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*Research Article*

# Correlations for Heat Transfer Coefficients and Wall-Temperatures for Laminar and Turbulent Free Convection from Plane Vertical Surfaces to Supercritical Fluids

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### Abstract

*Correlations for heat transfer coefficient and wall-temperature are proposed for free convection from a plane vertical surface to super critical water and super critical carbon-dioxide based on numerical solutions using modified Patankar-Spalding implicit finite difference scheme For turbulent flow, the proposed correlations are based on the numerical solutions using turbulent kinetic energy model with Van Driest's mixing length. In the papers published earlier by us, the numerical predictions presented cannot be used directly. Hence, in this paper correlations based on these predictions have been proposed for both laminar and turbulent flows for both constant wall-temperature and constant wall-heat flux conditions. The correlations cover a wide range of Rayleigh number, $Ra_{\infty x}$ from $10^{7}$ to $2.1 \times 10^{16}$ for a pressure ratio, $P/P_{cr}$ = 1.01 to 1.5.The maximum deviation of the proposed correlations are within ± 10 percent with respect to numerical predictions except for heat transfer coefficient for turbulent flow for which the maximum deviation is within ± 15 percent. The maximum deviations are found to occur whenever the pseudo-critical temperature (pseudo-critical temperature is the temperature at which the specific heat at constant pressure is maximum for the given supercritical pressure) is within the thermal boundary layer.*

***Keywords:*** *Super-critical fluid; pseudo-critical temperature; turbulent kinetic energy; mixing length*

## Nomenclature

$C_p$, specific heat at constant pressure (J/kg-K)
$\hat{C}_p$, Integrated mean value of $C_p = \frac{i_w - i_\infty}{T_w - T_\infty}$
e, turbulent kinetic energy,(J/kg)
g, acceleration due to gravity (m/s²)
h, heat transfer coefficient (W/m²-K)
i, specific enthalpy of the fluid (J/kg-K)
k, thermal conductivity of the fluid (W/m-K)
L , Total length of the plate (m)
P, Pressure of the fluid (bar)
q, heat flux (W/m²)
T, Temperature of the fluid (K)
ΔT, temperature difference between the wall and the ambient fluid, $(T_w - T_\infty)$, $(K)$
u , x-component of the fluid velocity for laminar flow (m/s)
v,y-component of the fluid velocity for laminar flow (m/s)
û, Time averaged x-component of fluid velocity for turbulent flow (m/s)
ṽ,Time averaged y-component of fluid velocity for turbulent flow (m/s)
x, Coordinate measured along the plate from the leading edge of the plate (m)
y, Coordinate measured normal to the plate from the surface of the plate (m)
β, coefficient of thermal expansion of the fluid,($K^{-1}$)
ρ, density of the fluid,(kg/m³)
μ, absolute viscosity of the fluid,(kg/m-s)
ν, kinematic viscosity of the fluid, (m²/s)
τ, shear stress,(N/m²)

*Subscripts*

Av, Average value
cp, conditions corresponding to constant property fluids
cr, properties corresponding to critical point
∞, properties evaluated in bulk fluid
l, quantities evaluated at x = l
x, quantities evaluated at a distance x from the leading edge of the plate
W, properties evaluated at the wall

*Superscripts*

*, conditions corresponding to the peak value of $C_p$

*Non-dimensional parameters*

$Nu_x$, Local Nusselt number, $hx / k_\infty$
$Nu_{av}$, Average Nusselt number, $h_{av}L / k$
Pr, Prandtl number of the fluid,$\left(\frac{\mu C_p}{k}\right)$
$Pr_t$, Turbulent Prandtl number
$Pr_{te}$, turbulent Prandtl number for turbulent kinetic energy
$Ra_{\infty x}$ , Rayleigh number at any x = $\frac{g\beta_\infty \Delta T\, x^3}{\nu_\infty^2} Pr_\infty$
$Ra^*_x$, modified Rayleigh number at any x = $\frac{g\beta_\infty q_w x^4}{k_\infty \nu_\infty^2} Pr_\infty$

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## 1. Introduction

The necessity to study the heat transfer to near-critical fluids has increased significantly in recent years due to its use in various industrial applications. These applications include supercritical pressure water oxidation systems for waste processing, the use of carbon dioxide at supercritical pressures for decaffeination and for power generation and liquid hydrogen-oxygen fuelled rockets and the development of supercritical pressure water-cooled nuclear reactors. Earlier investigations on the problem of heat transfer to or from near-critical fluids have clearly established the fact that the heat transfer process in the supercritical region would become more complex due to the severe variation of the thermo-physical properties of the fluid, especially near the pseudo-critical point(pseudo-critical point is defined as the thermodynamic state at which the specific heat at constant pressure is maximum for the given supercritical pressure).For example for carbon dioxide at 75 bar ($P/P_{cr}$ = 1.015) near the pseudo-critical temperature ρ,μ and k reduce by a factor of 3-6 where as $C_p$,β and i increase by a factor of 8-14.Earlier investigations (W.B.Hall,1971; S.Kakac,1987; A.F.Polyakov,1991; I.L.Pioro, R.B.Duffey,2003) on the problem of heat transfer in the supercritical region have shown that the conventional correlations based on constant property solutions have failed in predicting the heat transfer rates accurately in the supercritical region.

