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Research Article

An overview of Buckling Analysis of Cylinder Subjected to Axially Compressive Load

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Abstract

Buckling is a sudden failure of a structural member subjected to high compressive stress and it is a structural instability leading to a failure mode. Buckling strength of structures depends on many parameters like supports, linear material, composite or nonlinear material etc. This paper intends to study buckling behavior of cylindrical shell which is influenced by thermal loads and imperfections. ANSYS FE non-linear buckling analysis including both material and geometrical non-linearity is used to determine the critical buckling pressure.

Keywords: *Buckling analysis, Boundary conditions, Cylinder shell, Critical buckling pressure, FEA modeling.*

1. Introduction

Cylindrical shells are used in many types of structures which are subjected to various combinations of loading. One of the most critical loads which cause the instability of thin shells is axial compression. The usual failure mode associated with thin shell structures is buckling (Himayat Ullah, *et al*, 2007).

When a structure (which is usually subjected to compression) undergoes visibly large displacements transverse to the load then the structure is said to be buckled; for example by pressing the opposite edges of a flat sheet of cardboard towards one another. If a component or part therefore is prone to buckling then its design must satisfy both strength and buckling safety constraints (K. N. Kadam, *et al*, 2013).

Buckling occurs when most of the strain energy which is been stored as membrane energy gets converted into bending energy requiring large deformation resulting in the catastrophic failure (B. Prabu, *et al*, 2009).

2. Mathematical Modeling Approach

Chetan E. Kolambe, *et al*, (2016) gives one of the methods of performing theoretical analysis of thin shell cylinders is given below.

Assumptions are made to perform the calculations:

i. For linear buckling analysis theoretical analysis has been done, internal pressure on the walls is been neglected.
ii. Value of factor k=2 for one end fix and other end free condition.
iii. Homogeneous material is used throughout shell element.

2.1. Steps for performing the calculations:

a. Calculating critical buckling load

$$\sigma_{cr} = \frac{\pi^2 * E * t}{k * L^2} \tag{1}$$

Where; E= Modulus of elasticity of Aluminum
I= Moment of Inertia of thin shell = $\frac{\pi}{64} * (D^4 - d^4)$
L= Length of Shell Element

b. Calculation of Equivalent Stress

$$\sigma = \frac{2 * E * t}{D\sqrt{3(1-\upsilon^2)}} \tag{2}$$

Where, $\sigma_=$ Equivalent stress in GPa
υ = Poison's Ratio = 0.3
D = Outer Diameter of thin shell
E = Modulus of Elasticity
t = Thickness of thin shell in mm

c. Calculation of total deformation

$$\varepsilon = \frac{\delta l}{L} \tag{3}$$

Where; ε= Elastic Strain

δ= Change in Length or Deformation
L = Original length of thin shell

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3. Analytical Modeling

Himayat Ullah, *et al*, (2007) predicts that the analytical model is based on the Kirchhoff–Love hypothesis. The problem is analyzed by the Method of Equilibrium. The three sets of field equations: Equilibrium, Kinematic and Constitutive relations.

3.1 Differential Equations of Equilibrium

Pressure Vessels exemplify the axisymmetrically loaded cylindrical shell. Owing to symmetry an element cut from a cylinder of radius 'a' will act on it the internal pressures P_x, P_y, P_z, surface force resultants N_x, N_θ , $N_{\theta x}$, Q_x , Q_θ moment resultants M_x , M_θ , $M_{x\theta}$.

