International Journal of Current Engineering and Technology
E-ISSN 2277 – 4106, P-ISSN 2347 – 5161

Available at http://inpressco.com/category/ijcet

Research Article

Synthesis of WATT and Stephenson Mechanisms six bar mechanism using Burmester Theory

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Accepted 03 Jan 2017, Available online 11 Jan 2017, **Vol.7, No.1 (Feb 2017)**

Abstract

To date Burmester Theory, as an analytical method, was predominantly used to design four-bar mechanisms that prescribe four precision points. This theory was later extended to design four-bar mechanisms prescribing five precision points. There have been few attempts to use the Burmester Theory in design of mechanisms with more than four bars and/or the ones prescribing more than five precision points. However, not much was done towards the design of mechanisms with six, seven, or more bars that prescribe more than four precision points. This paper describes possible ways in which the Burmester Theory can be utilized in the synthesis of six-bar planar mechanism prescribing four or five precision points.

Keywords: *Burmester Theory, four-bar, six-bar, mechanism, synthesis.*

1. Introduction

By looking at the world around, approximately every few seconds a mechanism of some kind is encountered without being aware of its presence. For any object that's needs moving from one position to another a mechanism of some sort is required. Although mechanisms can be three dimensional, most of them are limited in tw+85o dimensions.

There are various types of mechanisms that form different kind of mechanism between different bars. The most common ones are sliding and rolling types of mechanisms. With the existence of different types of mechanisms there are numerous methods with which one can synthesise a mechanism.

The objectives of this paper are to utilize the Burmester Curves in combination with Inversion Method in a design of a WATT-II type of six-bar mechanisms that will prescribe five precision points as well as keep the desired angle of the element moved by the mechanism. This combination of these two methods will be shown using an extreme case where a mechanism has to prescribe a straight line. In addition, the Burmester Theory can be combined with other analytical methods including Freudenstein method in conjunction with Gauss Method of Least Squares.

2. Burmester Theory

When a body moves through two and three prescribed positions there are three respectively two infinities of solutions in the design of the mechanism; however, when the body passes through four and five prescribed positions then there is one-infinity respectively few possible designs of a mechanisms. Burmester found the solutions to problem of four prescribed positions of the plane using complex numbers, Fig.1(Sandor & Erdman, 1984). According to the authors, one can notice that the vectors defined above form a closed loop, including the first and j-th positions:

$$We^{i\beta_j}+Ze^{i\alpha_j}-\delta_j-Z-W=0 \quad (2.1)$$

By combining these terms,

$$\mathbf{W}(e^{i\beta_j}-1)+\mathbf{Z}(e^{i\alpha_j}-1)=\boldsymbol{\delta}_j \quad (2.2)$$

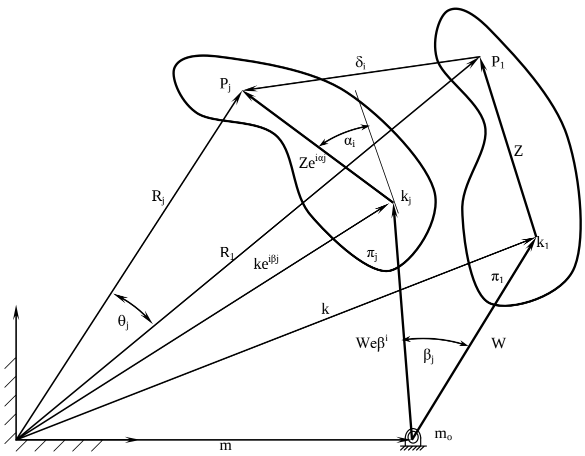

Fig.1 The unknown dyad's W,Z which can guide the moving plane π

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Note that δ_j and α_j are known from the prescribed positions of π. α_j is the angle that the coupler (plane π) turns through.

2.1 Four precision points

For four positions, the following equations for a dyad are obtained.

