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Research Article

Stress Analysis and Study of Fir-Tree Assembly of Turbine Disc

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Abstract

One of the major sources of the stress arising in the turbo-machinery blades are the centrifugal and thermal loads acting at any section of the airfoil. Accounting for this phenomena stress evaluation of the blade attachment region in the disc has to be performed in order to avoid blade failure. Turbo-machinery blades are generally twisted, and the cross-section area varies from root to tip. The blade root shape at the attachment region is of much concern. Stress concentrations are predictable at this contact region. Therefore, a Comprehensive three-dimensional finite element study is made of the effect of the critical geometric features and interface conditions in the fir--tree region of a gas turbine disc upon the structural integrity. The aspects of the work were accordingly examined:- The first was concerned with the modeling of the three dimensional model on software PRO-E, and the second was concerned with the structural analysis of the geometry and examining the various stress distribution on the male and female assemblies of turbine assembly,

Keywords: *Stress Analysis, Finite Element Method, Gas turbine blade, Fir-Tree joint.*

1. Introduction

Turbine Disc Assemblies

The safety of gas turbine engines has always been the main concern of aircraft certification authorities. Economic pressure resulting from the reduced availability of strategic materials and the high cost of engine components and the continued demand, by all engine suppliers/users, for longer life and higher thrust to weight ratio continue to provide a stimulating challenge for engine designers/developers. (Papanikos P. *et al*,, 1992, Venkatesh S, *et al*,1988)

The mechanical joint between either a compressor or a turbine blade and the disc represents the most critical load path between the blades and the disc. Contact stresses, interface conditions (friction, surface roughness, residual stress), and the detailed geometry of the joint determine the severity of the resulting stress field.

Cracks usually develop in this region and the designer must understand fully the consequence of varying the geometry or the interface conditions. Unfortunately, contact stresses are not constant, but vary depending upon power and speed requirements. The time variation of the stress field during service loads of a GTE can induce fatigue and/or fretting fatigue.

Fir-tree fasteners have been commonly implemented in turbines because they provide multiple areas of contact over which large thermal and centrifugal stresses can be accommodated. (Stjepanovic J *et al*, 1996, Venkatesh, S *et al*., 1998) The thermo mechanical integrity of turbine discs and the attached blades is crucial to the operational safety and service life of gas turbine engines. The failure mode of a primary member in an engine, such as turbine disc assemblies, is usually catastrophic, often resulting in loss of life and hardware. Aero engine designers are constantly faced with the challenge of establishing stress levels in these critical parts that will allow the use of suitable high strength heat resistant alloys operating in a safe thermo mechanical loading regimes. (Arvanitis S. T *et al*, 1987, Srinivasan J *et al,* 1989, Stjepanovic J *et al*, 1996)

At this stage, it is important to identify the pertinent parameters which influence the mechanical integrity of aero engine turbine disc assemblies. These include:
The quasi static thermo mechanical strength, toughness and rupture strength of the different constituents of the assembly,

The applied thermal and body forces associated with the temperature of the gases and the rotational speed of the disc, and Residual stress state of the components/assembly. In this thesis, we focus our attention to the study of the Structural, thermo-mechanical and coupled behavior of a turbine disc made from Inconel X-750.

2. Finite element analysis of turbine disc assembly

The finite element method involves discretising a physical domain into small sub-domains, known as

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elements, over which piecewise continuous field variables such as velocity, stress, pressure, or temperature can be approximated. Since the actual variation of the field variable inside the element is not known, some approximating functions are needed to describe this variation. These approximating functions are similar to trial functions since they interpolate the field values at the nodal points of each element. Knowing the geometric and material properties of each element, suitable field equations such as equilibrium or heat balance can be written, and the elemental stiffness matrix cm be obtained, usually by minimizing the potential energy of this unit system.

