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Research Article

Stability Analysis of Pipe Conveying Fluid Stiffened by Linear Stiffness

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Abstract

Stability analysis of pipe conveying fluid with internal flow is studied in this paper. Finite Element Analysis is used to study the dynamic analysis of pipe conveying fluid stiffened by linear spring. Several effective parameters play an important role in stabilizes the system, such as stiffness addition. The effect of stiffness addition (linear spring) and effect of spring location was studied. Also, effect of flow velocity on the dynamic stability of the system was taken into the consideration. There is a spring constant at which the dynamic behavior becomes more sensitive and the spring offers best results for frequency of the system. Spring locations depend on the flow velocity and spring constant itself.

Keywords*: Stability, Finite element method; Pipe conveyed fluid; linear spring stiffness.*

1. Introduction

One of the most important studies on the structure especially those supplied to dynamic external or internal applied loads is the dynamic behavior. So the great and deep studies on the dynamic behaviors of structure were accomplished. It was so important to explain all the effective parameters and understand how they affect the dynamic behavior itself. In the simple structures, one can predict some results of the dynamic response due to the accumulated experience or some time what related with the common experiments occurs during our lives. When the structure become more complex such as a pipe conveying fluid, the dynamic behavior become more difficult to predict. The complexities are arises for the combination of the different parameters resulting from pipe stiffness (structural stiffness) and flow stiffness (hydrodynamic stiffness resulting from the velocity of flow). The combination effect of these stiffnesses represents the major source of the complexity.

Flow induced vibration takes a great attention of the researchers. Interest in studying the dynamic behavior of fluid conveying pipes was stimulated when excessive vibrations were observed and subsequently analyzed first by Ashley and Haviland, 1950. Long, 1955, studied the influence of clamped-clamped and clamped-pinned boundary conditions on the critical velocity. Gregory and Paidoussis, 1966 presented results on the dynamic behavior of a cantilevered pipe conveying fluid. Chen, 1971, considered the critical flow velocity of a cantilevered pipe with a spring attached at its free end. Sundararajan, 1974 and Sugiyama *et al*, 1975discussed the effect of a spring support at the free end of elastic systems subjected to a follower force. Neimark, 1978 and Neimark *et al* 2003 used the D-composition method in the analytical solution of the pipe conveyed flow problem, and they mentioned that the main disadvantage of the method is the necessity to know the number of unstable mode. In addition they added that in the particular case, direct numerical calculation is not stable since it would give a big error through calculating the imaginary part. Becker, *et al* 1979, examined the dynamics of the system of end spring support. Sugiyama, et al, 1985, studied the dynamic behavior of the pipe conveyed fluid with spring support. They stated that the support location have a great effect on the dynamic characteristics of the structure.

In his book, Païdoussis, 1998, said An extension to studying the dynamic behavior with spring support take at the free ends (no intermediate support location) and any one didn't mention why?. The main reason was the complexities in the theoretical modeling or inaccuracy of the results. Also he mentioned that The case when there is an additional, intermediate spring support is a very complex problem, and it is an arduous task. Paidoussis, 2008 study the dynamic behavior of fluid conveying pipe. Jixing Yang, *et al*, 2009, studied the dynamic analysis of fluid-structure interaction on cantilever structure using ansys package to obtain the dynamic characteristics.

Lu, and Lee, 2009 studied the dynamic instability of the of pipe conveying fluid. They showed that the results gave good agreement with the analytical method. Stephanie, *et al*, 2010, accomplished the theoretical analysis of microscale resonators

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containing internal flow. Huang Yi-min, *et al* 2010, studied the natural frequency analysis of fluid conveying pipeline with different boundary conditions using eliminated element-Galerkin method. They explained that Galerkin method give good results with ineffective error.

Ni, *et al*, 2011, used **D**ifferential **T**ransformation **M**ethod (**DTM**) to analyze the free vibration problem of pipe conveying fluid. They demonstrated that the **DTM** method was gave good precision as compared with the analytical methods.

