma_k}$	$\bar{y} \cdot \bar{\rho}_p \cdot \bar{u}_{tp} \cdot \left(\frac{\partial \bar{U}_p}{\partial y} \right)^2 - \bar{y} \cdot \bar{\rho}_p \cdot \varepsilon_p^*$
$\bar{\varepsilon}$	$\bar{y} \cdot \bar{\rho}_g \cdot \bar{U}_g$	$\bar{y} \cdot \bar{\rho}_g \cdot \bar{V}_g - \frac{\partial}{\partial y} \left(\bar{y} \cdot \bar{\rho}_g \cdot \frac{\bar{u}_{ig}}{\sigma_\varepsilon} \right)$	$\bar{y} \cdot \bar{\rho}_g \cdot \frac{\bar{u}_{ig}}{\sigma_\varepsilon}$	$\frac{C_{\varepsilon 1} \cdot \bar{y} \cdot \bar{\rho}_g \cdot \bar{\varepsilon}}{\bar{K}_g} \left[\bar{u}_{ig} \cdot \left(\frac{\partial \bar{U}_g}{\partial y} \right)^2 + G \right] -$ $-\bar{\rho}_g \cdot \bar{y} \cdot \bar{\Phi}_p - \frac{\bar{\rho}_g \cdot \bar{y} \cdot \bar{\varepsilon}^2 \cdot (C_{\varepsilon 2} - C_{\varepsilon 3})}{\bar{K}_g}$

Conclusion

In the present work are made

1. It is composed mathematical model of vertical non-isothermal two-phase turbulent jet
2. Developed a new solution to the two-phase non-isothermal vertical jet based on the finite difference method.

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