Research Article

Accuracy Assessment of the Gravitational Geoid Model of Hilla Area, Iraq

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Abstract

Due to the importance of the subject and for the lack of such studies in Iraq, this research attempts to determine the accuracy of the geoid based on the available GPS and gravity data for the study area (Hilla, Iraq). To assess the geoid model accuracy, undulation measurements for terrestrial points in the area are compared with the undulation values resulted from the global model. Then, the standard deviations are determined which will be the indicator for the accuracy of the global model. The present analysis was carried on 21 gravity stations and 22 points observed by a GPS and level devices available in the study area. Topographic maps have been used to provide an accurate RTM for the study area instead of satellite data as they lack the required precision. After integrating the gravity data (Bouguer and Free air gravity data) with the global model EGM2008, the attained accuracies were ± 0.019 m for Bouguer data and ± 0.024 m for Free air data. Better results are achieved using Free air gravity data in calculation of the gravitational geoid as it is closer to the undulation accuracy extracted from the GPS/leveling method (± 0.034 m). Finally, as an evaluation, it can be stated from results that the computed geoid accuracy is acceptable and the present geoid model EGM2008 is considered as the optimal geoid so far for the study area. It is possible to produce a new geoid model by the method of interpolation using Free air gravity data but the scarcity of the gravity points of the study area and the lack of uniformly distributed points prevent us from producing a new geoid model.

Keywords: Geoid undulation, GPS, Bouguer and Free air Anomaly.

1. Introduction

Gravity data have many applications in earth related sciences such as geophysics. Gravity measurements made the basis of the knowledge of the gravity field which has an essential role in determining the shape of the earth's surface precisely which is known as geoid. As one of the major geosciences, geodesy is concerned with the continuous improvement of our knowledge about the size and shape of the earth and its gravity field (Saad, 1994).

The determination of the earth's surface and its external gravity field as well as the mean earth ellipsoid is the main problem currently facing the geodesists around the world. The earth's surface is irregular and consequently difficult to be represented by a mathematical surface. Instead this surface is approximately represented by an equipotential surface called geoid. Thus, as a matter of fact, the principal aim of all worldwide geodetic activities is the description of the surface of the earth and of the structure of its terrestrial gravity field, including the pertinent variation of time.

The need for a high resolution geoid model has grown-up significantly during the last few decades especially after foundation of Global Positioning System (GPS). Many countries have already developed their own geoidal model by means of deriving orthometric heights from GPS observations. So far, there is no available precise national geoid model that covers the entire country of Iraq. The main aim of this study is to determine the local gravimetric geoid model and computing the accuracy of the geoid undulation in the study area (Hilla area, Iraq).

2. Theoretical Background

Geoid and Ellipsoid

The geoid as equipotential surface of the gravity field, may be closely approximate the mean sea level (MSL). The geoid for any country is necessary for obtaining precise vertical positioning, since all geodetic measurements are closely related to the actual gravity field, as being taken on the physical surface of the earth. On the other hand, for geometrical relationships and associated mathematical computation among different terrain points, a mean earth reference

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ellipsoid is usually chosen to approximate the actual earth. Such ellipsoid generates, what is known in practice, the theoretical or normal gravity field. Also, it is practically accepted that the geoid represents the actual gravity filed of the earth. The difference between the geoid and mean earth ellipsoid is known as the anomalous gravity field. This anomalous field is the basic parameter for transformation of the geodetic measurements from the physical space of observation (the geoid) to the geometrical space of computation, the ellipsoid (Shaker *et al.*, 1997)

GPS /Leveling

GPS/leveling is the most recent and advanced method that is used in the determination of heights. The ellipsoidal heights obtained by GPS are not directly used for practical surveying. The ellipsoidal height has to be transformed to orthometric height, which is distance measured along the plumb line between the geoid and a point on the Earth's surface and taken positive upward from the geoid (National Geodetic Survey, 1986).

The determination of orthometric heights by traditional techniques, such as spirit leveling, is known to be a difficult task (Vergos and Sideris, 2002) Moreover leveling over areas with rough terrain is very tough and time consuming, in addition to distortion of significant number of benchmarks.

