Governing Heat and Mass Transfer Equation for Solid Desiccant Dehumidifier Wheel

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Abstract

Desiccant cooling system (DCS) is an alternate suitable option against conventional cooling system in a hot and humid climate. Solid Desiccants are natural or synthetic substances capable of absorbing moisture present in the outdoor air. Desiccant wheels are rotary desiccant dehumidifiers used in air conditioning for dehumidification applications. The modeling of concurrent heat and mass transfer in these components is crucial for estimating their performances. A steady state one-dimensional model is developing and resolves to obtain good accuracy and short computational times. Experimental data have to be gathered to resolve eventual missing phenomena and validate the model for all input parameters. The modeling result are used to develop simple correlations for the outlet air conditions of humidity and temperature of air through the desiccant wheel as a function of the physically measurable input variables. These correlations will be used to reproduce the solid desiccant wheel in an air conditioning system in order to define the year round efficiency. In solid desiccant system moist air enters the process side and passes over the desiccant and is dehumidify. As a result of decrees in humidity of the process air and increases its temperature.

In the other hand, the humidity ratio of the regeneration air is increased and its temperature decreases.

Keywords: Solid Desiccant, Dehumidification, Modeling of Desiccant Wheel.

1. Introduction

The Reduction of the environmental impact and energy saving are higher once the desiccant material is regenerated by means of "free" thermal energy (Giovanni Angrisani, 2011). The desiccant wheel is that the most vital part for a desiccant dehumidifier. A rotating cylindrical wheel may be a rotary regenerative desiccant wheel divided into two sections: regeneration sections and dehumidification sections. The dehumidification and regeneration air streams are sometimes in a counter-flow arrangement. The desiccant wheel turns slowly to reveal one portion of the desiccant material to the hot regeneration air stream whereas the opposite portion at the same time passes through the wet method air stream. A partition and versatile seals separate the process and regeneration air streams within the dehumidifier.

Hybrid desiccant air-conditioning system, which is combination of a rotary solid desiccant dehumidification and a vapor compression air-conditioning unit (C.X. Jia,2006). Solid desiccant wheel works on principle of adsorption, during which the desiccant materials like silica gel is adsorb additional quantity of molecules into pores on their surface. two air streams are passes over the desiccant wheel; one stream (Process air) loses moisture to the desiccant materials whereas the other (Regeneration air) removes that vapour due to vapour pressure difference. The porous desiccant medium of the matrix is cylindrically operated for adsorption and desorption. In process of the desiccant wheel, dehumidification of air takes place and within the reactivation a part of the wheel humidification of the air takes place. Rotation of the wheel causes periodic reactivation of the adsorption part. Consequently the solid desiccant wheel need source of warmth to dry out the process air throughout adsorption, heat discharged as desiccant adsorbs vapour.

The heat generated within the desiccant is transferred through the material that decreases the sorption capacity. Therefore, the heat and mass transfer inside the solid desiccant are coupled and will be considered commonly in developing mathematical model. The modeling of a desiccant wheel used for dehumidifying the ventilation air of an air-conditioning system, which predicts the temperature and humidity states of the outlet air from a desiccant wheel and the optimum speed of the wheel once used as a dehumidifier (Fatemeh Esfandiari Nia, 2006). The desiccant material is regenerated by means of heating coil (at temperature of concerning 1100C). The subsequent performance parameters are evaluated:
- The moisture Removal capacity of the wheel.
- The dehumidification effectiveness.
- The Dehumidification coefficient of Performance.
- The sensible Energy ratio, SER.

**Table 1. Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2a</td>
<td>Length of flow passage of one channel</td>
</tr>
<tr>
<td>2b</td>
<td>Pitch of flow passage of one channel</td>
</tr>
<tr>
<td>(A_f)</td>
<td>Cross sectional area of flow passage of channel</td>
</tr>
<tr>
<td>(A_t)</td>
<td>Total Cross sectional area of flow passage.</td>
</tr>
<tr>
<td>(\Delta)</td>
<td>Thickness of Channel wall</td>
</tr>
<tr>
<td>(D_x)</td>
<td>Length of control volume</td>
</tr>
<tr>
<td>(A_r)</td>
<td>Area ratio</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>Porosity in desiccant</td>
</tr>
<tr>
<td>(V_{pores})</td>
<td>Volume of pores</td>
</tr>
<tr>
<td>(V_{total})</td>
<td>Total volume of layer</td>
</tr>
<tr>
<td>(V_d)</td>
<td>Desiccant Volume</td>
</tr>
<tr>
<td>(V_m)</td>
<td>Volume of matrix material</td>
</tr>
<tr>
<td>(\Phi)</td>
<td>Volume ratio of desiccant in layer</td>
</tr>
<tr>
<td>(U)</td>
<td>Velocity of the air at the entrance of control volume</td>
</tr>
<tr>
<td>(\rho_a)</td>
<td>Density of air at entrance</td>
</tr>
<tr>
<td>(D_{comb})</td>
<td>Combined diffusivity</td>
</tr>
</tbody>
</table>