A number of theoretical investigations on the problem of heat transfer in the super critical region have been reported in the literature. Comprehensive reviews on previous works associated with variable property heat transfer and supercritical heat transfer are given in Ref. (W.B.Hall,1971; S.Kakac,1987; A.F.Polyakov,1991). An exhaustive review of literature published during the last five decades has been carried out by Ref. (I.L.Pioro, R.B.Duffey,2003) and is confined to forced convection through tubes for near-critical fluids. For free convection to supercritical fluids, the investigations of Ref. (C.A. Fritch,R.J. Grosh,1961; S.Hasegawa, K.Yoshioka,1966; H.Kato et.al.,1968; K.Nishikawa,T. Ito, 1969;E.S.Nowak, A.K.Konanur, 1970) have used the principle of similarity or integral method to solve the governing equations. All their predictions indicate that the heat transfer coefficient depends on the values of $T_\infty$ and $T_w$ individually. Though there is qualitative agreement between their analytical predictions and experimental values, the experimental values are generally found to be 10 to 30 percent higher than the theoretical predictions, the difference being more for large values of ΔT.All these investigations are carried out for ΔT < 16K. Ref.(T.R.Seetharam, G.K.Sharma,1977; G.S. Deshpande, G.K. Sharma, 1979; T.R.Seetharam, G.K.Sharma,1979) have numerically solved the governing equations for free convection from a plane vertical surface to near critical-fluids under laminar flow conditions using the Patankar-Spalding implicit finite difference scheme with suitable modifications to take into account the actual variation of the fluid properties in the near-critical region. They have obtained solutions for large values of ΔT (up to 30 K) and have proposed correlations to evaluate the heat transfer coefficients both for constant wall-temperature and constant wall-heat flux conditions. Their (T.R.Seetharam, G.K.Sharma,1979) predictions of wall temperature distribution with the predictions of Ref.(E.M.Sparrow, J.L Greg ,1956) for constant property fluid agree very well for small values of surface heat fluxes which give wall-temperatures such that T* does not lie within the thermal boundary layer. For higher values of the surface heat fluxes which gave values of $T_w$ close to T*, the predicted wall-temperatures are found to be significantly different from those predicted by Ref.(E.M.Sparrow, J.L Greg, 1956). Hence there is a need for a correlation to predict the wall temperature as a function of the distance x for supercritical fluids and this correlation shall take into account the variation of fluid properties in the supercritical region.

Very little information is available on the problem of turbulent free convective heat transfer from vertical surfaces to near-critical fluids.Ref. (Larson, J.R., Schoenhals,R.T.,1966) are the first to solve the problem analytically using integral method. They have accounted for the variation of the fluid properties by taking their integral mean values. Comparison of their predictions with their measurements has shown that the agreement between them is not satisfactory. They have found that for the cases when $T_w$ < T*, the theoretical predictions of heat transfer coefficients are higher than the measured values and vice-versa.In order to get better agreement they evaluated the Prandtl number appearing in the integral energy equation both at the wall-temperature and at the bulk fluid temperature. None of these methods improved the agreement with their measurements. They also proposed a correlation to evaluate the heat transfer coefficient based on their limited experimental data. The average deviation of the data points is found to be ± 15 percent with some individual deviations being as high as 40 percent whenever the pseudo-critical temperature is within the thermal boundary layer ($T_w$ <T* <$T_\infty$) Further the proposed correlation has a temperature explicitly appearing in it and this is not desirable. In addition, their investigations are confined to small temperature differences (ΔT < 20 K). Ref.(Beschastnov,S.P. *et al*,1973) have measured turbulent free convective heat transfer from vertical surfaces to near-critical carbon dioxide. Their experimental data is limited to pressures considerably away from the critical pressure (P = 78.5 to 88.2 bar). Based on their experimental data they have proposed a correlation to evaluate the average heat transfer coefficient. The scatter of the experimental data with respect to their proposed correlation is found to be ± 25 percent. Ref. (Protopopov,V.S.,Sharma,G.K., 1976) have carried out an exhaustive experimental investigation to measure the heat transfer coefficient from the outer surfaces of vertical tubes to near-critical carbon dioxide. They have covered a wide range of conditions (tubes: 8.2 mm and 19.6 mm dia; pressure: 75 – 100 bar; $\Delta T$ = 2 – 250 K; $q_w = 3.5 \times 10^3 - 1.1 \times 10^5$ W/m$^2$; $T_\infty$ = 15 – 54$^0$C) by varying the bulk fluid temperature and wall-heat flux.They have proposed a correlation to evaluate the heat transfer coefficient based on their measurements and have found that the maximum scatter of their data with respect to the proposed correlation are within ± 20 percent. They compared their data with the correlation suggested by Ref.(Larson,J.R., Schoenhals,R.T.,1966) and found satisfactory agreement only for small temperature differences ($\Delta T$ < 10 K).For large temperature differences the difference between the two correlations is found to be as high as 50 percent.

