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Research Article

Comparative Seismic Analysis of EL Centro and Japan Earthquakes using Response Spectra Method

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Abstract

This paper presents the application of Response Spectrum Method in the seismic analysis of structures employing time history record. In-built mathematical capabilities of MATLAB are utilized to solve the ordinary differential equation (ODE45) at 2 % damping ratio. Data regarding the time histories of EL Centro and Japan earth quakes is obtained from the websites of various observatories. A MATLAB code is developed to read the time versus acceleration data and solve the ODE45 and give various results like accelogram, displacement (SDOF), pseudo velocity, pseudo acceleration.

Keywords: *Seismic analysis, time history, acceleration, ordinary differential equation*

1. Introduction

In order to perform the seismic analysis and design of a structure to be built at a particular location, the actual time history record is required. However, it is not possible to have such records at each and every location. Further, the seismic analysis of structures cannot be carried out simply based on the peak value of the ground acceleration as the response of the structure depend upon the frequency content of ground motion and its own dynamic properties. To overcome the above difficulties, earthquake response spectrum is the most popular tool in the seismic analysis of structures. There are computational advantages in using the response spectrum method of seismic analysis for prediction of displacements and member forces in structuralsystems.

The method involves the calculation of only the maximum values of the displacements and member forces in each mode of vibration using smooth design spectra that are the average of several earthquake motions.

The 1940 El Centro earthquake (or 1940 Imperial Valley earthquake) occurred at 21:35 Pacific Standard Time on May 18 (05:35 UTC on May 19) in the Imperial Valley in southeastern Southern California near the international border of the United States and Mexico. It had a moment magnitude of 6.9 and a maximum perceived intensity of X (Extreme) on the Mercalli intensity scale. It was the first major earthquake to be recorded by a strong-motion seismograph located next to a fault rupture(Hough, S.E., 2004). The earthquake was characterized as a typical moderate-sized destructive event with a complex energy release signature (Trifunac & Brune 1970). It was the strongest recorded earthquake to hit the Imperial Valley, and caused widespread damage to irrigation systems and led to the deaths of nine people(Southern California Earthquake Data Center, 2016).

The Salton Trough is part of the complex plate boundary between the Pacific Plate and the North American Plate where it undergoes a transition from the continental transform of the San Andreas Fault system to the series of short spreading centers of the East Pacific Rise linked by oceanic transforms in the Gulf of California. The two main right lateral strike-slip fault strands that extend across the southern part of the trough are the Elsinore Fault Zone/Laguna Salada Fault to the western side of the trough and the Imperial Fault to the east(Mueller, 1995).

On June 14, the 2008 Iwate earthquake struck the Tōhoku region of northeastern Honshū in Japan.(Los Angeles Times, 1940) Japan Meteorological Agency (JMA) officially named this earthquake the Iwate–Miyagi Nairiku earthquake in 2008.

This earthquake occurred in the south of the inland of Iwate Prefecture at 8:43 JST on June 14 (23:43 UTC on June 13). The JMA magnitude was estimated at M▯ 7.2, and the moment magnitude by USGS was at Mw 6.9. The epicenter was located at 39°01.7′N 140°52.8′E / 39.0283°N 140.8800°E, about 85 kilometres (55 mi) north of Sendai and about 385 kilometres (240 mi) north-northeast of Tokyo (Japan Meteorological Agency, 2016).

The strongest shaking was measured in the cities of Ōshū (Iwate) and Kurihara (Miyagi), both of which

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were measured as strong 6 on the Japan Meteorological Agency seismic intensity scale, shindo. Peak ground acceleration readings were high, with a maximum vector sum (3 component) value of 4278 cm/s^2 (4.36g)(Mainichi Daily News, 2008).

This paper deals with response spectrum method and its application to various types of the structures. The codal provisions as per IS:1893 (Part 1)-2002 code for response spectrum analysis of building is also summarized.