In this paper, the simply supported pipe conveying fluid stiffness by addition (linear spring) analyzed by finite element method. A simply supported end conditions with change value of spring addition and its location on the dynamic behavior of pipe conveying fluid are discussed.

Equation of Motion

The straight pipe is very important in the engineering application especially that of simply supported. The main reason of study the simply supported system is that one of the ways transpose any type of fluid, the pipe is not conservative system, Sugiyama, *et al*, 1985.

Fig.(1) presents a simply supported pipe conveyed fluid with fluid velocity of U m/s and length L. represents *(x/L)* the distance from spring to the supported end.

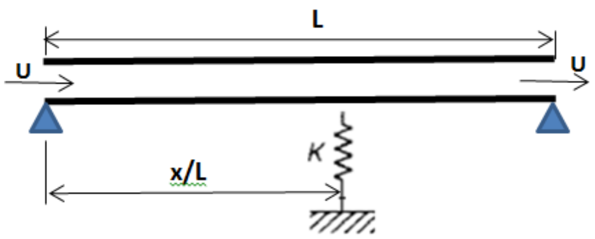


Fig.1: Simply supported pipe with intermediate spring support K

Pipe of (2 m) length. The fluid and pipe densities are 1000 kg/m^3 and 8000 kg/m^3, respectively. The pipe thickness is assumed to be 0.001 m with outer diameter of 0.01 m. Elastic modulus of pipe is 207 GPa.

Païdoussis 1998, and Païdoussis, *et al* 2011, and Mustafa, 2011, lists the step by step procedure to derive the governing differential equation of motion of the straight flexible pipe conveying fluid. The system consists of a uniform pipe of length (L), pipe mass per unit length (m), flexural rigidity (EI), conveying fluid of mass per unit length (M), flowing axially with fluid velocity (U). The cross-sectional flow area is (A), inner perimeter is (S) and the fluid pressure is (p). Fδx represents the reaction forces of the pipe on the fluid normal to the fluid element.

From Fig.(2), three partial differential equations parallel to the elements directions are resulted due to apply the equilibrium equations in the corresponding directions.

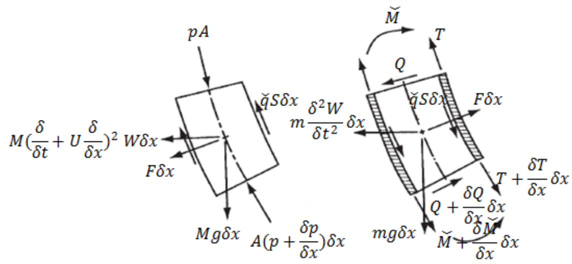


Fig.2: Presents fluid and pipe elements and reaction forces and moments (Mustafa, 2011)

It is so easy to carry out the three equilibrium equations and its mentioned in details in Païdoussis (1998, 2011).It is not important to mention them here, but the important is the overall equilibrium equation which resulted from combining the three equations as shown in Eq(1);

$$EI\frac{\partial^4 W}{\partial x^4} + MU^2\frac{\partial^2 W}{\partial x^2} + 2MU\frac{\partial^2 W}{\partial x \partial t} + (m+M)\frac{\partial^2 W}{\partial t^2} = \qquad (1)$$

The term $(EI\frac{\partial^4 W}{\partial x^4})$ represents a force component acting on the pipe as a result of pipe bending. The expression $(MU^2\frac{\partial^2 W}{\partial x^2})$ represents the force component acting on the pipe as a result of flow around a deflected pipe (curvature in pipe). Some researchers refer to this term as a centrifugal force of the fluid element due to its instantaneous velocity and instantaneous curvature of the deflected pipe. This term has a great effect on the pipe stability and accelerates the pipe to be unstable. The term $(2MU\frac{\partial^2 W}{\partial x \partial t})$ represents the inertial force associated with the Coriolis acceleration arising from the fluid flows with velocity U relative to the pipe. The inertia force for both fluid and pipe density is referred in term $((m+M)\frac{\partial^2 W}{\partial t^2})$.The boundary conditions for the simply supported pipe are

$$w|_{x=0,L} = 0 \qquad EI\frac{\partial^2 W}{\partial x^2}|_{x=0,L} = 0 \qquad (2)$$