Eliminating the shear forces Q_x and Q_θ and assuming $P_x = P_y = P_z$ $(P_r) = 0$, then the final equilibrium equations are;

$$a\,\partial N_x/\partial x + \partial N_{x\theta}/\partial\theta = 0 \quad (4)$$

$$\partial N_\theta/d\theta + a\,\partial N_{x\theta}/\partial x + a\; N_x \partial^2 v/\partial x^2 - \partial M_{x\theta}/\partial\theta - \frac{1}{a}\,\partial M_\theta/\partial\theta = 0 \quad (5)$$

$$a\; N_x \partial^2 w/\partial x^2 + N_\theta + a\;\partial^2 M_x/\partial x^2 + 2\,\partial^2 M_{x\theta}/\partial x\partial\theta + \frac{1}{a}\,\partial^2 M_\theta/\partial\theta^2 = 0 \quad (6)$$

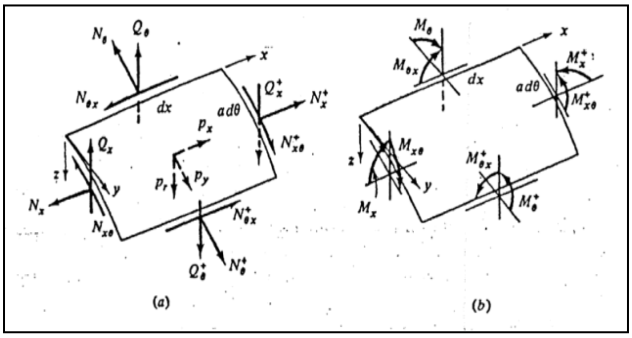

Fig.1 Cylindrical shell element (a) with internal force resultants and surface loads (b) With internal moment resultants

3.2 Kinematic Relationships

The strain components at any point through the thickness of the shell are written as;

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xo} \\ \varepsilon_\theta \\ \gamma_{xyo} \end{bmatrix} - Z \begin{bmatrix} \chi_x \\ \chi_\theta \\ \chi_{x\theta} \end{bmatrix} \quad (7)$$

Kinematic expressions relating the mid surface strains to the displacement are;

$$\varepsilon_{xo} = \partial u/\partial x \quad (8)$$
$$\varepsilon_{\theta o} = 1/a\;(\partial v/\partial\theta) - w/a \quad (9)$$
$$\gamma_{x\theta o} = 1/a\,(\partial u/\partial\theta) + \partial v/\partial x \quad (10)$$

3.3 Constitutive relations

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [0] \\ [0] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon\} \\ \{\chi\} \end{Bmatrix} \quad (11)$$

Where;
$\{N\} = \{Nx, N\theta, Nx\theta\}^T$ & $\{M\} = \{Mx, M\theta, Mx\theta\}^T$ being the resultant membrane forces and bending moments.

The elastic matrices are given by;

$$[A] = Et/1 - v^2 \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 1 - v/2 \end{bmatrix} \quad (12)$$

$$[D] = Et^3/12(1 - v^{2)} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 1 - v \end{bmatrix} \quad (13)$$

4. Buckling Analysis

The buckling analysis is defined as the analysis type and the analysis option in which the solution methods is chosen either subspace iteration method which is generally recommended for eigenvalue buckling because it uses the full system matrices, and the other method is the Householder method.

Two techniques are available in the ANSYS/Mechanical, programs for predicting the buckling load of a structure: nonlinear buckling analysis and eigenvalue (or linear) buckling analysis. Since these two methods frequently yield quite different results.

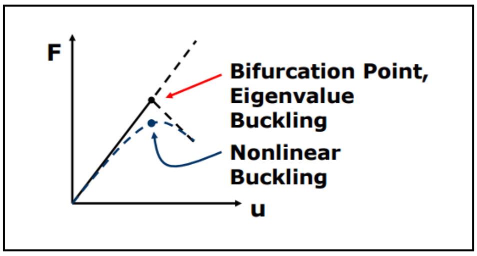

Fig. 2 Load vs. Deflection

Eigenvalue buckling analysis predicts the theoretical buckling strength (the bifurcation point) of an ideal linear elastic structure. This method corresponds to the approach of elastic buckling analysis for instance, an eigenvalue buckling analysis of a column will match the classical Euler solution. However, imperfections and nonlinearities prevent most real-world structures from achieving their theoretical elastic buckling strength (Dr. Hani Aziz Ameen, *et al*, 2010).