Ref.(T.R.Seetharam, G.K.Sharma ,1982) have solved the problem of turbulent free convection from a plane vertical surface to supercritical carbon dioxide at 75 bar by directly integrating the governing equations using Patankar-Spalding implicit finite difference scheme (Patankar, S.V., Spalding, D.B.,1970). They (T.R.Seetharam, G.K.Sharma ,1982) have used the turbulent kinetic energy model for specifying the turbulent viscosity and have taken into account the actual variation of the physical properties of the fluid taken from Ref.(Vukalovich, M.P., Altunin, V.V.,1968 for carbon dioxide;Rivikin, S.L.,1970, for supercritical water) . The scheme is suitably modified to account for the variation of the physical properties of the fluid. They (T.R.Seetharam, G.K.Sharma, 1982) have introduced the turbulent parameters at the transition Reynolds number as suggested in Ref. (Protopopov,V.S., Sharma,G.K., 1976) for near-critical fluids, i.e. $(Ra_\infty)_{Tr} = 5 \times 10^{10} (\check{C}_p / C_{p\infty})$ and have obtained the solution both for constant wall-temperature and constant wall-heat flux conditions. For constant wall-temperature conditions ,they (T.R.Seetharam,G.K.Sharma,1982) have obtained the solutions for two values of bulk fluid temperatures, $T_\infty < T^*$ and $T_\infty = T^*$ and for a wide range of temperature differences ($\Delta T$ = 2 – 100 K).They have found that for $T_\infty < T^*$ and for small values of $\Delta T$ the variation of wall-heat flux $q_w$ is very small. But as $\Delta T$ is increased, $q_w$ increases considerably. For the condition $T_w > T^* > T_\infty$ , $q_w$ increases with x , reaches a peak and then decreases. The decrease in $q_w$ persists for a certain distance beyond which $q_w$ increases continuously with x.They have attributed the following reasons for the decrease in $q_w$ with increase in x. For a specified value of $\Delta T$ the temperature gradient at the wall initially increases with x. If T* lies within the boundary layer, then the location of T* moves towards the wall with increase in x. Therefore, if initially T* lies within the turbulent zone of the boundary layer, the effective transport coefficients are mainly decided by the turbulent diffusivities. But as T* moves towards the buffer zone where the laminar and turbulent effects are of same order of magnitude, the effective transport coefficients like $\mu_{eff}$ and $Pr_{eff}$ are affected considerably by the physical properties of the fluid like viscosity and Prandtl number.In the turbulence model they have used Ref. (T.R.Seetharam,G.K.Sharma,1982) have assumed that the effective transport coefficients can be expressed as a sum of laminar and turbulent contributions.The large value of Pr occurring in the neighborhood of T* increases $Pr_{eff}$ locally, thereby lowering the effective thermal diffusivity. The low value of thermal diffusivity influences the temperature distribution resulting in low temperature gradient thereby decreasing the wall-heat flux. The decrease in $q_w$ persists till the location of T* moves into the viscous sub-layer. When T* does not lie within the boundary layer, the wall-heat flux increases with x continuously. The predicted heat transfer results is compared with the correlation proposed in Ref. (Protopopov,V.S.,Sharma,G.K., 1976) and found to agree with the correlation within ± 20 percent, the scatter of their data with respect to their correlation is being ± 20 percent. The predictions of Ref. (T.R.Seetharam, G.K.Sharma,1982) is found to be lower for lower values of $Ra_\infty$ (< $5 \times 10^{13}$) and for larger values of $Ra_\infty$, their predictions are found to be higher than the values obtained using the correlation proposed by Ref. (Protopopov,V.S.,Sharma,G.K., 1976) It is to be mentioned here that the correlation proposed by them (Protopopov,V.S.,Sharma,G.K., 1976) assumes that the heat transfer coefficient is independent of length where as the numerical predictions of Ref.(T.R.Seetharam, G.K.Sharma,1982) shows that heat transfer coefficient depends on x.They (T.R.Seetharam,G.K.Sharma,1982) have also obtained results for constant wall-heat flux conditions using wall values of the fluid properties in the damping term of the mixing length for carbon dioxide at 75 bar for $T_\infty < T^*$ for wide range of surface heat flux from 5000 to 25000 W/m$^2$. Their predictions of wall-temperature indicate that for small values of wall-heat flux (<15000 W/m$^2$), $T_w$ initially decreases with x and then remains practically constant. For large values of $q_w$ ( = 25000 W/m$^2$), $T_w$ decreases with x up to a certain distance and then it increases. The increase in $T_w$ indicating a decrease in the heat transfer coefficient is found to persist as long as T* lies within the buffer zone of the boundary layer where the laminar and turbulent effects are of same order of magnitude. With further increase in x,$T_w$ once again decreases and then remains practically constant. They (T.R.Seetharam, G.K.Sharma,1982) have compared their numerical predictions with the correlation proposed by Ref. (Protopopov,V.S., Sharma,G.K., 1976) based on their experiments. It is found that for $Ra_\infty > 10^{12}$ the numerical predictions are 10 to 40 percent higher than the correlation. They (T.R.Seetharam, G.K.Sharma,1982) have attributed this difference mainly to the following reasons: (i) using wall-temperature values of the fluid properties in the damping term of the mixing length results in high thermal diffusivities and hence higher heat transfer coefficients; (ii) the numerical predictions indicate that the heat transfer coefficient depends on x while the correlation of Ref. Protopopov,V.S.,Sharma,G.K., 1976) is arrived at by assuming that the heat transfer coefficient is independent of x.