$\mathbf{W}(e^{i\beta_2}-1)+\mathbf{Z}(e^{i\alpha_2}-1)=\boldsymbol{\delta}_2$

$\mathbf{W}(e^{i\beta_3}-1)+\mathbf{Z}(e^{i\alpha_3}-1)=\boldsymbol{\delta}_3$ (2.3)

$\mathbf{W}(e^{i\beta_4}-1)+\mathbf{Z}(e^{i\alpha_4}-1)=\boldsymbol{\delta}_4$

The above set of equations can be written in matrix form

$$\begin{bmatrix} e^{i\beta 2}-1 & e^{i\alpha 2}-1 \\ e^{i\beta 3}-1 & e^{i\alpha 3}-1 \\ e^{i\beta 4}-1 & e^{i\alpha 4}-1 \end{bmatrix} \begin{bmatrix} W \\ Z \end{bmatrix} = \begin{bmatrix} \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} \qquad (2.4)$$

The second column of the coefficient matrix, as well as the right hand side column, contain prescribed input data, while the first column contains unknown sets of rotations β. In order for this overdetermined system to have a solution, the rank of the "augmented matrix" of the coefficient is 2. The augmented matrix M is formed by adding the right-hand column of system (eqn.2.4) to the coefficient matrix of the left side. Hence mathematically this means that the determinant of the augmented matrix of the system must be equal to zero in order for the system to have a solution.

Hence by using this fact, the sets of β which make the three equations of the system compatible, can be found. Then for each set, there will be a different solution for the dyad **W**, **Z.**

$$Det \quad M = Det \begin{bmatrix} e^{i\beta 2}-1 & e^{i\alpha 2}-1 & \delta_2 \\ e^{i\beta 3}-1 & e^{i\alpha 3}-1 & \delta_3 \\ e^{i\beta 4}-1 & e^{i\alpha 4}-1 & \delta_4 \end{bmatrix} = 0 \qquad (2.5)$$

If the determinant is expanded about the first column then the following eqn is obtained.

$$\Delta_2 e^{i\beta_2} + \Delta_3 e^{i\beta_3} + \Delta_4 e^{i\beta_4} + \Delta_1 = 0 \qquad (2.6)$$

where $\Delta_1 = -\Delta_2 - \Delta_3 - \Delta_4$ (2.7)

and the Δj (j=2,3,4) are the cofactors of the elements in the first column.

There are four equations (eqn.2.3 and eqn.2.6) and five unknowns (W, Z, β_2, β_3 and β_4).

$$\Delta_2 = \begin{vmatrix} e^{i\alpha_3}-1 & \delta_3 \\ e^{i\alpha_4}-1 & \delta_4 \end{vmatrix}$$

$$\Delta_3 = -\begin{vmatrix} e^{i\alpha_2}-1 & \delta_2 \\ e^{i\alpha_4}-1 & \delta_4 \end{vmatrix} \qquad (2.8)$$

$$\Delta_4 = \begin{vmatrix} e^{i\alpha_2}-1 & \delta_2 \\ e^{i\alpha 3}-1 & \delta_3 \end{vmatrix}$$

The Δ's (thick vectors in Fig.2) are known since they contain only known input data. Equation (2.6) is termed the compatibility equation, because sets of β_2, β_3 and β_4, which satisfy this equation, will render system (2.3) "compatible". This means that the system will yield simultaneous solutions for **W** and **Z.**

In the compatibility equation, the unknowns are located in the exponents of exponentials. This transcendental equation can be simplified through a graphical solution procedure (Fig.2).

Equation (2.6) can be further simplified as follows:

$$(-\boldsymbol{\Delta}_1 - \Delta_2 e^{i\beta_2}) = (\Delta_3 e^{i\beta_3} + \Delta_4 e^{i\beta_4}) = -\Delta \qquad (2.9)$$

Then for an arbitrary choice of β_2 , $-\boldsymbol{\Delta}$ is known and hence, since Δ_3 and Δ_4 are already known, the rotation operators $e^{i\beta_3}$, $e^{i\beta_4}$ can be found, such that these two vectors form a closed loop with $\boldsymbol{\Delta}$.

Equation (2.6) may be regarded as the "equation of closure" of a four-bar linkage, the so-called "compatibility linkage", with "fixed link" Δ_1 , "movable links" Δ_j (j=2,3,4) and "link rotations" β_j , measured from the "starting position" of the linkage which is defined by eqn. (2.7). This concept is illustrated in the following figure.