Finite Element Discretization

In planar beam elements there are two degrees of freedom (DOFs) at a node in its local coordinate system. They are deflection in the y direction, v, and rotation in the x–y plane, θ z with respect to the z-axis. Therefore, each beam element has a total of four DOFs

Table 1: Effect number of elements on length of curved and frequency (U=0m/s, t=1mm, OD=10mm, L=2m and p=100kPa)

Number of Elements	Frequency (rad/s)
2	3.5768
4	3.5637
8	3.5628
12	3.5628

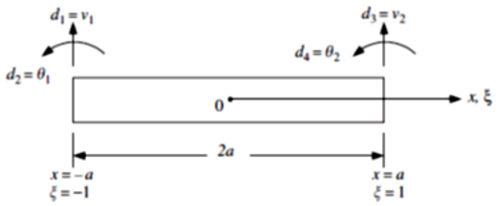


Fig 3: Beam element and its local coordinate system

Consider a beam element of length l = 2a with nodes 1 and 2 at each end of the element, as shown in Fig(3). The local x-axis is taken in the axial direction of the element with its origin at the middle section of the beam. Similar to all other structures, to develop the FEM equations,

To derive the four shape functions in the natural coordinates, the displacement in an element is first assumed in the form of a third order polynomial of x that contains four unknown constants:

Element of 4 nodes can be used. The displacement filed in element direction can be therefore written as follows (Raw, 2004 and Mustafa, 2011);

$$W(x) = \sum_{i=1}^{4} N_i(x) q_i \quad (3)$$

Where q_i is the generalized coordinates. The shape functions N_i are equal to

$$N_1 = \frac{1}{l^3}(2x^3 - 3lx^2 + l^3)$$
$$N_2 = \frac{1}{l^2}(x^3 - 2lx^2 + l^2 x)$$
$$N_3 = \frac{1}{l^3}(3lx^2 - 2x^3)$$
$$N_4 = \frac{1}{l^2}(x^3 - lx^2) \quad (4)$$

Where, l is the element length. The kinetic and potential energies of the pipe element can be expressed by

$$T = \frac{1}{2}\int_0^l (M+m)\left(\frac{\partial W}{\partial t}\right)^2 dx = \frac{1}{2}\sum_e \dot{q}^T (M+m) \int_0^l N^T N\, dx\, \dot{q} \quad (5)$$

$$V_1 = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 W}{\partial x^2}\right)^2 dx = \frac{1}{2}\sum_e q^T\, EI \int_0^l \bar{\bar{N}}^T \bar{\bar{N}}\, dx\, q\,. \quad (6)$$

Each prime sign that appear above the shape function symbol, i.e. N, represents one time derivative with respect to x-coordinate. Thus, mass $(\hat{m})$ and stiffness $(\hat{k}_1)$ matrices are equal to (Rao, 2004)

$$[\hat{m}] = \frac{(m+M)l}{420}\begin{bmatrix} 156 & 22l & -54 & -13l \\ 22l & 4l^2 & -13l & -3l^2 \\ 54 & 13l & -156 & -22l \\ -13l & -3l^2 & -22l & -4l^2 \end{bmatrix} \quad (7)$$

$$[\hat{k}_1] = \frac{2EI}{l^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix} \quad (8)$$

The term ($MU^2 \frac{\partial^2 W}{\partial x^2}$) has a potential energy that can be represented in terms of displacement shape function derived for the pipe as;

$$V_2 = \frac{1}{2}\int_0^l M\, U^2 \left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial x}\right) dx = \frac{1}{2}\sum_e q^T M U^2 \int_0^l \bar{N}^T \bar{N}\, dx\, q \quad (9)$$

The stiffness matrix that comes from flow around the deflected pipe is

$$[\hat{k}_2] = \frac{MU^2}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \quad (10)$$