The objectives of this paper are: (i) to propose correlations for wall-temperature distribution for both laminar and turbulent free convection to near critical fluid from a plane vertical surface with uniform heat flux conditions based on the numerical predictions of Ref.(T.R.Seetharam,G.K.Sharma,1979 and 1982] and (ii) to propose correlations to determine the heat transfer coefficients for turbulent free convection to near-critical fluid from a plane vertical surface with uniform wall-temperature and uniform wall-heat flux conditions based on the numerical predictions of Ref.(T.R.Seetharam, G.K.Sharma, 1982).

## 2. Governing Equations

### *2.1 Governing equations for laminar flow*

A semi infinite vertical flat plate with prescribed uniform heat flux is chosen as the physical model. For steady, two-dimensional stable laminar boundary layer flow conditions the equations for conservation of mass, momentum and energy can be written in the following form: Ref.(T.R.Seetharam ,G.K.Sharma,1979) :

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (1)$$

$$\rho u\left(\frac{\partial u}{\partial x}\right) + \rho v\left(\frac{\partial u}{\partial y}\right) = g(\rho_\infty - \rho) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \quad (2)$$

$$\rho u\left(\frac{\partial i}{\partial x}\right) + \rho v\left(\frac{\partial i}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{k}{C_p}\frac{\partial i}{\partial y}\right) \quad (3)$$

with the boundary conditions of

(i) at $y = 0$, $u = v = 0$ and $-\left(\frac{k}{C_p}\frac{\partial i}{\partial y}\right) = q_w =$ a constant

(ii) as $y \to \infty$, $u \to 0$ and $i \to i_\infty$ (4)

### *2.2 Governing equations for turbulent flow*

For two-dimensional steady turbulent flow with isotropic turbulence the governing equations are (T.R.Seetharam and G.K.Sharma ,1982):

Continuity equation: $\frac{\partial}{\partial x}(\rho\tilde{u}) + \frac{\partial}{\partial x}(\rho\tilde{v}) = 0$ (5)

Momentum equation : $\rho\tilde{u}\frac{\partial\tilde{u}}{\partial x} + \rho\,\tilde{v}\frac{\partial\tilde{u}}{\partial y} =$

$$\frac{\partial}{\partial y}\left[(\mu + \mu_t)\frac{\partial\tilde{u}}{\partial y}\right] + g(\rho_\infty - \tilde{\rho}) \quad (6)$$

Energy equation: $\rho\tilde{u}\frac{\partial\tilde{\imath}}{\partial x} + \rho\,\tilde{v}\frac{\partial\tilde{\imath}}{\partial y} = \frac{\partial}{\partial y}\left[\left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t}\right)\frac{\partial\tilde{\imath}}{\partial y}\right]$

$$\frac{\partial}{\partial y}\left[\left(\mu - \frac{\mu}{Pr}\right)\left\{\frac{\partial}{\partial y}\left(\frac{\tilde{u}^2}{2}\right) + \frac{\partial}{\partial y}\left(\frac{u'^2}{2}\right)\right\}\right] - \tilde{\rho}\tilde{u}g \quad (7)$$

With the following boundary conditions:

(i) at $y = 0$, $\tilde{u} = \tilde{v} = 0$ and $\tilde{\imath} = i_w$ = a constant or $-\left(\frac{k}{C_p}\frac{\partial\tilde{\imath}}{\partial y}\right) = q_w$ = a constant

(ii) as $y \to \infty$, $\tilde{u} \to 0$ and $\tilde{\imath} \to i_\infty$ (8)

### *2.3 Turbulence model for eddy diffusivities*

Closure to the system of equations (5) to (8) describing the turbulent boundary layer flow requires the specification of turbulent viscosity, $\mu_t$ and turbulent Prandtl number $Pr_t$. As at present the information available on turbulent transfer mechanism in free convection flows for near-critical fluids is very limited. They (T.R.Seetharam,G.K.Sharma ,1982) have assumed that the turbulent diffusivities for near-critical free convective flows are described by equations which have been used for analyzing constant property free convection flows with suitable modifications to account for the variation of the fluid properties in the near critical region. They have used the turbulent kinetic energy model to specify the eddy viscosity, $\nu_t$ (= $\mu_t / \rho$):

$$\nu_t = C_v\, l\sqrt{e} \quad (9)$$

In Eq.(9) 'e' is the turbulent kinetic energy, 'l' is a length scale representing energy containing eddies and $C_v$ is a constant.A transport equation for e can be derived from the Navier-Stokes equations. At high Reynolds numbers of turbulence where the dissipation process is essentially isotropic, the transport equation for 'e' can be written as follows (Launder, B.E., Spalding, D.B.,1972):

$$\rho\tilde{u}\frac{\partial e}{\partial x} + \rho\,\tilde{v}\frac{\partial e}{\partial y} = \frac{\partial}{\partial y}\left[\left(\mu + \frac{\mu_t}{Pr_{te}}\right)\frac{\partial e}{\partial y}\right] + \mu_t\left(\frac{\partial\tilde{u}}{\partial y}\right)^2 - \frac{C_D\,\rho\,e^{3/2}}{l_D} + \frac{g\beta\mu_t}{Pr_t}\frac{\partial\check{T}}{\partial y} \quad (10)$$