Nonlinearity arises when the load displacement graph is nonlinear. The cause of nonlinearity may be material or geometric. Material nonlinearity may be due to the nonlinear stress strain relation and geometric nonlinearity due to nonlinear strain displacement relation. The critical load could not be determined with sufficient accuracy if pre-buckling nonlinearity is neglected. Normally the loss of stability occurs at the limit point, rather than at the bifurcation point. In such cases the critical load must be determined through the solution of non-linear system of equations. ANSYS employs Newton Raphson approach to solve nonlinear problem (Sreelatha P.R, *et al*, 2012).

5. FEA Modeling

The structural static analysis capabilities in the ANSYS program are used to determine the displacements, stresses, strains and forces that occur in a structure or component when the load is been applied to it. Static analysis is an appropriate way to solve problems in which the time dependent effects of inertia and damping don't affect the structure's response. Nonlinearities such as plasticity, creep, large deflection, large strain and contact surfaces are also included in the ANSYS program. A nonlinear static analysis is usually performed by applying the load so as to obtain an accurate solution (K. N. Kadam, *et al*, 2013).

Finite element is an essential and powerful tool for solving structural problem. FEM can be used for a variety of linear, nonlinear and structural stability problems. Finite element package ANSYS is used for modeling and analysis of the structure. ANSYS is general purpose software used for different types of structural analysis mainly for marine structures .It provides a powerful pre and post processing tool for mesh generation from only geometry source to produce almost any element type (Sreelatha P. R, *et al*, 2012).

6. Types of Analysis

1. Eigen or linear or bifurcation buckling analysis
2. Non-linear buckling analysis

6.1. Eigen Buckling Analysis

B. Prabu, *et al*, (2009) shows Eigen buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure. This analysis is used to predict the bifurcation point using linearized model of elastic structure. The other name for this Eigen buckling analysis is "Bifurcation Analysis". The bifurcation buckling refers to unbounded growth of new deformation pattern. This analysis involves calculating the points at which the primary pressure deflection path is bifurcated by a secondary pressure deflection path.

The basic form of the Eigen buckling analysis is given by;

$$[K]\{\phi_i\} \equiv \lambda_i [S]\{\phi_i\} \qquad (14)$$

Where;

[K] = Structural stiffness matrix

$\{\phi_i\}$ = Eigen vector

λ_i = Eigen value

{S} = Stress stiffness matrix

Michael Bak, (2014) shows that Eigenvalue buckling generally yields non-conservative results and should be used with caution. There are two advantages to performing an eigenvalue buckling analysis:

1. Relatively inexpensive (fast) analysis.
2. The buckled mode shapes can be used as an initial geometric imperfection for a nonlinear buckling analysis in order to provide more realistic results.

In Fig.3 K. N. Kadam, *et al*, (2013) shows the deformation plot of linear elastic Buckling analysis. The maximum deformation for the linear elastic buckling was shown in Fig. 3. Maximum deformation can be observed at the free end and minimum displacement can be observed at the constraint end.

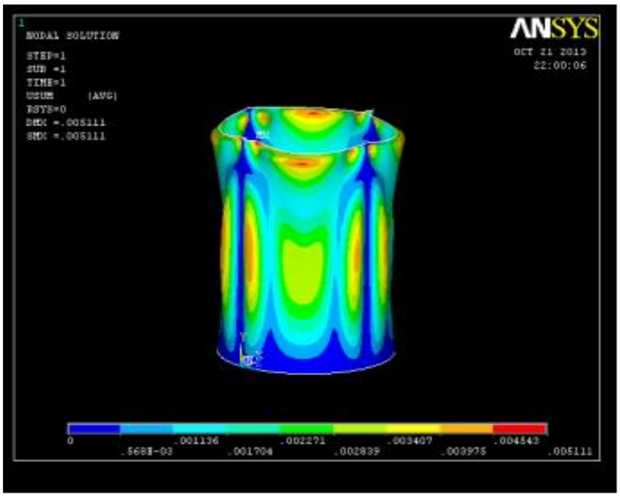

Fig.3 Deformation plot

6.2. Non-linear buckling analysis

Michel Bak, (2014), predicted Nonlinear buckling analysis is usually the more accurate approach and is therefore recommended for design or evaluation of actual structures. This technique employs a nonlinear static analysis with gradually increasing loads to seek the load level at which a structure becomes unstable. There are three different types of non-linearity such as;

i. Material Non-linearity
ii. Large deflection
iii. Contact non linearity

In a nonlinear buckling analysis, the goal is to find the first limit point (the maximum load before the solution becomes unstable).