2.1 Fundamentals of the Finite Element Method

The fundamental concept of the finite element method is that a physical domain is discretised into a small number of sub domains, known as elements, over which a continuous field variable such as velocity, stress, pressure, or temperature can be approximated. These elements are connected at specific points known as nodes or nodal points. Since the actual variation of the field variable is not known inside the domain, approximating functions are needed to describe this variation. These approximating functions interpolate the values of the field variable at the nodal points of each element. Since the geometric and the required material properties of each element are known, suitable field equations such as equilibrium or heat balance can be written for each element. Using the principal of minimum potential energy of each unit, the elemental Stiffness matrix can be obtained for each element. (Meguid S. A *et al*, 1987, 1996)

Each element is connected to the other element at nodal points to form a continuum of the entire model. The global stiffness matrix $[K^g]$ is obtained by assembling the element stiffness matrices for each element for the entire domain. The new unknowns obtained by the assemblage of elements are the nodal values of the field variable. Boundary conditions are taken into account, which help to modify the overall equilibrium equations. This then yields the global equilibrium in terms of banded matrices:

$$[K^g]\{u^g\} = \{F^g\}$$

Where,

$\{u^g\}$ represents the global displacement vector, and $\{F^g\}$ represents the global applied load or force vector.

The general solution of an engineering problem can be described in a step-by-step procedure. This sequence of steps describes the actual solution process that is followed in setting up and solving equilibrium or heat balance equations. A step-by-step approach adopted in finite element problems is summarized below:

Idealization of the structure: The geometrical features of the structure are simplified in order to accommodate sensible discretisation.

Discretisation of the structure: In this case, the body is subdivided into an equivalent system of finite elements. The type, size and number of elements is dictated by the geometrical features of the component, applied loads and restraints, accuracy needed and CPU floating point power.

Choice of interpolation or displacement function: The assumed displacement function approximates the actual or exact distribution of the displacement field within the continuum. In general, the interpolation function is taken in the form of a polynomial; the number of terms that can be retained in the polynomial is limited by practical considerations.

Derivation of the element stiffness matrix: The stiffness matrix is composed of the coefficients of the equilibrium equations derived from the material and the geometric properties of an element and obtained by the use of the principle of minimum potential energy (equilibrium condition). The stiffness $[K^{(e)}]$ relates the displacements at the nodal points $\{u^{(e)}\}$to the applied forces at the nodal points $\{F^{(e}\}$ where *(e)* denotes the element number, namely,

$$[K^{(e)}]\ \{u^{(e)}\}=\{F^{(e}\},$$

Assembly of element equations for the overall discretised body: This process includes the assembly of the global stiffness matrix $[K^g]$ for the entire body from the individual element stiffness matrices $[K^e]$ and the global vector $\{F^g\}$from the element nodal force vectors

$$\sum_{e}^{n}[\,K(e)]$$

$\{F^e\}$ such that $[K^g]$ = with *n* being the total number of elements.

Solution for the unknown nodal displacements: the overall equilibrium equations have to be modified to account for the boundary conditions of the problem. After the incorporation of the boundary conditions, the global equilibrium equations can be expressed as $[k^g]$ $\{u^g\}$ = $\{F^g\}$. For linear elastic problems, the displacement vector can be easily obtained. But for non-linear problems, the solution is obtained in a sequence of steps, each step involving the updating of the global stiffness matrix $[k^g]$ and /or load vector $\{F^g\}$.

Computation of element strains and stresses from nodal displacements: having determine the primary unknowns (nodal displacements), it is often necessary to use these nodal displacements to determine the element strains and stresses by using the appropriate solid mechanics equations.

With the availability of many powerful linear and nonlinear finite element packages, it was felt unnecessary to develop the solution programs for the current study. The work concentrates on the mechanics and design aspects of the fir-tree joint in an aero engine turbine disc rather than the programming aspect of the work.

Throughout this work, use has been made of ANSYS 11.0; a finite element package that contains a pre-processor, a number of solvers and a post-processor. The pre-processor allows the user to rapidly create the three-dimensional finite element model and prepares the model for analysis through automatic model checking routines. The frontal solver and the pre conditional conjugate gradient solver (three-dimensional model) were used in the present problem to accommodate the geometrical non-linearties occurring in the fir-tree joint. The general post-processor allows the user to review results over the entire model at specific load steps and sub-steps. (Papanikos P. *et al*,1992 ,Bathe K. J *et al,* 1996)

2.2 Description of the Problem

Turbo-machines such as turbines, compressors and pumps involve the use of fast rotating disc assemblies. In order to optimize their geometrical features for maximum efficiency and to ensure safety and reliability, a precise knowledge of their performance under loading is necessary. Conventional methods of mere strength assessment are no longer considered to be sufficient, when taking into account the increasing demands for safety, reliability and high strength requirements of such rotating discs.