### 2. Mathematical Modeling

A Rotary regenerative solid desiccant wheel, cross section of sinusoidal channel and control volume in model as shown in the following fig.1 One sinusoidal channel with length ‘\(dx\)’. Where cross sectional area of flow passage ‘\(A_f\)’, Perimeter of air flow passage of one channel ‘\(P_e\)’ and total cross sectional area of one channel ‘\(A_t\)’.

#### 2.1 For the simplicity following assumption has been considered:

1. All air channels assumed to be adiabatic.
2. Axial heat conduction and mass diffusion in the moisture are neglected.
3. The thermodynamic properties in the solid desiccant, dry air and matrix material are constant.
4. Rotary speed is constant and low enough to neglect the effect of centrifugal forces.

#### 2.2 Governing Equation

Cross sectional area of flow passage of one channel
\[ A_f = \frac{1}{2}(2a)(2b) = 2ab \]

Total cross sectional area of flow passage of one channel
\[ A_t = \frac{1}{2}(2a+\delta)(2b+\delta) \]

The perimeter and hydraulic diameter is given by (Zhang et al, 2013)
\[ P_e = 2b + 2\sqrt{ab + \left(\frac{a}{P}\right)^{2} + \frac{b}{P}^{2}} \]

Hydraulic diameter of flow passage of one channel
\[ D_h = 4A_f/V_{pores} \]

Area ratio
\[ A_r = \frac{A_f}{A_t} \]

Porosity in desiccant
\[ \varepsilon = \frac{V_{pores}}{V_{total}} \]

Volume ratio in desiccant
\[ \Phi = \frac{V_d}{V_{total}} \]

Cross sectional area of layer (desiccant + pores + matrix material) of one channel
\[ A_{layer} = (1-\varepsilon)A_t \]

Cross sectional area of pores in layer of one channel
\[ A_{pores} = \varepsilon(1-\varepsilon)A_t \]

Cross sectional area of layer without pores (desiccant + pores + matrix material) of one channel
\[ A_{layer} = (1-\varepsilon)(1-\varepsilon)A_t \]

Cross sectional area of desiccant in layer of one channel
\[ A_{d} = (1-\varepsilon)(1-\varepsilon)A_t \]

Cross sectional area of matrix material in layer of one channel
\[ A_{m} = (1-\varepsilon)(1-\varepsilon)A_t \]

#### a) Mass Conservation in control volume of air

Rate of accumulation of mass in control volume
\[ \frac{\partial m}{\partial t} = \frac{\partial}{\partial t}(\rho_a \times \text{control volume}) \]

Inflow
\[ \dot{m}_x = \rho_a A_r A_t u \]

Outflow
\[ \dot{m}_x + \frac{\partial}{\partial x}(\dot{m}_x)dx + h_m P_e dx (\omega_a - \omega_d) \]

Rate of accumulation of mass in control volume

---

**Fig.1 Schematic Diagram of (a) Desiccant Wheel (Fatemeh Esfandiari Nia, et.al. 2006), (b) cross section of channels (c) differential control volume.**
b) Mass Conservation in control volume of Desiccant

In desiccant layer moisture will be absorbed by desiccant surface and trapped in pores of desiccant

Rate of accumulation of mass in control volume

\[
\frac{\partial ma}{\partial t} + \frac{\partial m\text{desiccant} + \text{water}}{\partial t}\text{Desiccant} = \varepsilon (1 - A_v) A_t \frac{\partial da}{\partial t} + (1 - \varepsilon)(1 - A_v)A_t \varphi \frac{aw}{\partial t} \rho_a
\]

The diffusion takes place by mechanism

1) Surface diffusion

\[
\dot{N} x(surface\ diffusion) = -D_s \frac{\partial}{\partial x} (\rho_a - \rho_d)\text{desiccant}\times\text{Area}\text{ of desiccant}
\]