Eq.(10) which is appropriate to two-dimensional boundary layer flows is to be solved simultaneously with equations (5) to (8). Before solving these equations the values for the constants $C_v$, $C_D$ and $Pr_{te}$ must be assigned and the length scale for viscosity l and that for dissipation $l_D$ must be prescribed. Ref.(T.R.Seetharam, G.K.Sharma,1982) have assumed that the length scales l and $l_D$ are same and have chosen them to be Prandtl's length scale of turbulence with the Van Driest's modification (Van Driest, E.R.,1956) for the wall layers:

$$l = l_D = l_0\left[1 - \exp\left(-\frac{\sqrt{(\rho\tau_w)}}{\mu A^+}\right)\right] \quad (11)$$

where

$l_0 = ky$ for $ky \le a\delta$ and $l_0 = a\delta$ for $ky > a\delta$ with $k = 0.4$ and $a = 0.09$ (12)

The local values of density , viscosity and wall value of the shear stress are used in the damping term of Eq. (11). This follows the procedure adopted for forced flow of near-critical fluids by Ref.(Sastry,V.S. and Schnurr ,N.M.,1976).As no unique form of $A^+$ has been found to give better results than the value suggested by Van Driest, E.R.(1956).Ref. (T.R.Seetharam, G.K.

Sharma,1982) have used the same value ($A^+$ = 26) suggested by Ref.(Van Driest,E.R.,1956)The values for the constants $C_v$ and $C_D$ are chosen to be 0.5 and 0.1252 respectively as suggested by Ref. Mason,H.B., Seban,R.A.,1977) for the analysis of turbulent free convection for constant property fluids.

Experimental results (Reynolds, A.J., 1975) have shown that the turbulent Prandtl number for heat, $Pr_t$ is not constant across the boundary layer. Measurements by Ref.(Simpson,R.L., *et al*,1970) have indicated that near a wall the local values of $Pr_t$ are greater than unity and are dependent on the laminar Prandtl number Pr. The linear extrapolation of Smith's data by Ref. (Plumb,O.A.,Kennedy, L.A., 1977) yielded a value of $Pr_t$ at the wall which is as high as 2.5, whereas for free convection flows Ref. (Mason, H.B., Seban, R.A.,1977) have prescribed the values of 0.85 and 0.5 for the inner region (ky ≤ aδ) and outer region respectively. Ref. (Cebeci,T., Khattab,A., 1975) have prescribed a continuous variation of $Pr_t$ taking into account the effect of Pr on $Pr_t$. For near critical fluids, in the neighborhood of T*, Pr changes by a factor of 10 to 12.Hence Ref. (T.R.Seetharam,G.K. Sharma,1982) have used the expression proposed by Ref. (Cebeci,T., Khattab,A., 1975) for $Pr_t$ which is as follows:

$$Pr_t = \frac{0.4}{0.44}\,\frac{\left[1-\exp(-\frac{y^+}{A^+})\right]}{\left[1-\exp(-\frac{y^+}{B^+})\right]} \qquad (13)$$

Where

$$B^+ = B^{++} \,/\, \sqrt{Pr} \quad \text{and} \quad B^{++} = \sum_{i=1}^{5} C_i\,(\log_{10} Pr)^{i-1} \qquad (14)$$

The values of the constants chosen by Ref.(Cebeci,T.,Khattab,A., 1975) are as follows: $C_1$ = 34.96, $C_2$ = 28.79, $C_3$ = 33.95, $C_4$ = 6.33 and $C_5$ = −1.186. The TKE model also requires the specification of $Pr_{te}$. Ref. (T.R.Seetharam,G.K. Sharma,1982) have used the following values as suggested by Ref. (Mason,H.B.,Seban,R.A.,1977);

$$Pr_{te} = 1.7 \text{ for } ky \le a\delta \text{ and } = 0.7 \text{ for } ky > a\delta \qquad (15)$$

The equations for momentum (Eq.(2) and Eq.(6) ), for energy (Eq.(3) and Eq.(7)) and for turbulent kinetic energy (Eq.(10)) can be written in a common form

$$\rho u \left(\frac{\partial \phi}{\partial x}\right) + \rho v \left(\frac{\partial \phi}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\mu_{eff}}{Pr_{eff}}\,\frac{\partial \phi}{\partial y}\right) + d \qquad (16)$$

where ϕ stands for u,i or e, and 'd' represent the source term. $Pr_{eff}$ represents effective Prandtl number unique to each equation.