2. Seismic Analysis Method

In order to perform the seismic analysis and design of a structure to be built at a particular location, the actual time history record is required. However, it is not possible to have such records at each and every location. Further, the seismic analysis of structures cannot be carried out simply based on the peak value of the ground acceleration as the response of the structure depend upon the frequency content of ground motion and its own dynamic properties. To overcome the above difficulties, earthquake response spectrum is the most popular tool in the seismic analysis of structures. There are computational advantages in using the response spectrum method of seismic analysis for prediction of displacements and member forces in structural systems. The method involves the calculation of only the maximum values of the displacements and member forces in each mode of vibration using smooth design spectra that are the average of several earthquake motions.

2.1 Response Spectra

Response spectra are curves plotted between maximum response of SDOF system subjected to specified earthquake ground motion and its time period (or frequency). Response spectrum can be interpreted as the locus of maximum response of a SDOF system for given damping ratio. Response spectra thus helps in obtaining the peak structural responses under linear range, which can be used for obtaining lateral forces developed in structure due to earthquake thus facilitates in earthquake-resistant design of structures.

Usually response of a SDOF system is determined by time domain or frequency domain analysis, and for a given time period of system, maximum response is picked. This process is continued for all range of possible time periods of SDOF system. Final plot with system time period on x-axis and response quantity on y-axis is the required response spectra pertaining to specified damping ratio and input ground motion. Same process is carried out with different damping ratios to obtain overall response spectra.

Consider a SDOF system subjected to earthquake acceleration,$x_{g(t)}$ the equation of motion is given by

$$mx''(t) + cx'(t) + kx(t) = -mx_g''(t) \tag{a}$$

Substitute $\omega_{o=\sqrt{k/m}\ and\ \xi=\frac{c}{2m\omega_o}\ and\ \omega_d=\omega_o\sqrt{1}-\xi^2}$

The equation (a) can be written as

$$x_t'' + 2\xi\omega_o x'(t) + \omega_o{}^2 x(t) = -x_g''(t) \tag{b}$$

Using Duhamel's integral, solution of SDOF system initially at rest is given by

$$x(t) = -\int_0^t x_g''(\tau)\frac{e^{-\xi\omega_o(t-\tau)}}{\omega_d}\ ins\omega_d\,(t-\tau)d\tau \tag{c}$$

The maximum displacement of the SDOF system having parameters of ξ and ω_o and subjected to specified earthquake motion, $x_g''(t)$ is expressed by

$$|x(t)|_{max} = \left|\int_0^t x_g''(\tau)\frac{e^{-\xi\omega_{o(t-\tau)}}}{\omega_d}\ in\omega_d(t-\tau)d\tau\right|_{max} \tag{d}$$

The relative displacement spectrum is defined as,

$$s_d(\xi,\omega_o) = |x(t)|_{max} \tag{e}$$

Where $S_d(\xi,\omega_o)$ is the relative displacement spectra of the earthquake ground motion for the parameters of ξ and ω_o.

Similarly, the relative velocity spectrum, S_v and absolute acceleration response spectrum, and absolute acceleration response spectrum, and absolute acceleration response spectrum, S_a are expressed as,

$$S_v(\xi,\omega_o) = |x'(t)|_{max} \tag{f}$$

$$S_a(\xi,\omega_o) = |x_a''(t)|_{max} = |x''(t) + x_g''(t)|_{max} \tag{g}$$

The pseudo velocity response spectrum, S_{pv} for the system be defined as

$$S_{pv}(\xi,\omega_o) = \omega_o S_d(\xi,\omega_o)\text{(h)}$$

Similarly, the pseudo acceleration response, S_{pa} is obtained by multiplying the $S_d$to ω_o^2, thus

$$S_{pa}(\xi,\omega_o) = \omega_o^2 S_d(\xi,\omega_o) \tag{i}$$

Consider a case where $\xi = 0\ i.e\ x''(t) + \omega_o^2 x(t) = -x_g''$(t)

$$\begin{aligned} S_a &= |x''(t) + x_g''(t)|_{max} \\ &= |-\omega_o^2 x(t)|_{max} \\ &= \omega_o^2 |x_{max}| \\ &= \omega_o^2 S_d \\ &= S_{pa} \end{aligned} \tag{j}$$

The above equation implies that for an un-damped system,$S_a = S_{pa}$.