Therefore the use of careful GPS survey procedures coupled with high resolution geoid models, to obtain orthometric heights is known by GPS leveling. On the other hand, heights obtained by GPS are above an ellipsoid and are fundamentally different from traditionally obtained heights which are given with respect to the geoid. Mathematically, there is a simple relation between the two concepts and may be given by:

$$H = h - N \qquad (1)$$

In practice, Equation 1 reflects the possibility of GPS leveling, because it states that if the geoidal height N is known, the orthometric height H can be obtained from ellipsoidal height h determined by GPS. Obtaining orthometric heights this way, could in certain circumstances, depending on the required accuracy, replace conventional spirit leveling and thus make the leveling procedure cheaper and faster which is necessary for both practical surveying and scientific applications (Martensson, 2002). The accuracy of classical leveling is a complex subject in itself, and for engineering applications it may be taken 6 mm√km (Rummel, 1992).

In order to satisfy this accuracy using GPS leveling, there are two different things which we have to take into account: the accuracy of the GPS itself and the accuracy of the geoid model we need to transform heights above the ellipsoid to orthometric. Differential GPS measurements, which are used for ellipsoidal height determination, are now known for their high-accuracy due to improved data processing techniques (Zilkoski *et al.*, 2001). The critical part of this method is the geoidal height, which normally is obtained with a lower accuracy than the ellipsoidal height and thus affecting the accuracy of the orthometric height (El-Ashquer, 2010).

3. Gravity Data Reduction

Gravity Correction

There are many corrections for gravity observations. There are four main corrections as follows (Sharma, 1997; Lowrie, 2007):

1) Latitude correction: This correction is made to remove the effect of the increase of gravity from equator to poles due to a decrease in the earth's radius and centrifugal force. Latitude corrections are usually made by subtracting the normal gravity, calculated from the International Gravity Formula, from the observed or absolute gravity.
2) Free air correction: This correction takes into account the vertical decrease of gravity with the increase of elevation. It is equal to 0.3086 mgal/meter.
3) Bouguer correction: It accounts for the excess mass underlying observation points located at elevations higher than the elevation datum (sea level or the geoid). Conversely, it accounts for a mass deficiency at observation points located below the elevation datum.
4) Terrain correction: It accounts for variations in the observed gravitational acceleration caused by variations in topography near each observation point.

Gravity Anomalies

Suppose that we can measure gravity on the reference ellipsoid. If the distribution of density inside the earth is homogeneous, the measured gravity should agree with the theoretical gravity given by the normal gravity formula. The gravity corrections compensate for the usual situation that the point of measurement is not on the ellipsoid. A discrepancy between the corrected, measured gravity and the theoretical gravity is called a gravity anomaly. It arises because the density of the earth's interior is not homogeneous as assumed.

The most common types of gravity anomaly are the Bouguer and Free-air anomalies:

1) Bouguer anomaly

The Bouguer gravity anomaly (Δg_B) is defined by applying all the corrections.

$$\Delta g_B = g_m + (\Delta g_{FA} - \Delta g_{BP} + \Delta g_T + \Delta g_{tide}) - g_n \qquad (2)$$

In this formula g_m and g_n are the measured and normal gravity values; the corrections in parentheses are the free air correction (Δg_{FA}), Bouguer plate correction (Δg_{BP}), terrain correction (Δg_T) and tidal correction (Δg_{tide}).

2) Free air anomaly

The free-air anomaly (Δg_F) is defined by applying only the free-air, terrain and tidal corrections to the measured gravity:

$$\Delta g_F = g_m + (\Delta g_{FA} + \Delta g_T + \Delta g_{tide}) - g_n \qquad (3)$$

The next step in interpretation is the removal (subtraction) of the regional field anomaly from Bouguer (or Free air) anomaly resulting in residual (local) anomaly. Several analytical methods of residual-regional field analysis are available and include trend surface analysis (fitting a polynomial to the observed data, and low-pass filtering, etc.) (Sharma, 1997; Beltrão *et al.*, 1991; Kearey *et al.*, 2002).