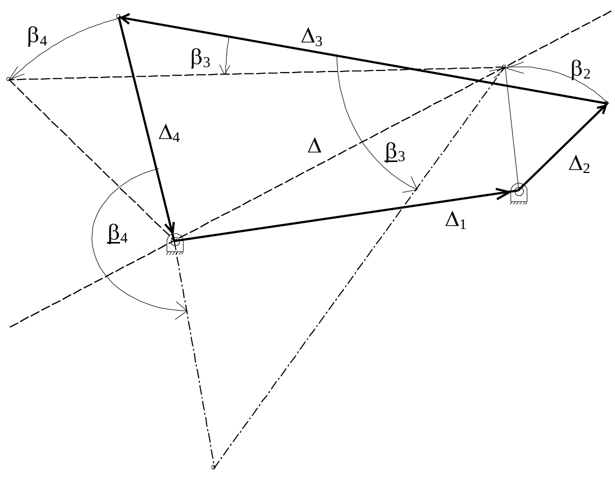

Fig.2 Geometric solution of the compatibility equation

It can be seen that for a given value of β_2 , there will be two sets of values for β_j (β_j and β_j , where j=3,4). The range, within which β_2 may be assumed, is determined by the limits of mobility of the compatibility linkage. The solution of the compatibility linkage problem, (to find compatible sets of β_j) can be obtained analytically. After the compatible sets of β_2 are obtained, then any method for solving two simultaneous complex equations, can be used to find **Z** and **W**. Then the circle point **k** and the center point **m** are given by the following expressions (with reference to Fig.2).

$$\mathbf{k} = \mathbf{R}_1 - \mathbf{Z}$$

$$\mathbf{m} = \mathbf{k} - \mathbf{W}$$

The above pair of these two points, is termed **Burmester Point Pair** (B.P.P.), and the locus of these pairs for different values of β_2 are forming the **Burmester Curves** (Sandor & Erdman, 1984).

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3. Stephenson and Watt six-bar mechanisms

In theory, direct application of Burmester Theory is in the synthesis of four-bar mechanisms prescribing four precision points. However, there are more types of mechanisms (more than four-bar ones) that prescribe more than four precision points. The way that the Burmester Curves can be used in the design of four-bar mechanisms prescribing five positions is there will be two sets of dyads of the four-bar mechanisms created. One set prescribing first four precision points (1-2-3-4) and than the second set of dyads prescribing the first set off three points and the fifth precision point (1-2-3-5) (Sandor & Erdman, 1984) (Chan. 1994).

That in mind, one can look beyond four-bar mechanisms and analyse options of using the Burmester Curves in synthesis of mechanisms with more than four bars. There are several types of six-bar mechanisms, however STEPHENSON (Fig.3) and WATT (Fig.4) six-bar mechanisms are the only one where all the joint of the bars (kinematic pairs) are rolling ones (no sliding kinematic pairs).

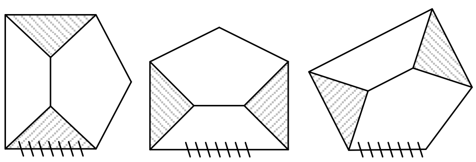

Fig.3 Three types of Stephenson six-bar mechanisms

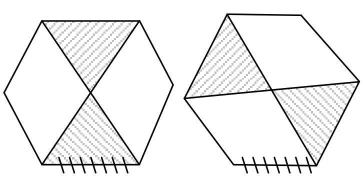

Fig.4 Three types of WATT six-bar mechanisms

3.1 Stephenson type of six-bar mechanisms

There are three types of six-bar mechanisms: Stephenson I, II and III (Fig.3).

3.1.1 Stephenson III type of six-bar mechanisms

Stephenson I type of the mechanisms is a six-bar mechanis where the Burmester Theory can be easily applied. This mechanism can be split in to one four-bar mechanisms and one dyad (Fig.5).