Aero-engine compressor discs subjected to high loading demands contain inherently high stressed components. While many of these components can be contained within the engine casing on failure, such as blade loss, the catastrophic failure of a turbine disc on the other hand causes the larger fragments of the disc to puncture the engine casing. The consequences of such a failure are particularly costly resulting in the destruction of the engine and ultimately in the loss of Me.

The distribution of stresses around the regions of high stress concentration such as dovetail or fir-tree regions of the disc is sources of great concern to the designer. The high stresses due to geometry and loading become particularly severe when fretting at the contact surfaces is experienced by the assembly. Indeed, it has ken found that fatigue cracks tend to initiate in regions of combined high stress and cumulative fretting damage.

It is generally believed that in the case of the disc, the cracks are associated with an intermittent high frequency engine resonance and/or load fluctuations due to engine power or speed changeover. Since the operational steady state load is already high, only moderate fluctuations are required for cracks to initiate followed by subsequent crack propagation.

The geometry adopted for non-linear finite element analysis of 3-D fir-tree joint is as shown in the figure and is as same as that used in the works by Venkatesh (1988) and Meguid *et al* (2000).

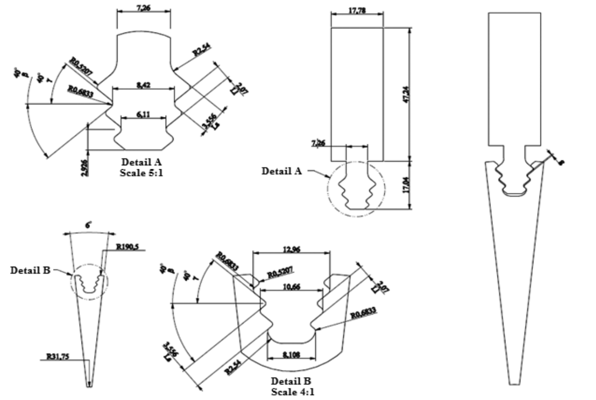

Figure 1: Disc and Blade Geometry for Fir-Tree joint

3. Three Dimensional Modeling of Turbine Disc Assembly and Finite Element Analysis

Strictly speaking, the distribution of stresses in a fir-tree joint is three-dimensional. However, if it is assumed that the variation of load in the thickness direction is not significant and that there is no dramatic thickness variation near the fir-tree region, then it is possible to mode1 only a slice of the blade and disc assembly as a two-dimensional plane stress problem. The three-dimensional results will then characterize the stress distribution of the blade and disc assembly to a certain extent. This can result in a substantial saving of effort and cost and still can provide a significant insight into the problem. In this study, the 3D Model is prepared on software PRO-E and imported in Ansys for the further results. FE analysis was carried out on turbine disc assembly.

Although eight-noded quadrilateral elements could have been employed for the blade and disc assembly, ANSYS does not accommodate such elements accurately in the contact regions. For this reason, four noded quadrilateral plane stress elements, with reduced integration, were used.

The validity of the results obtained by FE analysis was ensured by checking the condition of equilibrium of forces. The values of the reaction forces, *Fx* and *Fy,* were summed over the restrained nodes in contact. In reality, the nodes are not restrained in the y-direction, so that the summation of forces is necessarily zero in this direction. The material properties used for the modeling the blade and the disc assembly were that of a typical Nickel alloy used in aero-engine component design; namely, INCONEL X-750. This material is creep resistant.

Table 1: Material Properties of the disc assembly

Property	Value	Unit
Poisson's Ratio	0.32	
Young's Modulus	212.4	KN/mm^2
Density	8303	Kg/m^3
Specific Heat	432	J/Kg K
Coefficient of Expansion	12.6	μ/mK
Bulk Temperature	873	K
Thermal Conductivity	20.5	W/m at 873K
Film Coefficient	20	W/m^2K

Three Dimensional Details of the Geometry

In this case, the number of teeth n = 3,
Contact angle α = *20°,*
Bottom Flank angle β =40°,
Top flank angle γ=40° and
Flank length *l* = 2.5 mm are defined and shown in the figure.