Geopotential Geoid Computations: Basic Theory

The basic method of the will be spherical Fast Fourier Transform (FFT). The remove-compute-restore techniques are used to eliminate the terrain effects on land. The RTM (residual terrain model) reduction is used. The geoidal height N is split into three parts (Heiskanen and Moritz, 1993; Schwarz *et al.,* 1990):

$$N = N_{EGM} + N_{RTM} + N_{res} \qquad (4)$$

where:

N_{EGM}: The geoidal height of the EGM global field.
N_{RTM}: The geoidal height generated by the Residual Terrain Model, RTM: i.e. the high- frequency part of the topography. This part is computed by prism integration.
N_{res}: The geoidal height residual, i.e. corresponding to the un-modeled part of the residual gravity field. This part of the field is computed by spherical FFT.
N_{res} computed from Δg_{res} using Stoke's integration extending in principle all around the earth:

$$N_{res} = \frac{R}{4\pi\gamma} \iint \Delta g_{res}(S(\psi) + g_1)\, d\sigma \qquad (5)$$

where:
R: The mean earth's radius
ψ: The geocentric angle
Δg_{res}: The residual anomaly
$d\sigma$: An infinitesimal surface element of the unit sphere σ
g_1: This term is the Molodensky term which can be neglected when terrain reductions
γ: Normal gravity on the reference ellipsoid, and
$S(\psi)$: The Stokes function

The function S is Stokes' function:

$$S(\psi) = \frac{1}{sin(\frac{\psi}{2})} - 6\sin\frac{\psi}{2} + 1 - 5\cos\psi - 3\cos\psi \; log(sin\frac{\psi}{2} + \frac{\psi}{2}) \qquad (6)$$

Here Δg_{res} is the residual free-air anomaly or residual Bouguer anomaly, i.e. what is left in the gravity data after the contributions of the residual terrain effect Δg_{RTM} and the global field Δg_{EGM} are subtracted.

To prevent the influence of local gravity data on the longest wavelengths (where spherical harmonic models are most accurate) we use modifications of the Stoke's kernel $S(\psi)$. The technique is a modified Wong-Gore method, in which the modified kernel $S_{mod}(\psi)$ has the following expression (Dawod, 2012):

$$S_{mod}(\psi) = S(\psi) - \sum_{n=2}^{N_2} \alpha(n) \frac{2n+1}{n-1} P_n \cos(\psi) \qquad (7)$$

where the coefficients $\alpha(n)$ increase linearly from 0 to 1 between degrees N1 and N2.

$$\alpha(n) = \begin{cases} 1 & for\ 2 \leq n \leq N_1 \\ \frac{N_2 - n}{N_2 - N_1} & for\ N_1 \leq n \leq N_2, \; n = 2, \dots, N \\ 0 & for\ N_2 \leq n \end{cases} \qquad (8)$$

The coefficients N_1 and N_2 are in principle to be determined by trial and error or experience, and represent a trade-off between when use to satellite fields fully (for $n < N_1$) and when to use local gravity data fully ($n > N_2$).

The residual gravity term are transformed into the residual geoid by multi-band spherical FFT (Forsberg and Sideris, 1993) which provides a virtually exact implementation of Stokes formula on a sphere. The geoid is obtained by a number of band-wise operations of form:

$$N_{res} = S(\Delta\varphi, \Delta\lambda) * [\Delta g_{res}(\varphi, \lambda)\sin\varphi] = F^{-1}[F(S)F(\Delta g_{res}\sin\varphi)] \qquad (9)$$

This technique is recommended by the US Geological Survey (NGS) in surveying works achieved in small local areas. The main idea of this technique depends upon trying to discover the errors of EGM2008 and increase the accuracy in the local area through Combination or modifying the local measurements (Dawod, 2012).

4. Study Area

Location

The study area is located in central Iraq almost within Babylon Governorate. It is located between 405523 E to 498739 E and 3540836 N to 3641517 N meter (or 43° 59' 59" λ to 44° 59' 31" λ and 31° 59' 58" φ to 32° 54' 42" φ degree). This region occupies an area around area 10568 km²computed by using software ArcGIS. Figure (1) shows map of Iraq, the study area and gravity points distributed on the study area.
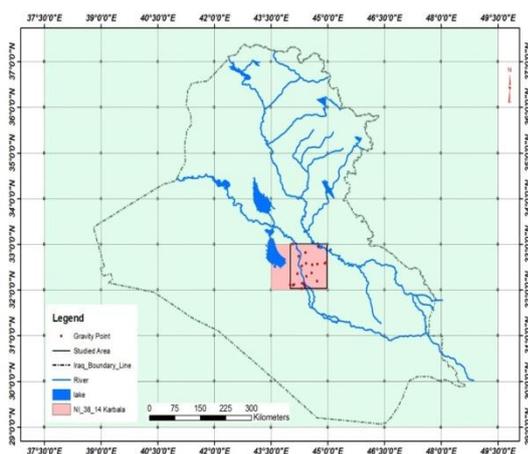
Fig.1 Map of Iraq with the study area

Available Data

Different sorts of data are available at the selected area of study, which thought to enable the achievement of the mentioned proposed objective. The available data can be classified into three groups:

- Geoid models, GPS data (GPS utilizes coordinates determination and ellipsoid height);
- Gravity data, leveling data (Precise leveling is used to determine the difference of the orthometric height); and
- Digital elevation model (DEM) and topographic maps.