## 3. Numerical Solution

The finite difference form of Eq.(16) and its solution are obtained using the Genmix-4 program of Ref. (Patankar, S.V., Spalding, D.B.,1970). This program requires three major modifications for solving problems involving free convection to near-critical fluids,viz., (i) the addition of buoyancy force term in the momentum equation, (ii) the revised Couette flow solution to take into account the variation of the fluid properties in the near-critical region and (iii) addition of turbulent kinetic energy equation. Some minor modification like the addition of a subroutine to interpolate the property values from the available property data is also required. The details of these modifications and some of the results like velocity profiles, temperature profiles, heat transfer coefficients, wall-temperature distribution etc. for both laminar and turbulent flow conditions under constant wall-temperature and constant wall-heat flux conditions are given in Ref. (T.R.Seetharam, G.K.Sharma, 1977; T.R.Seetharam, G.K.Sharma, 1979 ; T.R.Seetharam, G.K.Sharma,1982).

## 4. Proposed Correlations

A number of correlations have been proposed to determine the heat transfer coefficient in the supercritical region by various investigators: Ref. (Larson, J.R., Schoenhals, R.J.,1966;Beschatnov, S.P.,et.al.,1973;Protopopov,V.S.,Sharma,G.K.,1976;T.R.Seetharam,G.K.Sharma,1982;Hahne,E.W.P.,et.al.,1974; Beschastnov, S.P.,Petrov, V.P.,1974;Ghajar, A.J., Parker, J.D.,1981;Hilal,M.A.,Boom,R.W.,1980).How well these correlations predict the available experimental data will help in identifying an appropriate heat transfer correlation.Ref.( Saeed Ghorbani-Tari, Afshin J. Ghajar, 1985) have compared all these correlations based on the same physical property inputs in order to have a meaningful comparison.For this purpose, they (Saeed Ghorbani-Tari, Afshin J. Ghajar,1985) have determined the constants in these correlations by curve fitting the equations to the experimental data based on correct values of the physical property inputs.They have compared the predicted results not only with the experimental data of those authors who have developed the specific correlations, but also with the experimental data of others.After sufficient amount of study, they Saeed Ghorbani-Tari, Afshin J. Ghajar,1985) have come to the conclusion that the correlation in the form given in Eq.(17) predicted all the experimental data better than the rest of the correlations.

$$Nu_x = a\,(Ra_\infty)^b \left(\frac{\hat{C}_p}{C_{p\infty}}\right)^c \left(\frac{\rho_w}{\rho_\infty}\right)^d \left(\frac{\mu_w}{\mu_\infty}\right)^e \left(\frac{k_w}{k_\infty}\right)^f \qquad (17)$$

where a,b,c,d,eand f are curve-fitted constant

*4.1Laminar flow: Correlation for wall temperature distribution*

A form of correlation similar to the one given in Eq.(17) is assumed to determine the dimensionless wall-temperature as a function of dimensionless distance as well as of property ratios and is given by

$$\frac{\Delta T_x}{\Delta T_l} = n\,(x/L)^{p} \left(\frac{\hat{C}_p}{C_{p\infty}}\right)^{q} \left(\frac{\rho_w}{\rho_\infty}\right)^{r} \left(\frac{\mu_w}{\mu_\infty}\right)^{s} \left(\frac{k_w}{k_\infty}\right)^{t} \quad (18)$$

The constants n,p,q,r,s and t are obtained using multiple linear regression analysis.

Using 272 data points of wall-temperatures based on numerical predictions of Ref.(T.R.Seetharam, G.K.Sharma, 1979) for supercritical carbon dioxide at pressures of 75,80 and 100 bar and for water at pressures of 225 and 245 bar, the correlation obtained is

$$\frac{\Delta T_x}{\Delta T_l} = 1.085\,(x/L)^{0.235} \left(\frac{\hat{C}_p}{C_{p\infty}}\right)^{-0.007} \left(\frac{\rho_w}{\rho_\infty}\right)^{-0.006} \left(\frac{\mu_w}{\mu_\infty}\right)^{0.121} \left(\frac{k_w}{k_\infty}\right)^{0.009} \quad (19)$$

with $R^2$ value equal to 0.84.The deviation was found to be less than ±10 percent for 256 data points which is less than 6 percent of total data points.For the remaiming 16 data points the deviation lies between 10 and 30percent and found to occur for cases for which $T_w > T^* > T_\infty$

For small temperature differences and for the cases where T* does not lie within the boundary layer such that the variation of fluid properties are not severe, it is interesting to compare Eq.(19) with the constant property correlation proposed by Ref. (E.M.Sparrow, J.L.Greg, 1956) which is of the form

$$\left(\frac{\Delta T_x}{\Delta T_l}\right)_{cp} = (x/L)^{0.2} \quad (20)$$

For this comparison a correlation is obtained assuming that the dimensionless temperature distribution $\frac{\Delta T_x}{\Delta T_l}$ varies with (x/L) in the same way as that proposed by Ref. (E.M.Sparrow, J.L.Greg, 1956) and the correlation obtained is as follows:

$$\frac{\Delta T_x}{\Delta T_l} = 1.05\,(x/L)^{0.2} \left(\frac{\hat{C}_p}{C_{p\infty}}\right)^{-0.018} \left(\frac{\rho_w}{\rho_\infty}\right)^{-0.03} \left(\frac{\mu_w}{\mu_\infty}\right)^{0.133} \left(\frac{k_w}{k_\infty}\right)^{0.004} \quad (21)$$