Outflow:

\[
\dot{N} (x+dx)\text{ (surface diffusion)} = \dot{N} x(surface\ diffusion) + \frac{\partial}{\partial x} \left[\dot{N} x(surface\ diffusion)\right] dx
\]

Rate of mass inflow due to convective mass transfer

\[
\dot{N} = \dot{N} x(surface\ diffusion) + \dot{N} x(comb\ diffusion) + \dot{N} x(comb\ diffusion) \frac{\partial}{\partial x} \left[\dot{N} x(comb\ diffusion)\right] dx
\]

Rate of accumulation of mass in control volume

\[
\dot{N} = \dot{N} x(surface\ diffusion) + \dot{N} x(comb\ diffusion) + \dot{N} x(comb\ diffusion) \frac{\partial}{\partial x} \left[\dot{N} x(comb\ diffusion)\right] dx
\]

\[
\rho_a A_t D_p \frac{\partial}{\partial x} \left(\frac{\varphi P}{\rho_d}\right) dx + \rho_d A_p \text{pores} D_{\text{comb}} \frac{\partial}{\partial x} \left(\frac{\varphi P}{\rho_d}\right) dx + \rho_m P_e dx (\omega_a - \omega_d)
\]

Rate of change of stored energy in control volume

\[
\epsilon (1 - A_v) A_t \rho_da \frac{\partial da}{\partial t} + (1 - \varepsilon)(1 - A_v)A_t \varphi \frac{aw}{\partial t} \rho_a + h_m P_e \omega_a
\]

\[
\epsilon (1 - A_v) A_t \rho_ad \frac{\partial da}{\partial t} + (1 - \varepsilon)(1 - A_v)A_t \varphi \frac{aw}{\partial t} \rho_a + h_m P_e \omega_a
\]

\[
\dot{Q}\Convective\ heat\ transfer = h P_e dx \left(T_a - T_d\right)
\]

Rate of energy Transfer due to convection mass transfer

\[
\dot{Q}\ Convective\ mass\ transfer = h P_e dx \left(\omega_a - \omega_d\right) c_p \left(T_a - T_d\right)
\]

Rate of change of stored energy in control volume

\[
\rho_d \left(c_{pda} + \omega_a c_p\right) \left(A_t A_t \frac{\partial\varphi}{\partial t} + \frac{\partial\varphi}{\partial x} \frac{\varphi}{\rho_d}\right) = h P_e dx \left(T_a - T_d\right) - h_m P_e dx \left(\omega_a - \omega_a\right) c_p \left(T_a - T_d\right) - \cdots
\]

d) Energy conservation in control volume of desiccant

Rate of change of stored energy

\[
\dot{Q} = \rho_d dx c_d Ad \left(\frac{\partial\varphi}{\partial t}\right) + \rho_m dx c_m A_m \left(\frac{\partial\varphi}{\partial x}\right)
\]

Rate of energy Transfer due to conduction

\[
\dot{q}_\text{conduction} = \left(-k_A Ad\right) \left(\frac{\partial\varphi}{\partial x}\right)
\]

Outflow:

\[
\dot{q} (s+dx)\text{ (conduction)} = \dot{q} (s)\text{ (conduction)} + \frac{\partial}{\partial x} \left[\dot{q} (s)\text{ (conduction)}\right] dx
\]

Rate of energy Transfer due to convection

\[
\dot{q} \text{convective heat transfer} = h P_e dx \left(T_a - T_d\right)
\]

Rate of energy Transfer due to convection mass transfer

\[
\dot{q} \text{ convective mass transfer} = h P_e dx \left(\omega_a - \omega_d\right) c_p \left(T_a - T_d\right)
\]

Rate of energy Transfer due to Heat of adsorption

\[
\dot{q}_\text{adsorption} = h_m P_e dx \left(\omega_a - \omega_d\right) h_{ads}
\]

Energy conservation n desiccant is given by

\[
\rho_c cm \left(1 - e\right)(1 - A_v)A_t \left(\varphi + \omega_a c_p\right) \frac{\partial\varphi}{\partial t} + \rho_d c_d \left(1 - e\right)(1 - A_v)A_t \varphi \frac{aw}{\partial t} + \rho_d \frac{\partial^2 da}{\partial x^2} + h_m c_p \left(\omega_a - \omega_d\right)
\]