Geology of the Study Area

The area is covered by more than 80% of Quaternary deposits, while the pre- Quaternary rocks are Upper Eocene to Upper Miocene in age. Tectonically the map area belongs partly to the Stable Shelf (Salman Zone), and partly to the Mesopotamian Zone (Tigris Subzone). Geomorphologically, the area belongs to three different morpho-genesis area: Mesopotamian plain, Desert plain and Bahr AL-Najaf; and Razzaza low lands. Hydrogeologically, the area has been subdivided into two Zones, in which the aquifers belong to Quaternary,

Miocene and Eocene deposits; the water is in general highly mineralized (Barwary and Slewa, 1995).

5. Methodology

GPS/Leveling

In this study, the Differential Global Positioning method (GPS) and high precise leveling are used to compute the values of the local geoid undulation. The difference between ellipsoid height observed by GPS and orthometric height observed by precise level instrument as well as the undulation value were determined.

The data of the difference between GPS (ellipsoid height) and leveling (orthometric height) were calculated as presented in Table 1. The highest value is -1.783 m, while the lowest value is -4.654 m with an average value of -3.113 m. The undulation contour values for the calculated points are presented in a map shown in Figure (2).

Table 1 The difference between GPS data and level data

Point No.	Spheroid height (m)	Orthometric height (M.S.L) (m)	N-undulation (m)
1	25.565	28.318	-2.753
2	23.064	25.736	-2.672
3	24.941	27.530	-2.590
4	28.546	31.031	-2.486
5	24.062	27.226	-3.164
6	31.441	33.402	-1.961
7	20.384	24.210	-3.827
8	22.061	25.750	-3.689
9	26.481	29.554	-3.082
10	17.600	22.254	-4.654
11	21.729	24.449	-2.720
12	20.697	25.017	-4.320
13	28.200	30.275	-2.075
14	29.851	31.634	-1.783
15	26.357	29.003	-2.646
16	22.405	25.469	-3.064
17	23.109	26.760	-3.651
18	22.192	25.771	-3.579
19	22.752	26.982	-4.230
20	23.594	26.851	-3.257
21	21.832	25.272	-3.440
22	25.371	28.210	-2.839

From Figure (2), it can be noted that the trend of contours indicate that the undulation heights increase from the southwest to the northeast direction as it is known that the elevations in Iraq are increasing from south to north. It is worthy to mention that the method of GPS/Leveling is the most accurate method for calculating the value of undulations as it uses the best reliable and most accurate devices. Thus, one can get more accuracy if more number of points in the study area is provided.
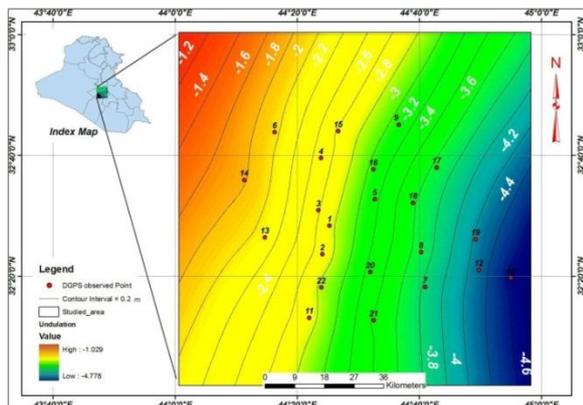
Fig.2 N- undulation contour map for GPS/leveling

Surveyed Data Corrections and Anomalies

Four main corrections were applied in this research as they are considered the most effective corrections on the gravity values, these are: latitude, Free air, Bouguer, and terrain corrections. For precise determination of terrain correction, Hammer chart was used.