The quantity S_{pv} is used to calculate the maximum strain energy stored in the structure expressed as

$$E_{max} = \frac{1}{2}kx_{max}^2 = \frac{1}{2}m\omega_o^2 S_d^2 = \frac{1}{2}mS_{pv}^2 \tag{k}$$

The quantity S_{pa} is related to the maximum value of base shear as

$$V_{max} = kx_{max} = m\omega_o^2 S_d = mS_{pa} \qquad (l)$$

The relations between different response spectrum quantities is shown in table 1.1

As limiting case consider a rigid system i.e. $\omega_o \to \infty \; or \; T_o \to 0$, the values of various response spectra are

$$\lim_{\omega_o \to \infty} S_d \; \to 0 \qquad (m)$$
$$\lim_{\omega_o \to \infty} S_v \to 0 \qquad (n)$$
$$\lim_{\omega_o \to \infty} S_a \to |x_g''(t)|_{max} \qquad (o)$$

The three spectra i.e. displacement, pseudo velocity and pseudo acceleration provide the same information on the structural response. However each one of them provides a physically meaningful quantity (refer equations (k) and (l)) and therefore, all three spectra are useful in understanding the nature of earthquake and its influence on the design. A combined plot showing all three of the spectral quantities is possible because of the relationship that exists between these three quantities. Taking the log of equations (h) and (i)

$$\log S_{pv} = \; \log S_d + \; \log \omega_o \qquad (p)$$

$$\log S_{pv} = \; \log S_{pa} - \log \omega_o \qquad (q)$$

From the Equations (p) and (q), it is clear that a plot on the logarithmic scale will $\log S_{pv}$ as ordinate and $\log \omega_o$ as abscissa, the two equations are straight lines with slopes $+45^o$ and -45^o for constant values of $\log S_d$ and $\log S_{pa}$, respectively. This implies that the combined spectra of displacement, pseudo velocity and pseudo acceleration can be plotted in a single graph.

Table 1: Response Spectrum Relationship

Relative displacement, $\lvert x(t) \rvert_{max}$	$= S_d$	$= \frac{S_v}{\omega_o}$	$\overset{*}{=} \frac{S_a}{\omega_o^2}$	$= \frac{S_{pv}}{\omega_o}$	$= \frac{S_{pa}}{\omega_o^2}$
Relative velocity, $\lvert x'(t) \rvert_{max}$	$= \omega_o S_d$	$= S_v$	$= \frac{S_a}{\omega_o}$	$= S_{pv}$	$= \frac{S_{pa}}{\omega_o}$
Absolute acceleration, $\lvert x_a''(t) \rvert_{max}$	$\overset{*}{=} \omega_o^2 S_d$	$= \omega_o S_v$	$= S_a$	$= \omega_o S_{pv}$	$\overset{*}{=} S_{pa}$

(* if ξ= 0 these relations are exact and the sign ≅ is valid up to 0 < ξ < 0.2)

3. Methodology

The response spectrum for a given ground motion component e.g, a(t) is developed using the following steps :

(1) Obtain the ground motion (a(t)) for an earthquake. Typically the acceleration values should be defined at time steps of 0.02 second, or less.

(2) Select the natural vibration period, T_n , and damping ratio,ξ ,for SDOF system. (Usually 5 percent damping is selected.)

(3) Determine the maximum displacement response for a SDOF structure with the selected percent damping for a given period or frequency of vibration.

To do this, the following differential equation must be solved

$$mM^2 \, u/dt^2 + cM \, u/dt + ku = F(t)$$

This can be solved using:

(a) Closed form solution (linear systems)

Solution is only valid for initial conditions at rest

(b) Duhamel's integral (linear systems)

Based on treating the periodic motion as a series of short impulses

(c) Domain method (linear systems)

Fourier transform
Inverse Fourier transform

(d) Numerical methods (linear and nonlinear systems)
Numerical time stepping methods

(4) Repeat step 3 and vary the fundamental period of the structure by changing the mass (m), the stiffness (k), or both. Plot the new results.