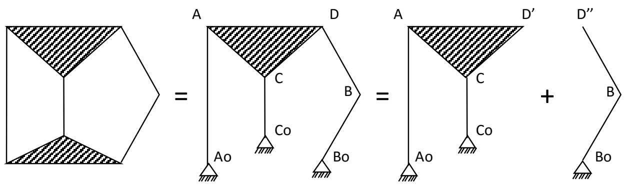

Fig. 5 Stephenson III type of six-bar mechanism Split into a four-bar and a dyad

The position C' of the main four-bar is the precision point that the mechanisms has to prescribe. The C'' in the remaining dyad is the corresponding point C whose coordinate is the same as point C' of the four-bar mechanism. Now, the Burmester Theory can be applied and three sent of Burmester Curves generated.

3.1.2 Stephenson I type of six-bar mechanisms

In contrast to the Stephenson I type of the mechanisms, in the Stephenson II type of a six-bar mechanism the Burmester Theory cannot be easily applied, not in a straight forward way (Fig.6). One way of solving this problem is to split the mechanisms in two sets of four bar mechanisms, where each mechanism will have a **dummy** bar in addition to their main dyads. The precision point C and its coordinates are the same as C' and C''.

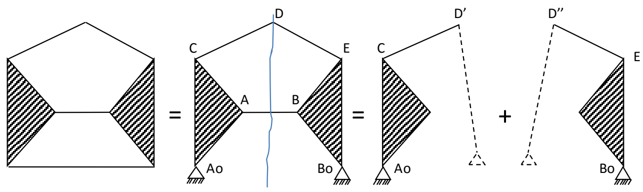

Fig.6 Stephenson I type of six-bar mechanisms Split in two sets of four-bars with a dummy bar

Now, Burmester Theory can be used and four sets of Burmester Curves generates. From these curves, two set of dads can be synthesised, and that dyads Ao-C-E and Bo-D-E that form the five-bar mechanism Ao-C-E-D-Bo. If we move the point E along its five precision points, there will be a five positions for the five-bar (Fig.7).s

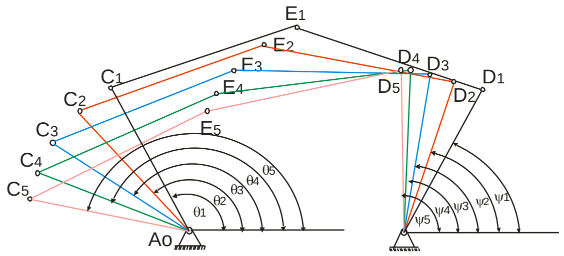

Fig.7 Five positions of the five-bar mechanism

In addition to this split of the mechanisms, another linkage to synthesise is the central four-bar A0-A-B-Bo. This four-bar can be found by using Freudenstein method in conjunction with Gauss Method of Least Squares (since Freudenstein method can be applied only for up to three precision points) (Pira, 1999).

For Freudenstein method and Gauss Method of Least Squares the following elements are known:

- The points Ao and Bo (distance Ao-Bo) are found using Burmester Theory from earlier,
- The angles that the crank and the follower are known (Fig.8).
-

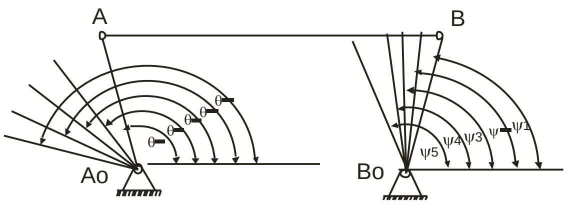

Fig.8 Five positions of the five-bar mechanism

By using the Freudenstein method and Gauss Method of Least Squares. The lengths of bars Ao-A, A-B and Bo-B can be determined along with the starting coupler and follower angles. Thus, the STEHPENCON I type of the mechanisms can be synthesised using Burmester Theory in conjunction with the Freudenstein method and Gauss Method of Least Squares.

3.1.3 Stephenson II type of six-bar mechanisms

The Stephenson II type of the mechanisms is unique. What makes it unique is the fact that it has no four-bar construction in its shape (Fig.9). There have been several attempts to synthesise a mechanism STEPHENSON II type using Burmester Theory and all so far have produced no results. Further research is recommended to be undertaken to address this issue.