After completing all the required corrections, by using equations 1 and 2, two kinds of anomalies are accounted those are Bouguer and Free air anomalies. Two maps are prepared for each anomaly (Figs. 3 and 4).

The Bouguer map of the area (Fig. 2) shows a general negative background with a regional gravity field trending from NW-SE. In general, the regional Bouguer anomaly is decreased toward north and northeast. The Bouguer gravity field is generally decreased from -10 mgal at the middle right edge to -74 mgal at the southwest. While, the Free air anomaly map (Fig. 3) shows the same anomalies appeared in Bouguer anomaly but with relatively less values. The Free air gravity field is generally decreased from -6.5 mgal at the middle right edge to -70 mgal at the southwest.

Next, the residual Bouguer and residual Free air anomalies have been computed by the removal of the regional field to isolate the residual anomalies.

Gravitational Geoid Computations

For calculation the geoid undulation, the residual (Δg_{res}) Bouguer anomaly or residual Free-air anomaly are calculated to be used later in Stoke's equation. Here Δg_{res} is the residual free-air anomaly or residual Bouguer anomaly, i.e. what is left in the gravity data after the contributions of the residual terrain effect Δg_{RTM} and the global field Δg_{EGM} are subtracted.

Fast Fourier Transform (FFT) method was used to compute the geoid height residual (N_{res}) from the residuals (Δg_{res}) of both residual Bouguer and residual Free air anomalies using Oasis Montaj Geosoft. N_{res} was computed from Δg_{res} using Stoke's integration. The Remove- Compute- Restore (RCR) techniques are used to eliminate the terrain effects on land.

The residual gravity term are usually transformed into the residual geoid by FFT technique which is

recommended by the US Geological Survey (NGS) in surveying works of small local areas. It is successfully proven in a number of scientific researches around the world. This method increases the global model accuracy after local measurements being combined to make it more compatible with the local real geoid surface in the study area. Better results are attained with increasing the number of known points with systematic distribution on the study area. Thus, this procedure is the best available methods in the field of geoid model and converting heights for surveying and mapping application (Dawod, 2012).