Time history record data with regard to two earthquakes under consideration is obtained from the observatories located in the two countries.

4. Results and Discussions

The results obtained from solving ODE45 are presented to arrive at a comparative analysis of two earthquakes- EL Centro 1940 and Japan 2008. A MATLAB program is developed to solve the ODE45 by utilizing the time history records of the two earthquakes.

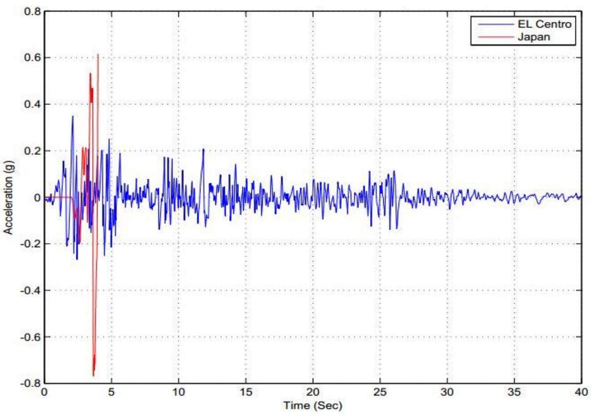

Figure 1: Accelogram

Based on the seismic inputs of EL Centro 1940 and Japan 2008 earthquakes, the acceleration response of a single degree of freedom system is obtained.

The maximum acceleration of 34.87 mm/sec^2 and a minimum displacement of $-26.82 mm/sec^2$ in respect of El Centro earthquake.

The maximum acceleration of $61.61 mm/sec^2$ and a minimum acceleration of $-76.78 mm/sec^2$ in respect of Japan earthquake. Figure 1 shows the accelogram comparison of the two earth quakes.

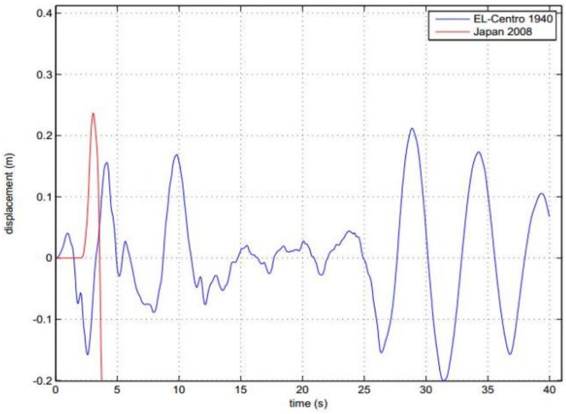

Figure 2: SDOF Response

Based on the seismic inputs of EL Centro 1940 and Japan 2008 earthquakes, the displacement response of a single degree of freedom system is obtained.

The maximum displacement of 208.7 mm and a minimum displacement of -200.8 mm in respect of El Centro earthquake.

The maximum displacement of 235.4 mm and a minimum displacement of -234 mm in respect of Japan earthquake.

An example of single degree of freedom system based on the two earthquakes is solved to compute-the maximum displacement, the maximum base shear, the maximum strain energy, considering mass, m = 2 × 10^3 kg, stiffness, k = 60 kN/m and damping, c = 0.44 kN.sec/m. Using the response spectra of El-Centro, 1940 and Japan 2008 earthquakes. Figure 2 shows the single degree of freedom response of the two earthquakes.