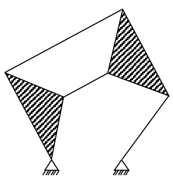

Fig.9 Stephenson II type of six-bar mechanism

3.2 Watt type of six-bar mechanisms

There are two types of six-bar mechanisms: WATT I and II (Fig.4).

3.2.1 WATT I type of six-bar mechanisms

The WATT I type of the mechanisms is a six-bar mechanisms consisted of two four-bar mechanisms linked together with an intermediately bar (triangle shaped) (Fig.10).

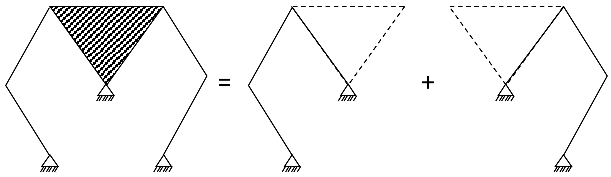

Fig.10 WATT I type of six-bar mechanism

This type of mechanisms is used to transmit the motion one four bar to the other using Function Generation methods. That in mind, there was no use of Burmester Theory to be used in the synthesis of the WATT I type of the six-bar mechanisms.

3.2.1 WATT II type of six-bar mechanisms

The WATT II type of the mechanisms is a six-bar mechanisms consisted of one four-bar mechanisms Ao-A-B-Bo and a dyad C-D-E. WATT-II has got only two ground points (binary link as base) (Fig. 11). This means that as it is, there is no solution to the existing problem of the cutting mechanism using Burmester Theory. In fact, the Burmester theory can be applied only for the main four bar and coupler point C, while the remaining dyad C-D-E, the Burmester Theory is difficult to be applied straight on.

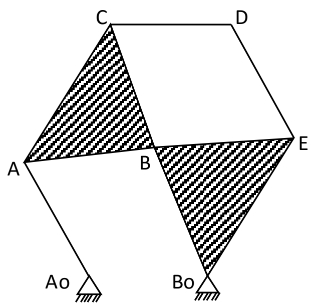

Fig.11 Watt II type of six-bar mechanism

One way of synthesising the Burmester Theory in the case of WATT-II type of six-bar mechanism is inverting the mechanism about point Bo (respectively about coupler Bo-B-E) only after the first main four bar has been synthesised using Burmester Theory.

Therefore, the following steps should be taken in order to synthesise the WTT II mechanism yusing Burmester Theory:

- First – synthesise the main four bar (A0-A-B-Bo) using Burmester Theory,
- Second – invert the WATT II mechanism about point Bo to a new mechanism type WATT I. With the inversion of the mechanisms, the precision points also need to be inverted (Fig.12).
- Third – even though the inverted mechanism turns into a WATT I type of a mechanism, where the Burmester Theory cannot be applied, we are interested only to synthesize the dyad E-D-C. Using Burmester Theory this can be done.
- Finally – the WATT II type of mechanism is synthesised.

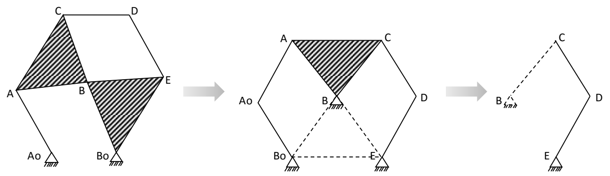

Fig.11 Inverted Watt II type of six-bar mechanism

Conclusions

There have been numerous attempts to use Burmester Theory (or rather Burmester curves) in the synthesis of four-bar mechanisms, however only few have attempted to use it in the synthesis of the six-bar mechanism. Apart from the Stephenson III type of the six-bar mechanisms, where Burmester Theory could be applied directly, in two other cases the Burmester Theory had to be used in conjunction with other methods and that:

- Inversion method, and
- Freudenstein method in conjunction with Gauss Method of Least Squares.

On the other hand, there were two instances (Stephenson II and WATT I) where the Burmester Theory could not be applied despite several attempts. None of the above mentioned methods produced any results in this matter. The theoretical application of the Burmester Theory is shown to be successful, however, more case studies and practical applications have to be undertaken until one can say that the combination of these methods is applicable through ought.

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