The geoid height residual contour maps by both residual Bouguer and Free air anomalies are presented in Figures 5 and 6.
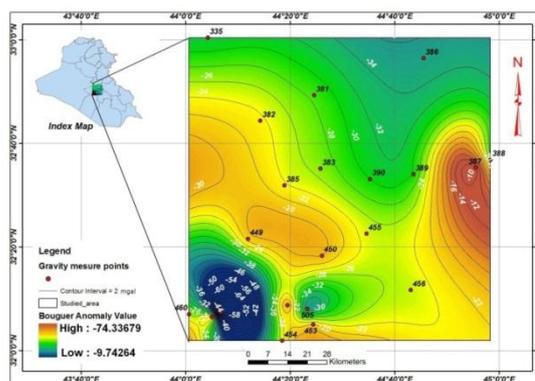
Fig.3 Contour map for Bouguer anomaly
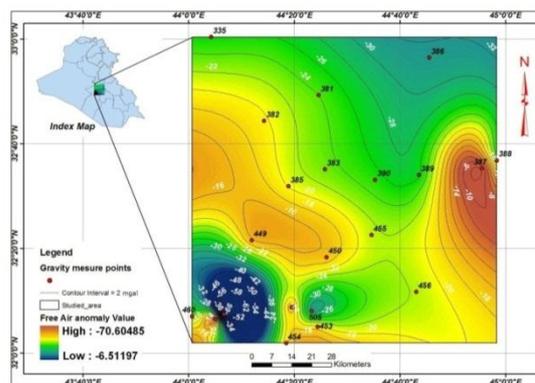
Fig.4 Contour map for Free air anomaly
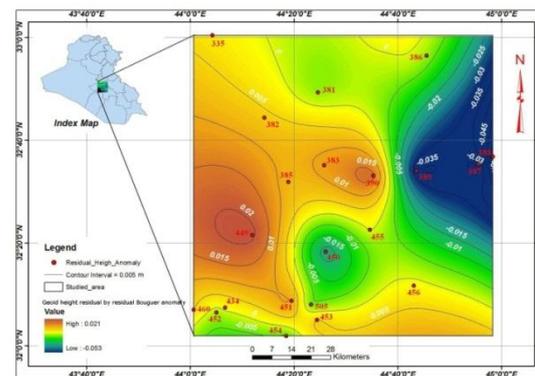
Fig.5 Contour map for geoid height residual (N_{res}) from residual Bouguer anomaly
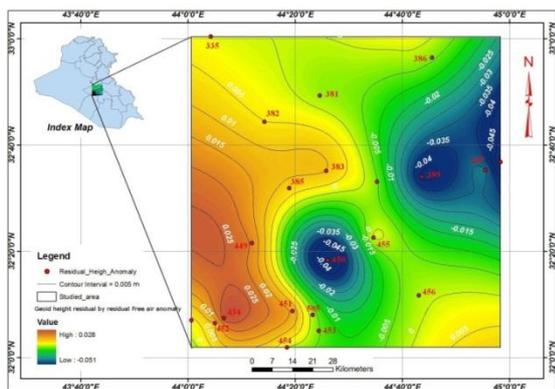
Fig.6 Contour map for geoid height residual (N_{res}) from residual Free air anomaly
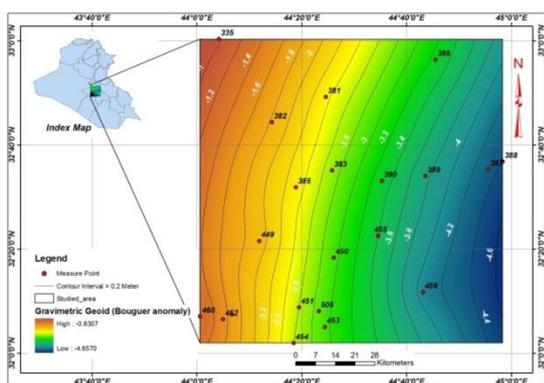
Fig.7 The final geoid undulation (N) using residual Bouguer gravity data
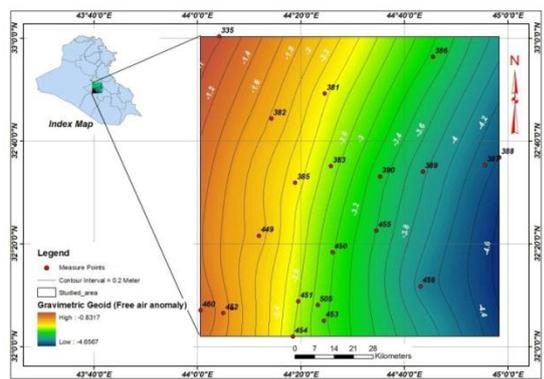
Fig.8 The final geoid undulation (N) using residual Free air gravity data

In this study, the RCR techniques used gravity data to compute and improve the values of gravimetric geoid undulation (N). The geoid undulation (N) using residual Bouguer anomaly (or residual Free air anomaly) and merge them with the global model EGM2008 by applying equations 4 mentioned in previous section are presented in Figures 7 and 8.

6. Results and Discussion

To assess the accuracy of the geoid model in a specific geographical area, the measurements of undulation

(N=h-H) of some terrestrial points are collected and compared with the values of undulation resulted from the global model. After calculating the difference between the values at each point, the standard deviation of all the differences is determined this will be considered as the indicator for the accuracy of the global model.

As mentioned above, the DGPS and high precise leveling (GPS/leveling) are used to compute the values of the local geoid undulation. The outputs of the GPS/leveling method are compared with geoid undulation EGM2008 model as shown in Table (2). The comparison results show that the geoid undulation accuracy is within ± 0.034 m. The geoid undulation accuracy is almost very good as the area of study is about 10434 km² which is relatively small region compared to the geoid surface.

Table 2 The GPS/leveling method compared with geoid undulation EGM2008 model

Model	Maximum (m)	Minimum (m)	Mean (m)	Standard deviation (m)
GPS/Leveling	-1.783	-4.654	-3.113	0.744
EGM2008	-1.767	-4.675	-3.133	0.744
Difference	0.078	-0.038	0.020	0.034

After computation the residual Bouguer and residual Free air anomalies by the removal of the regional field, the Δg_{res} values for both anomalies were determined (Figs. 5 and 6). The Δg_{res} is the residual Free-air anomaly or residual Bouguer anomaly, i.e. what is left in the gravity data after the contributions of the residual terrain effect Δg_{RTM} and the global field Δg_{EGM} is subtracted.