4. Results

Stress Analysis is done for the fir tree joint and distribution is analyzed for two different loads i.e. 20 KN and 50 KN. The following contour plots explain the stress distribution.

1. In figure 2 the FEA of (female assembly) is analyzed and explains the stress distribution for the turbine blade and disc with 20 KN load applied.
2. In figure 3 the stress distribution for the turbine blade and disc assembly (male assembly) with 20 KN load applied; we conclude that stress is equally distributed in the assembly.
3. In figure 4 the stress distribution for the turbine blade and disc assembly (female assembly) with 50 KN load and coefficient of friction= 0.3 is shown.
4. In figure5 the stress distribution for the turbine blade and disc assembly (male assembly) with 50 KN load and coefficient of friction= 0.3 is shown.

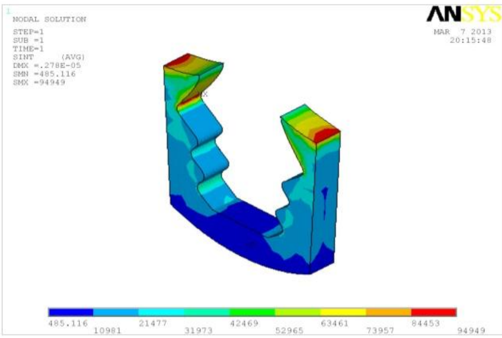

Figure 2: Stress Distribution Contour Plot (female assembly) for n=3, α =20°, β=40°, γ =40°, μ=0.0

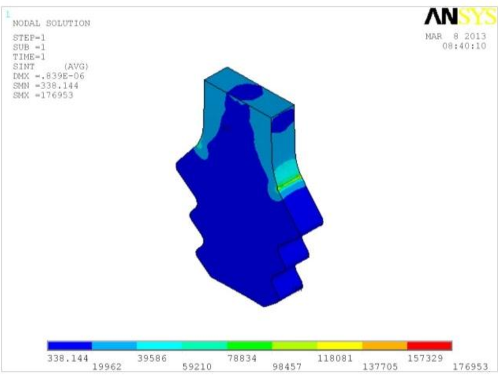

Figure 3: Stress Distribution Contour Plot (male assembly) for n=3, α =20°, β=40°, γ =40°, μ=0.0

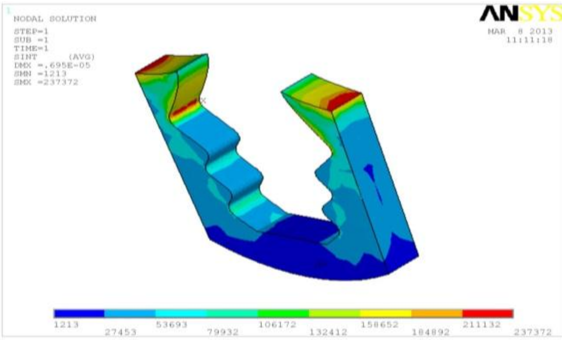

Figure 4: Stress Distribution Contour Plot (female assembly) for n=3, α =20°, β=40°, γ =40°, μ=0.3

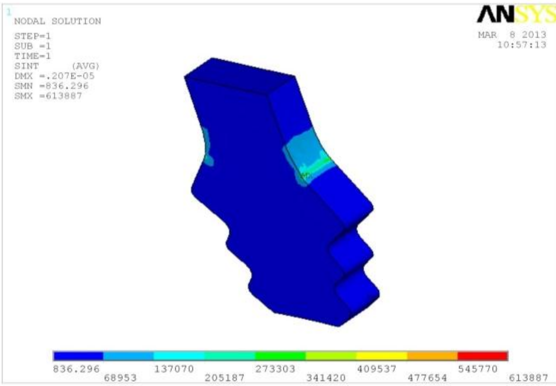

Figure 5: Stress Distribution Contour Plot (male assembly) for n=3, α =20°, β=40°, γ =40°, μ=0.3

Conclusions

A number of conclusions can be deduced from the current study. They can be summarized as follows:

Effect of Geometry, Interface Conditions and loads:

The results reveal that:

1. The maximum stress concentration occurs at just below the lower contact point between the blade and the disc for separate centrifugal loading as well as and for thermal loading,
2. Increase in load variates the stresses in both the geometries, there can be increase or decrease in stress distribution.

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