The natural frequency, time period and damping ratio of the SDOF system are

$$\omega_o = \sqrt{\frac{k}{m}} = \sqrt{\frac{60 \times 10^3}{2 \times 10^3}} = 5.48\, rad/sec$$

$$T_o = \frac{2\pi}{\omega_o} = 1.15 sec$$

$$\xi = \frac{c}{2m\omega_o} = \frac{0.44 \times 10^3}{2 \times 2 \times 10^3 \times 5.48} = 0.02$$

From the response spectrum curve of EL-Centro, 1940 and Japan 2008 earthquake ground motions for 3000 entries of time history records and damping ratio of 0.02

$$S_{d(EE)} = 0.2087\, m \;\; and \;\; S_{a(EE)} = 12.91 \frac{m}{sec^2}$$
$$S_{d(JE)} = 0.2354\, m \;\; and \;\; S_{a(JE)} = 23.7\, m/sec^2$$

(a) The maximum displacement

$$x_{\max(EE)} = S_{d(EE)} = 208.7\, mm$$
$$x_{\max(JE)} = S_{d(JE)} = 235.4\, mm$$

(b) The maximum base shear

$$V_{\max(EE)} = mS_{a(EE)} = 2 \times 10^3 \times 12.91 = 25.82\, kN$$
$$V_{\max(JE)} = mS_{a(JE)} = 2 \times 10^3 \times 23.7 = 47.4\, kN$$

Alternatively, $V_{\max(EE)} = kx_{\max(EE)} = 60 \times 10^3 \times 0.2087 = 12.52\, kN$

$$V_{\max(JE)} = kx_{\max(JE)} = 60 \times 10^3 \times 0.2354 = 14.12\, kN$$

(c) The maximum strain energy

$$E_{\max(EE)} = \frac{1}{2} k x^2_{\max(EE)} = \frac{1}{2} \times 60 \times 10^3 \times (0.2087)^2 = 1306.6\, N.m$$
$$E_{\max(JE)} = \frac{1}{2} k x^2_{\max(JE)} = \frac{1}{2} \times 60 \times 10^3 \times (0.2354)^2 = 1662.3\, N.m$$

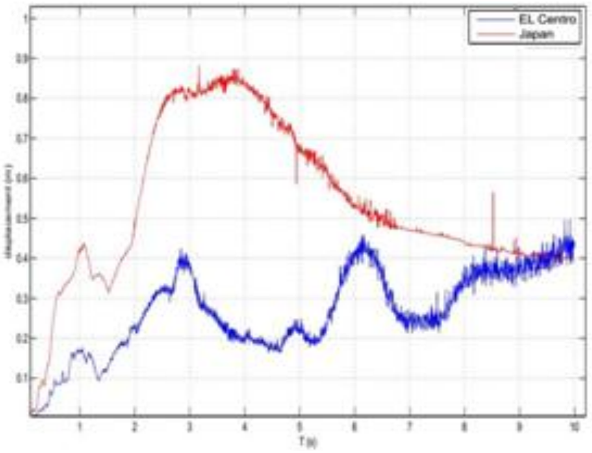

Figure 3: Displacement Response Spectra

Based on the seismic inputs of EL Centro 1940 and Japan 2008 earthquakes, the displacement response Spectrum of a single degree of freedom system is obtained.

The maximum displacement of 0.4975 m and a minimum displacement of 0.007186 m in respect of El Centro earthquake.

The maximum displacement of 0.8811 m and a minimum displacement of 0.01093 m in respect of Japan earthquake. Figure 3 shows the displacement response spectra of the two earthquakes.

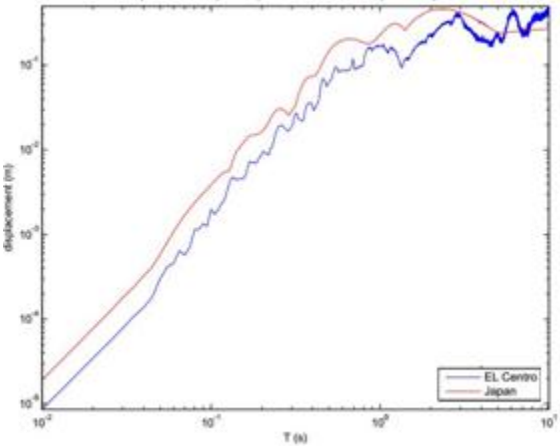

Figure 4: Log Response Spectra

Based on the seismic inputs of EL Centro 1940 and Japan 2008 earthquakes, the displacement log response spectrum of a single degree of freedom system is obtained.