It is important to clear that stiffness matrix $[\hat{k}_2]$ leads to weaken the overall stiffness of the pipe system (Païdoussis, 1998). The dissipation energy can be represented in term of the Coriolis force expression $(2MU\frac{\partial^2 W}{\partial x \partial t})$ as;

$$\mathcal{R} = \frac{1}{2}\int_0^l 2MU\left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial t}\right) dx = \frac{1}{2}\sum_e q^T\; 2MU \int_0^l \bar{N}^T N\, dx\, \dot{q} \quad (11)$$

This gives the unsymmetrical damping matrix

$$[\hat{C}] = \frac{MU}{30}\begin{bmatrix} -30 & -6l & -30 & 6l \\ 6l & 0 & -6l & l^2 \\ 30 & 6l & 30 & -6l \\ -6l & -l^2 & 6l & 0 \end{bmatrix} \quad (12)$$

Dynamic Analysis

The standard equation of motion in the finite element form is

$$[m+M]\{\ddot{q}\} + [\hat{C}\;]\{\dot{q}\} + [k_{total}]\{q\} = \{0\} \quad (13)$$

Where $k_{total} = \hat{k}_1 - \hat{k}_2$.Since the above equation has a damping term with skew-symmetric characteristic, thus the solution of eigenvalues problem should be executed to the characteristic matrix [Ω] (Meirovitch, 1980), which is equal to

$$[\Omega] = \begin{bmatrix} [0] & [I] \\ -[m+M]^{-1}[k_{total}] & -[m+M]^{-1}[C] \end{bmatrix} \quad (14)$$

The solution of eigenvalue problem yields complex roots. The imaginary part of these roots represents the natural frequencies of damped system. The real part indicates the rate of decay of the free vibration.

Results and Discussion

In this section, the out of plane results of simply support pipe conveying fluid were presented. Table (1) presents the number of elements, its represents the major parameter for accuracy of the results and the consumed time to solve the problem. Some types of error are increase or decrease with increasing or decreasing the number of elements and it is noted that, 256 elements gave a convergence in the results and then they will be used to discretize the curved pipe system.

Table (2) presents the comparison with other researcher. From the table, it was noted that there are good agreements between the results as compared with Jixing Yang, *et al*, 2009 in his study about the dynamic Analysis of fluid-structure interaction on cantilever structure without adding a spring.

Table 2: Presents the comparison of results

Length (h) Radius (r)	Dimensionless Natural frequency at different geometrical ratios	
r/h	**Present work**	**Jixing Yang, *et al*, 2009**
1/5	7.2471	7.2241
1/10	2.4155	2.4034
1/20	0.6279	0.6229

Figs.(4-7) present effect of spring location and dimensionless spring constants ($K=kl^3/EI$) on the natural frequency of simply supported pipe conveying fluid. The figures are remarked as data points for easier comparison. As a general view, it was noted that the frequency is increased with increasing the dimensionless spring constant. For the same value of spring, the frequency is seemed to be increased with increase x/L ratio (spring location measured from the supported end). This behavior was dominated for a specific spring constant started from K=100 to 1000 so the maximum frequency occurred at the center length of pipe. With increasing the dimensionless spring constant a different behavior was observed. Where an inflection point, at which the frequency has a maximum value then after it will decrease, is brought to light. Up to now. One can observe that the critical spring location (x/L) moved from the supported end and directed to the center length of pipe with increasing spring constant above its critical value.

Figures (4-7) present the relationship of the frequency and spring location for different spring values at 0,8,12 and15 m/s of fluid flow, respectively. The general sight is that, the different frequency(frequency of system at U=0 subtracted the frequency at flow velocity U) was increase with increase the flow velocity. The frequency of the system is largely depends on the structural stiffness of the pipe. With increasing the flow velocity, the hydrodynamic stiffness resulted from increasing the flow velocity (as mentioned in Eq.(10) is increased also due to the direct relationship between them. The hydrodynamic stiffness leads to weaken the structure stiffness of the pipe. Then increasing the flow velocity means increasing in hydrodynamic stiffness and thereby decreasing in the pipe stiffness which leads to decreasing in the frequency.