The difference in values of Bouguer anomaly and Free air anomaly is belonged to the difference in both Δg_{res} (residual Bouguer anomaly) and Δg_{res} (residual Free air anomaly). This difference is so small which is due to the relatively flat terrain of the study area with some isolated or simple highs and also due to the small undulation values compared to the rest of the world and also the lack of the terrain correction or residual terrain model in the study area. But many previous researches show larger difference in values due to the differences in the study area and also its terrain.

In this study, the Remove – Compute – Restore techniques (RCR) was used to compute the gravitational geoid undulation (N). Then, the outputs of RCR technique were compared with the geoid model data extracted from global model. The statistics of the global model_EGM2008 differences and heights from gravimetric geoid model using Bouguer and Free air gravity data show accuracy within ± 0.019 m and ± 0.024 m for the global model EGM2008, as presented in Tables 3 and 4. The final geoid undulation (N) using residual Bouguer anomaly (or residual Free air anomaly) are presented in Figures 7 and 8.

When the Bouguer and Free air gravity data are used and integrated with the global model EGM2008, the results of gravimetric geoid undulation were improved through the difference between GPS/leveling method and the gravimetric geoid giving accuracy differences of about ± 0.015 m and ± 0.01 m respectively (with respect to ± 0.034 m). This improvement is not enough due to many reasons, scarcity and random distribution of the observed points in the whole study area, lack of the precision of some points particularly those old ones observed in the years between 1963-1984, as well as some surveyed gravity stations are not the same points observed by GPS and level. On the other hand, the Bouguer gravity anomaly has a huge indirect effect and, therefore, is not recommended for geoid determination.

Table 2 Comparison of the gravimetric geoid (Bouguer gravity data) with EGM2008 model

Model	Maximum (m)	Minimum (m)	Mean (m)	Standard deviation (m)
Gravimetric Geoid (Bouguer gravity data)	-0.976	-4.421	-2.712	0.958
EGM2008	-1.008	-4.382	-2.730	0.946
Difference	0.039	-0.043	-0.018	0.019

Table 3 Comparison of the gravimetric geoid (Free air gravity data) with EGM2008 model

Model	Maximum (m)	Minimum (m)	Mean (m)	Standard deviation (m)
Gravimetric Geoid (Free air gravity data)	-0.978	-4.418	-2.713	0.961
EGM2008	-1.008	-4.382	-2.730	0.946
Difference	0.037	-0.067	-0.017	0.024

From above results and discussion, it can be stated that a geoid model is essential for transforming the GPS based ellipsoidal heights into MSL based orthometric heights usually used in geomantic, GIS, surveying, and mapping applications. So far, there is no available precise national geoid model that covers the entire territories of Iraq. Consequently, that geoid model EGM2008 is considered the optimal geoid so far for study area until an accurate national study area geoid model is developed. It is recommended to apply this geoid in all geomantic applications within study area for converting GPS heights into MSL heights. Moreover, the presented processing strategy is also recommended to be applied in other regions within Iraq. It is possible to produce a new geoid model by the method of interpolation using free air anomaly but the scarcity of the gravity points of the study area and the lack of uniformly distributed points prevent us from producing a new geoid model.

Conclusions

Conclusion remarks may be summarized as follows

1) A comparison between the GPS/leveling methods with geoid undulation EGM2008 model for the study area revealed an accuracy of around ± 0.034 m for the geoid undulation.
2) By using the Bouguer and Free air gravity data and integrating them with the global model EGM2008, the attained accuracies were around ± 0.019 m and ±0.024 m respectively. Better results are achieved using Free air gravity than Bouguer data in calculation of the gravitational geoid as it is closer to the undulation accuracy extracted from the GPS/leveling method (± 0.034 m).
3) The difference between the gravimetric geoid and GPS/leveling is due to the relatively few gravity points and due to the fact that the study area is considerably small with almost negligible terrain effect. Better results can be obtained if there are more gravity points available with regular distribution.
4) The accuracy attained in this study is good enough and acceptable as study area is small compared with geoid surface.
5) There is no available precise national geoid model that covers the entire territories of Iraq. Consequently, the present geoid model EGM2008 is considered the optimal geoid so far for the study area until complete and accurate national study geoid model is developed.
6) It is possible to produce a new geoid model by the method of interpolation using Free air anomaly but the scarcity of the gravity points of the study area and the lack of uniformly distributed points prevent us from producing a new geoid model.

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