The maximum displacement of 0.4639 m and a minimum displacement of 8.644e-06 m in respect of El Centro earthquake.

The maximum displacement of 0.4602 m and a minimum displacement of 1.1913e-05 m in respect of Japan earthquake. Figure 4 shows the respective log response spectra of the two earthquakes.

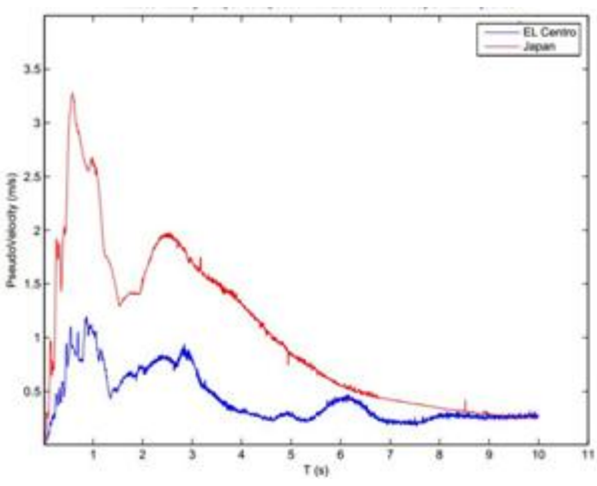

Figure 5: Pseudo Velocity Response Spectra

Based on the seismic inputs of EL Centro 1940 and Japan 2008 earthquakes, the pseudo velocity of response spectrum of a single degree of freedom system is obtained.

The maximum pseudo velocity of 1.195 m/s and a minimum pseudo velocity of 0.0181 m/s in respect of El Centro earthquake.

The maximum pseudo velocity of 3.255 m/s and a minimum pseudo velocity of 0.06349 m/s in respect of Japan earthquake. Figure 5 shows the respective pseudo velocity response spectra of the two earthquakes.

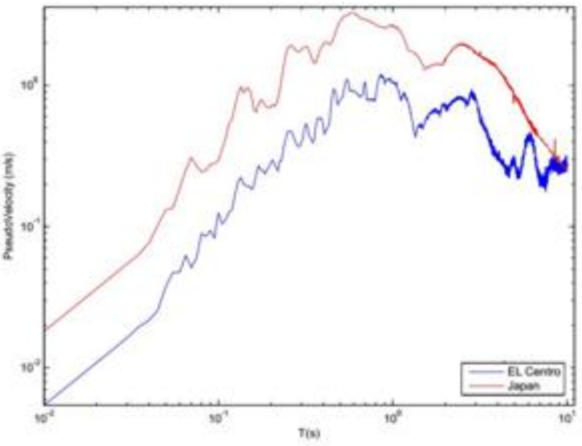

Figure 6: Log Pseudo Velocity Response Spectra

Based on the seismic inputs of EL Centro 1940 and Japan 2008 earthquakes, the log pseudo velocity of response spectrum of a single degree of freedom system is obtained.

The maximum log pseudo velocity of 1.195 m/s and a minimum log pseudo velocity of 0.005431 m/s in respect of El Centro earthquake.

The maximum log pseudo velocity of 3.256 m/s and a minimum log pseudo velocity of 0.01811 m/s in respect of Japan earthquake. Figure 6 shows the log pseudo velocity response spectra of the two earthquakes.