The more distinctive Fig (8) shown that all spring constants have an effect of the frequency at values of x/L, where the different frequency tend to increase with the flow velocity at closed the ratio x/L to the center length of pipe.

Conclusions

Some conclusions can be drawn from the results, the best of them are:

1- Spring addition plays an important role in changing of the dynamic behavior of the simply supported pipe conveying fluid.
2- Spring location has a great effect on the results of the system stability.
3- The Finite Element Method appears good agreement when comparison with numerical approach.

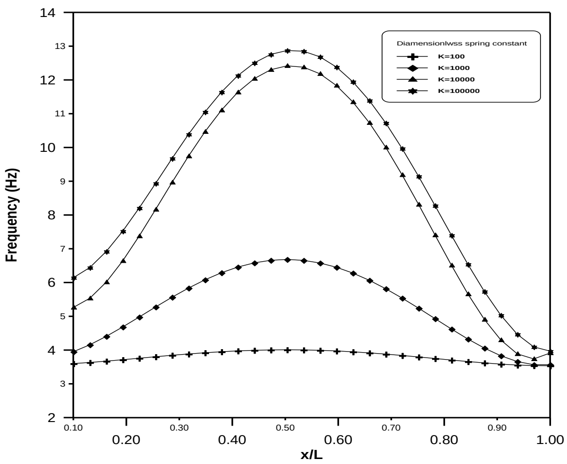


Fig.4: Effect of dimensionless spring constant and spring location on the frequency with no fluid velocity (U=0)

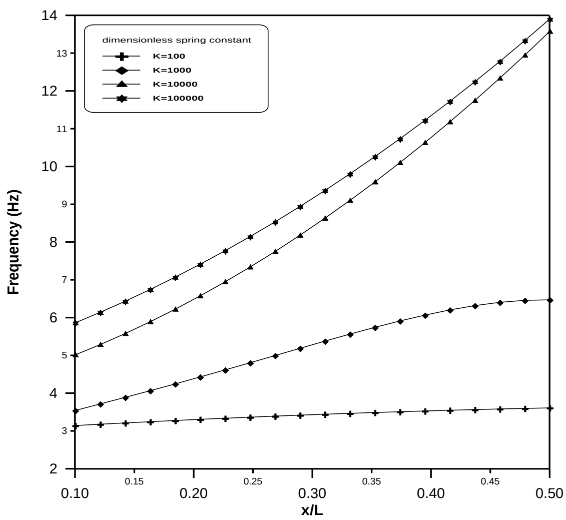


Fig.5: Effect of dimensionless spring constant and spring location on the frequency with fluid velocity (U=8 m/sec)

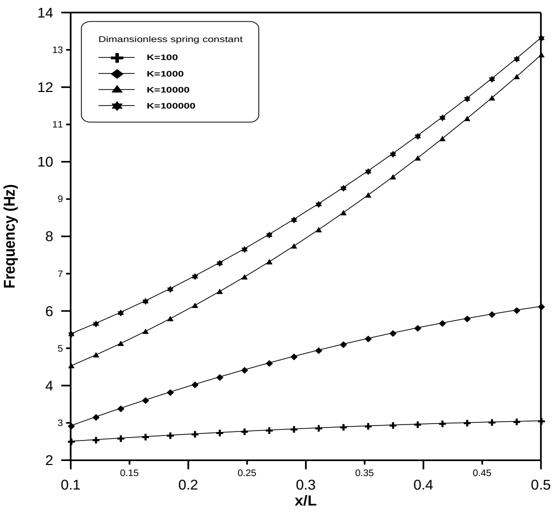


Fig.6: Effect of dimensionless spring constant and spring location on the frequency with fluid velocity (U=12 m/sec)