C. Jiang, *et al*, (1996), shows that in finite element analysis, the second-order behavior of a structure can be expressed by the Following equation

$$(K + K_g)a = F \qquad (15)$$

The geometric stiffness matrix Kg depends on the stresses u caused by the external forces F. In both linear and non-linear stability analyses, the applied load on the structure is regarded as a fixed loading pattern multiplied by some factor A. The condition can be written mathematically as;

$$(K + \lambda K_G)a = 0 \qquad (16)$$

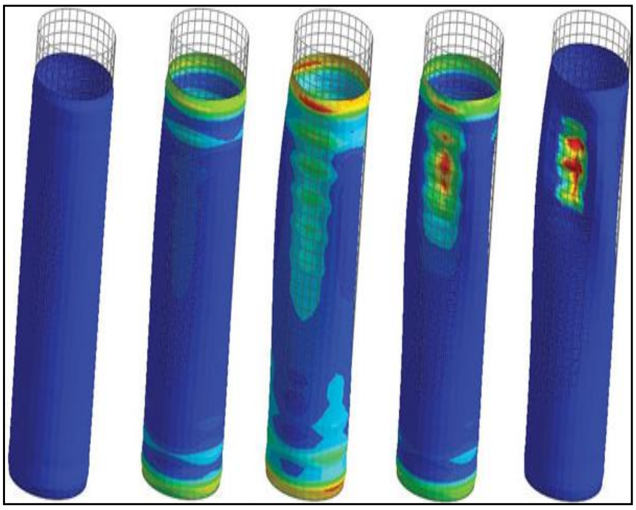

Fig. 4 Progressive stages in nonlinear cylinder buckling

Tony Abbey, (2015) shows the nonlinear buckling analysis in Fig. 4. The cylinder configuration and the level of eccentricity assumed result in a very stable structure that resists buckling until a mode occurs, similar in nature to the linear mode. There is then a transition to a highly localized mode.

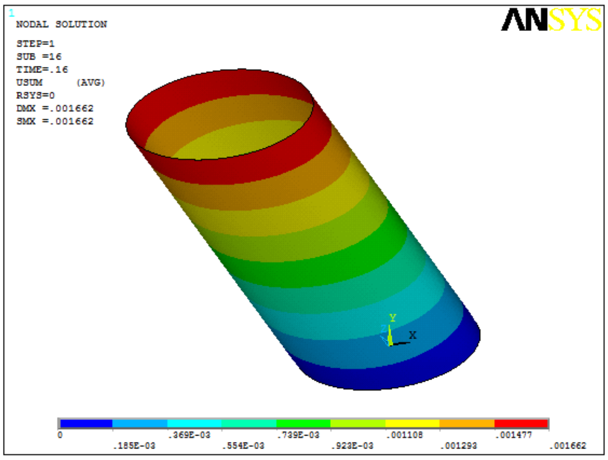

Fig. 5 Non-linear Deformation plot

Conclusions

1. It is observed that, as we go on increasing the number of nodes the total deformation goes on increasing drastically.

2. From fig. 3 we can say that maximum Deformation is obtained in linear elastic buckling.

3. Deformation, displacement and stress are observed to be maximum at free end rather than constrained end.

4. Buckling mode shape is the same for linear, perfect and imperfect nonlinear models. Linear buckling theory can only be used for determination of mode shapes.

5. Non-linear buckling analysis with geometric imperfections is usually more accurate approach and therefore, it is recommended for design or evaluation of actual cylindrical structures.

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