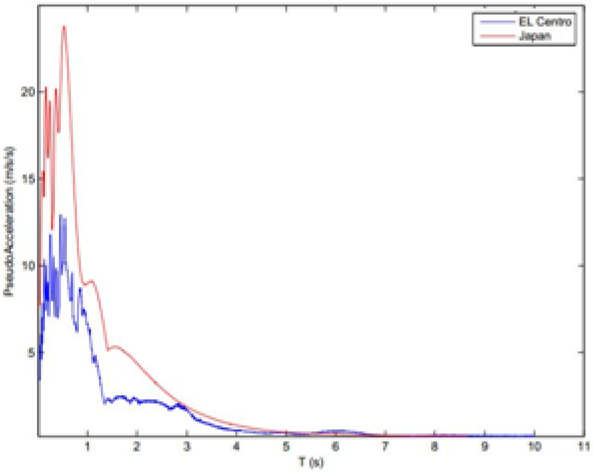

Figure 7: Pseudo Acceleration Response Spectra

Based on the seismic inputs of EL Centro 1940 and Japan 2008 earthquakes, the pseudo acceleration of response spectrum of a single degree of freedom system is obtained.

The maximum pseudo acceleration of 12.91 m/s^2 and a minimum pseudo acceleration of 0.1883 m/s^2 in respect of El Centro earthquake.

The maximum pseudo acceleration of 23.79 m/s^2 and a minimum pseudo acceleration of 0.1883 m/s^2 in respect of Japan earthquake. Figure 7 gives the respective pseudo acceleration response spectra of the two earthquakes.

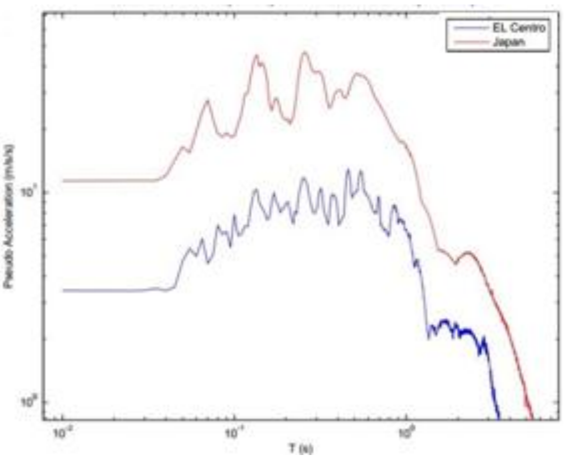

Figure 8: Log Pseudo Acceleration Response Spectra

Based on the seismic inputs of EL Centro 1940 and Japan 2008 earthquakes, the log pseudo acceleration of response spectrum of a single degree of freedom system is obtained.

The maximum log pseudo acceleration of 12.91 m/s^2 and a minimum log pseudo acceleration of 0.86 m/s^2 in respect of El Centro earthquake.

The maximum log pseudo acceleration of 46.42 m/s^2 and a minimum log pseudo acceleration of 0.85 m/s^2 in respect of Japan earthquake. Figure 8 shows the respective log pseudo acceleration response spectra of the two earthquakes

Conclusions

An introduction to response spectra has been presented, illustrating procedures that may be useful to professional engineers as an aid to design and evaluation of buildings and other structures. When earthquake ground motion data is available, the use of response spectra can be very useful in understanding how buildings perform and to identify deficiencies and damage potential. However, response spectra, as in any other technique, must be used with caution and a good understanding of the process. For single-degree-of-freedom systems responding in a linearly elastic manner, response spectra give good credible results, assuming that the data is credible. For a measured earthquake response spectrum with sharp peaks and valleys, the variations due to uncertainty in actual structural period of vibration is visually apparent. For multi-modal systems, the combination of modes is generally done by SRSS (square root of the sum of the squares) or CQC (complete quadratic combination) rule. Although these rules are based on probability approximations, the results are generally reasonable. The more technical time-history method is generally considered more exact; however, due to sensitivity to small variations in accuracy of structural periods of vibration, there are also uncertainties in this procedure. When analysis is extended into the inelastic nonlinear realm of structural response, complexities of analysis multiply. Response spectrum techniques allow engineers to visually imagine how buildings will perform during major damaging earthquakes. It is recommended that researchers and design professionals put more effort into detailed examinations of individual building response records. By deconstructing individual recorded floor motions into individual modes of vibration, there is the potential of better understanding how buildings perform during earthquake ground motions. This could lead to developing better methods of using response spectra.

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