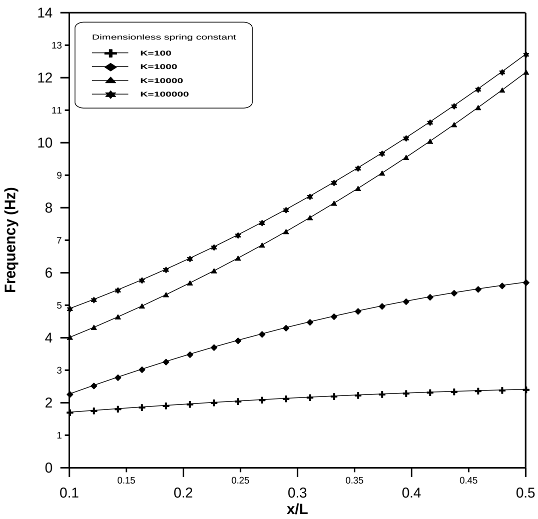
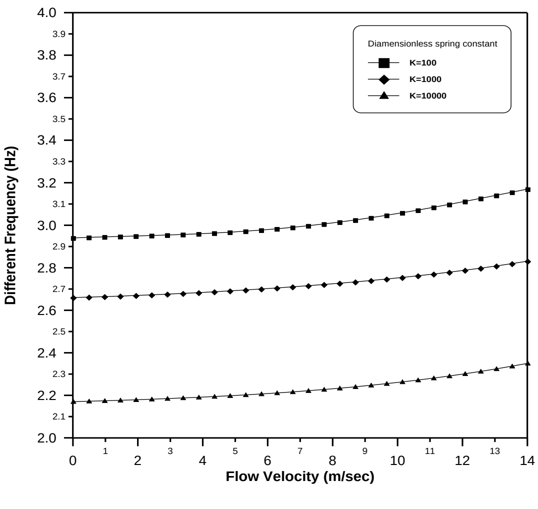
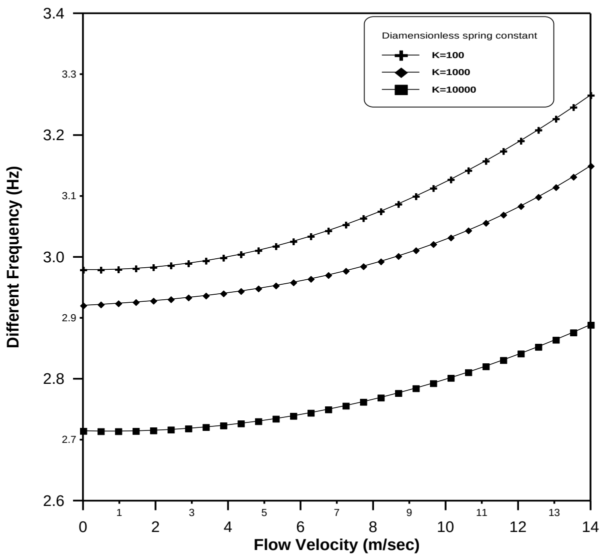


Fig.7: Effect of dimensionless spring constant and spring location on the frequency with fluid velocity (U=15m/sec)



(a): x/l=0.3



(b): x/l=0.4

Fig.8 (a-b) Effect the flow velocity on the different frequency for different dimensionless spring constants and different location

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Nomenclatures

Symbol	Definition	Units
A	Cross-sectional flow area	m^2
E	Modulus of elasticity of pipe	N/m^2
F	Reaction force inside the pipe	N
g	Acceleration constant	m/s^2
I	Pipe second moment of area	m^4
$[I]$	Unity matrix	-
$[\hat{k}_1]$	Stiffness matrix of pipe	-
$[\hat{k}_2]$	Stiffness matrix comes from flow around deflected pipe	-
L	Length of the pipe	m
M	Fluid mass per unit length	kg/m
M	Pipe mass per unit length	kg/m
$\breve{M}$	Bending moment	N/m
N_i	Shape function	m/s
P	Pressure inside the pipe	m/s
Q	Shear force	N
$\check{q}$	Wall shear stress	N/m^2
q	Lateral displacement of pipe	m
$\dot{q}$	Lateral velocity of pipe	m/s
$\ddot{q}$	Lateral acceleration of pipe	m/s^2
S	Pipe inner perimeter	m
T	Tension force in the pipe	N
t	Time	s
U	Fluid velocity relative to the pipe	m/s
x,W	Cartesian axes	-