The numerical predictions for supercritical fluid Ref. (T.R.Seetharam, G.K.Sharma,1982) agree with the correlation given by Eq.(21) within ± 10 percent for 94 percent of the data points and for the remaining data points the deviation is within ±30 percent. The comparison of Eq.(21) with that proposed by Ref. (E.M.Sparrow, J.L.Greg, 1956) indicates that for constant property fluids where the product of property ratios appearing in Eq.(21) is almost equal to unity, the proposed correlation gives values of $\frac{\Delta T_x}{\Delta T_l}$ which are within 5 % of those obtained from the correlation proposed by Ref. (E.M.Sparrow, J.L.Greg, 1956) [Eq.(20)].

### 4.2 Turbulent flow

#### 4.2.1 Correlation for wall temperature distribution

For turbulent free convection, a correlation for the dimensionless wall-temperature distribution as a function of dimensionless distance, x/L is obtained as follows:

$$\frac{\Delta T_x}{\Delta T_L} = 0.97 \left(\frac{x}{L}\right)^{-0.157} \left(\frac{\hat{C}_p}{C_{p\infty}}\right)^{-0.072} \left(\frac{\rho_w}{\rho_\infty}\right)^{-0.584} \left(\frac{\mu_w}{\mu_\infty}\right)^{0.425} \left(\frac{k_w}{k_\infty}\right)^{0.118} \quad (22)$$

with $R^2$ value of 0.82. The deviation of the predictions from those obtained using Eq.(22) is found to be within ± 9 percent except for four data points for which the deviation is found to be between 11 and 15 percent. This correlation cannot be compared with experimental data as no such data is available at present.

#### 4.2.2 Correlation for heat transfer coefficient for constant wall-temperature conditions

Ref. (T.R.Seetharam,G.K.Sharma,1982) have obtained numerical results for near critical carbon dioxide at 75 bar ($p/p_{cr}$ =1.015 ; T* = 305 K) for two values of $T_\infty$, $T_\infty$ = 300 K (< T*) and $T_\infty$ = 305 K ( = T*) and for seven values of ΔT ranging from 2 K to 100 K to cover the three cases viz , $T_w < T^*$, $T_w = T^*$ and $T_w > T^*$. Based on these predictions a correlation of the form given in Eq.(19) is assumed. Based on 99 data points and using multiple linear regression analysis the correlation is found to be

$$Nu_x = 0.016\,(Ra_{\infty x})^{0.393} \left(\frac{\hat{C}_p}{C_{p\infty}}\right)^{1.1} \left(\frac{\rho_w}{\rho_\infty}\right)^{-0.063} \left(\frac{\mu_w}{\mu_\infty}\right)^{0.9} \left(\frac{k_w}{k_\infty}\right)^{-0.393} \quad (23)$$

The $R^2$ value for this correlation is found to be 0.964. Except for nine data points, the deviation between the numerical predictions and values obtained using the above correlation [Eq.(23)]is within ± 15 percent. The deviations for the nine data points is between −15 and −17 percent. The reason for this deviation can be attributed to the fact for these data points the pseudo-critical temperature may lie in the buffer zone of the boundary layer where the contributions of laminar and turbulent viscosities are of the same order of magnitude and the variation of the thermo-physical properties of the fluid near T* are very severe.The proposed correlation is not compared with correlations based on experimental data as all the experimental data obtained so far are confined to constant wall-heat flux conditions. Further the investigators have assumed that for turbulent free convection in the supercritical region the heat transfer coefficient is independent of 'x' whereas the numerical predictions of Ref. (T.R.Seetharam,G.K.Sharma,1982) indicate that the heat transfer coefficient increases with x and this is reflected in the present correlation [Eq.(23)]i.e. $h_x$ is proportional to $x^{0.179}$.

*4.2.3 Correlation for heat transfer coefficient for constant wall-heat flux conditions*

Ref. (T.R.Seetharam,G.K.Sharma,1982) have also made numerical predictions of heat transfer coefficient for turbulent free convection to near-critical carbon dioxide at 75 bar from a vertical plane surface with uniform surface-heat flux for a wide range of wall heat flux from 2000 W/m$^2$ to 50,000 W/m$^2$ for one value of $T_\infty$ = 301K ($T_\infty < T^*$).Based on these results for 54 data points a correlation of the form given in Eq.(17) is obtained using multiple linear regression analysis. The correlation obtained is

$$Nu_x = 0.03\,(Ra_{\infty x})^{0.395} \left(\frac{\hat{C}_p}{C_{p\infty}}\right)^{0.324} \left(\frac{\rho_w}{\rho_\infty}\right)^{2.738} \left(\frac{\mu_w}{\mu_\infty}\right)^{-2.394} \left(\frac{k_w}{k_\infty}\right)^{-0.439} \quad (24)$$