2.3 Performance Parameter

In order to experimentally evaluate the performance of the desiccant wheel as a function of the outdoor air temperature (tout) and humidity ratio (\(\omega_t\)), the regeneration temperature (\(T_{reg}\)) and the ratio between the regeneration and process air Volumetric flow rates, the performance parameters here in described have been analyzed.

\[
\eta_{deh} = \frac{\omega_a - \omega_d}{\omega_a}
\]

(2) The Moisture Removal Capacity, MRC, represents the mass flow rate of moisture removed by the wheel (Slayzak SJ et.al, 2000).

\[
\text{MRC} = \rho c_{\text{proc}} (\omega_a - \omega_d)
\]
(3) The Dehumidification Coefficient Of Performance, DCOP is the ratio between the thermal power related to the air dehumidification and the thermal power supplied for the regeneration process (Ge TS et al., 2010).

\[
DCOP = \frac{\rho_l V_{\text{Proc}} \Delta h_{\text{w}} (\omega_a - \omega_d)}{\rho_l V_{\text{Reg}} (h_a - h_d)}
\]

The latent heat of vaporization of water, has been evaluated by means of the following empirical cubic function (Rogers RR et al., 2000).

\[
\Delta h_{\text{w}} = -0.614342 \times 10^{-4} t_1^3 + 0.158927 \times 10^{-2} t_1^2 - 0.236418 \times 10^{-1} t_1 + 0.250079 \times 10^4
\]

(4) The Sensible Energy Ratio, SER, represents the ratio between the thermal power related to the air heating through the wheel on the process side and the thermal power supplied for the regeneration process (Enteria N et al, 2009)

\[
SER = \frac{\rho_l V_{\text{Proc}} C_p (T_2 - T_1)}{\rho_l V_{\text{Reg}} C_p (T_4 - T_2)}
\]

**Solution method and Result**

The governing equation are solve by using the simple forward finite difference Numerical solution method and applying boundary condition. The outside air enters the wheel from one end \(x=0\). The building exhaust air enters it from the other end \(x=L\). In this design parameters of the desiccant wheel given in Table 2 are vary for a fixed range of operating parameters given in Table 3 Optimization of the design parameters has been done to improve the performance of the desiccant wheel.

**Table 2:** Operating and structural parameters for optimization of design parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Fixed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational speed, (N) (rph)</td>
<td>20</td>
</tr>
<tr>
<td>Sector angle of process air, (\theta_p)</td>
<td>180°</td>
</tr>
<tr>
<td>Sector angle of regeneration air, (\theta_r)</td>
<td>180°</td>
</tr>
<tr>
<td>Inlet temperature of process air, (T_p) (°C)</td>
<td>40°</td>
</tr>
<tr>
<td>Inlet temperature of regeneration air, (T_r) (°C)</td>
<td>100°</td>
</tr>
<tr>
<td>Inlet humidity ratio of process air, (Y_{p,ln}) (kg water vapour /kg dry air)</td>
<td>0.020</td>
</tr>
<tr>
<td>Inlet humidity ratio of regeneration air, (Y_{r,ln}) (kg water vapour /kg dry air)</td>
<td>0.020</td>
</tr>
<tr>
<td>Velocity of process air, (u_p) in (m/s)</td>
<td>2</td>
</tr>
<tr>
<td>Velocity of regeneration air, (u_r) in (m/s)</td>
<td>2</td>
</tr>
</tbody>
</table>

**Table 3** Design parameter of solid desiccant wheel

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel length (m)</td>
<td>0.3</td>
</tr>
<tr>
<td>Pitch of flow passage of one channel, (2b) (m)</td>
<td>0.004</td>
</tr>
</tbody>
</table>

**Conclusion**

A one-dimensional transient model was given in this paper for a solid desiccant wheel. First, some assumptions were created to simplify the governing equations. Then, the governing equations of heat and mass transfer within the control volumes of the process air and the desiccant felt are given. This equation predicts the temperature and humidity states of the outlet air from a desiccant wheel and the optimum speed of the wheel when used as a dehumidifier. Solving these four governing equation by finite difference method further modeling of desiccant wheel is possible. The solid desiccant wheel performs well in a climate with moderate temperature. At higher humidity ratio, the solid desiccant wheel will get higher moisture removal and better DCOP.

**References**


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Fatemeh Esfandiari Nia et al. (2006), Modeling and simulation of desiccant wheel for air conditioning system, Energy and Buildings, 38, 1230–1239.