With $R^2$ value of 0.995.Except for two data points, the deviation between the numerical predictions and values obtained using Eq.(24) is < ± 10 percent. The use of modified Rayleigh number $Ra^*_{\infty x}$ ($Ra^*_{\infty x} = \{g\beta q_w x^4\}/\{k\nu^2\}$)in the correlation for Nusselt number is customary for constant wall heat-flux conditions. Correlation using modified Rayleigh number is obtained as

$$Nu_x = 0.079\,(Ra^*_{\infty x})^{0.284} \left(\frac{\hat{C}_p}{C_{p\infty}}\right)^{0.228} \left(\frac{\rho_w}{\rho_\infty}\right)^{1.981} \left(\frac{\mu_w}{\mu_\infty}\right)^{-1.74} \left(\frac{k_w}{k_\infty}\right)^{-0.315} \quad (25)$$

with $R^2$ value of 0.997. The deviation between the predicted values and the values obtained using Eq.(25) is found to be within ± 9 percent.

The correlations given in Eq.(23) and Eq.(24) are not compared with the correlations suggested by Ref. (Larson, J.R., Schoenhals, R.J.,1966;Beschastonov et.al.,1973; Protopopov,V.S., and Sharma,G.K., 1976) as all these three correlations assume that heat transfer coefficient is independent of 'x' where as the numerical predictions of Ref. (T.R.Seetharam,G.K.Sharma,1982) indicate that heat transfer coefficient depends on 'x'.However, for the purposes of comparison, a correlation similar to the one proposed by Ref.(Protopopov,V.S., Sharma,G.K., 1976) is also obtained as

$$Nu_x = 0.193(Ra_x)^{0.333}(Cp^*)^{0.528}(\rho^*)^{0.389} \quad (26)$$

Comparison of Eq.(26) with that proposed by Ref. (Protopopov,V.S.,Sharma,G.K., 1976) given in Eq.(27)

$$Nu_x = 0.135\,(Ra_x)^{0.333}(Cp^*)^{0.75}(\rho^*)^{0.4} \quad (27)$$

indicates that for all the data points the proposed correlation [Eq.(26)] gives higher heat transfer coefficients. The deviation is as high as up to 23 percent for nine data points and for the rest of the data points the deviation is less than 20 percent. This difference can be attributed to the following factors: (i) using wall values in the damping term of the mixing length results in high thermal diffusivities and higher heat transfer coefficients, (ii) the numerical predictions of Ref. (T.R.Seetharam,G.K.Sharma,1982) indicate that the heat transfer coefficient depends on x while the correlation proposed by Ref. (Protopopov,V.S., Sharma,G.K., 1976)) assumes that it is independent of x, (iii) some of the constants used in turbulence models are the same as those used for constant property fluids which may affect the temperature profile in the boundary layer and hence the heat transfer coefficient and (iv) the scatter of the experimental data of Ref. (Protopopov,V.S., Sharma,G.K., 1976) with respect to their proposed correlation is ± 20 percent.

## Conclusions

Correlations to predict wall-temperature distribution for constant surface heat flux condition for laminar free convection to supercritical carbon dioxide and supercritical water have been proposed based on the numerical solution of the problem using modified Patankar-Spalding implicit scheme reported by Ref.(T.R.Seetharam,G.K.Sharma,1982).The deviations of the data points from the proposed correlation is found to be within ± 10 percent for majority of the cases, whereas for the other cases where the pseudo-critical temperature was within the boundary layer the deviation is between ±10 to ±15 percent. For temperature differences away from the pseudo-critical temperature where the property variations are not severe the proposed correlations agree very well with the constant property correlation of Ref. (E.M.Sparrow, J.L.Greg, 1956) The proposed correlation to determine the wall-temperatures for constant wall-heat flux conditions also takes into account the variation of all the thermo-physical properties of the fluids. This correlation cannot be compared with experimental data as no such data is available at present.

Correlations to find the heat transfer coefficient for turbulent free convection to near-critical carbon dioxide from a plane vertical surface have been proposed both for constant wall-temperature and constant wall-heat flux conditions based on numerical predictions of Ref. Ref. (T.R.Seetharam, G.K.Sharma,1982).The proposed correlations take into account the variations of all the thermo physical properties and cover range of Rayleigh number ($Ra_\infty$) from $1*10^{10}$ to $3*10^{15}$. For constant wall-temperature conditions the deviation between the numerical predictions and values obtained using the correlation given by Eq.(27) was within ± 15 percent except for nine data points. The deviations for these nine data points are between 15 and 17 percent. It is observed that for these nine data points the wall temperatures are found to be close to the pseudo-critical temperature where the fluid properties vary very severely. For constant wall-heat flux conditions the correlations based on Rayleigh number as well as on modified Rayleigh number, $Ra^*_{\infty x}$. have been proposed.

The deviation between the predicted values and the values obtained using the proposed correlations are found to be within ± 9 percent. A correlation [Eq.(26)]based on Rayleigh number on similar lines as proposed by earlier investigators [Eq.(27)] is obtained and the proposed correlation [Eq.(26)] gives higher heat transfer coefficients than those obtained using Eq.(27).The deviation is as high as up to 23 percent for nine data points and for the rest of the data points the deviation is less than 20 percent.

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