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Research Article

Comparison of harmonic analysis of cantilever beam using different finite elements

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Abstract

In this article, comparison of vibration response of a cantilever beam to harmonic forcing using different types of finite elements is performed. The study has been performed using a finite element model of the beam. While modeling the beam, different types of elements are used. Results are compared with the result obtained analytically.

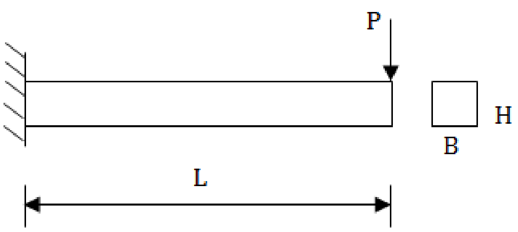
Keywords: *Finite element methods, harmonic analysis, natural frequency, tetrahedral, hexahedral*

1. Introduction

Various mechanical components are subjected to vibration loading such as chassis in automobiles which are subjected to vibrations of engines and many other components. If the frequency of the applied load matches with the natural frequency of component, component might fail because of resonance. Hence it is important to study the behavior using harmonic analysis.

These components can be analyzed by using finite element methods by approximation of the component with various finite elements such as 1 dimensional element, 2 dimensional triangular/rectangular elements and 3 dimensional tetrahedral/hexahedral elements. According to the type of element selected the variation in result is analyzed.

For analysis considering a cantilever beam subjected to harmonic loading at its free end.



Where,

P is applied load	L is length of beam
B is breadth of beam	H is height of beam
E in modulus of elasticity	ρ is density

For this analysis taking following values:
P= cyclic load 100N, frequency range= 0 to 100
L= 1m
B= H= 0.01m
E= 206.8e9N/m2
ρ=7830 kg/m3

3. Theory

In analysis of a component subjected to harmonic loading the maximum deflection of the component will occur when the applied frequency matches the natural frequency of the component.

In this analysis the component is cantilever beam and natural frequency of cantilever beam for different modes of vibration can be found.

3.1 Natural Frequency

Using Euler-Bernoulli Beam Theory we find

$$EI\frac{d^4y}{dx^4} + \rho A\frac{d^2y}{dt^2} = 0 \qquad (1)$$

Normal mode solution to solve the above equation is

$$Y(x,t) = X(x)T(t) \qquad (2)$$

This makes equation (1):

$$\frac{EI}{\rho A}\frac{d^4X}{dx^4}T(t) - X(x)\frac{d^2T}{dt^2} = 0 \qquad (3)$$

Solution for displacement is:

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$X = C_1\cos(x\lambda) + C_2\sin(x\lambda) + C_3\cosh(x\lambda) + C_4\sinh(x\lambda)$ (4)

Where,

$$\lambda = \left(\frac{\rho A}{EI}\omega^2\right)^{1/4} \quad (5)$$

For a cantilever beam, the displacement and slope are zero at the fixed end, while at the free end; the moment and shear are zero. Thus the boundary conditions are:

At x=0,
y=0 and $\frac{dy}{dx} = 0$
At x=L,
$\frac{d^2y}{dx^2} = 0$ And $\frac{d^3y}{dx^3} = 0$

This proves that C_2=-C_4 and C_1=-C_3
Solving for C1 and C2 we get:

$$\cos(\beta L)\cosh(\beta L) = -1 \quad (6)$$

The roots of this equation are

$$\beta_n L = \frac{(2n-1)\pi}{2} \quad (7)$$

The equation of time brakes down into

$$T(t) = b_1 \sin\left[\left(\beta_n^2\sqrt{\frac{EI}{\rho A}}\right)t\right] + b_2\cos\left[\left(\beta_n^2\sqrt{\frac{EI}{\rho A}}\right)t\right] \quad (8)$$

So the frequency in rad/s is:

$$\omega_n = \frac{\beta_n^2}{L^2}\sqrt{\frac{EI}{\rho A}} \quad (9)$$

Converting to Hz, we get the natural frequency as below,

$$f_n = \frac{\omega_n}{2\pi} = \frac{\beta_n^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \quad (10)$$

After substituting the values in above equation the natural frequencies are:

Table 1 Natural frequencies of cantilever beam

Frequency mode	ω_n	f_n
1	52.162	8.302
2	326.974	52.04

3. Analysis

Beam is analyzed for harmonic excitation for frequency from 0 to 100 in steps of 100 by considering various elements as given below.

The element size is kept same for all elements for comparison between element types.

3.1 1D element

1D element

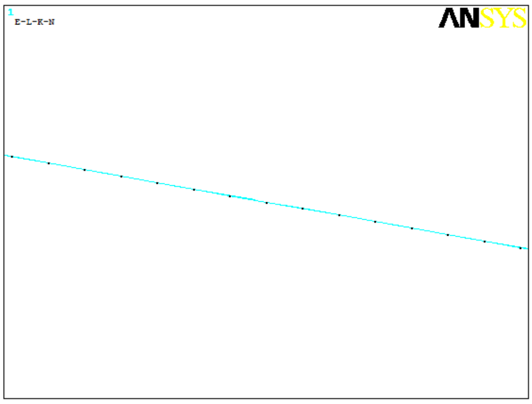

Fig.1 Beam as 1D element

3.2 2D Element

Quadrilateral element

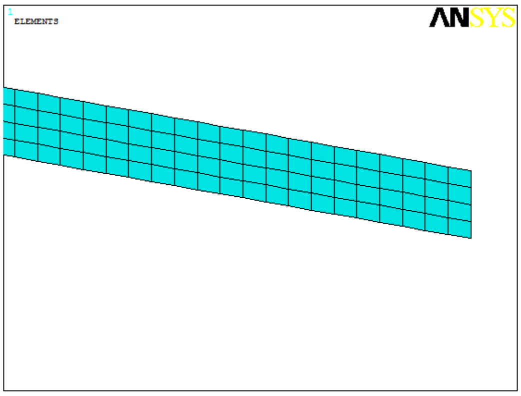

Fig.2 Beam as 2D quadrilateral element

Triangular element

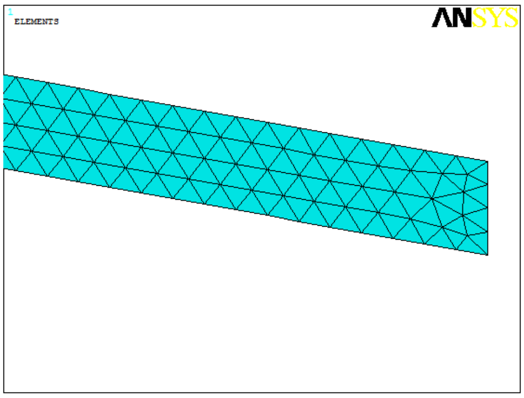

Fig.3 Beam as 2D triangular element

3.3 3D Element

Hexahedral elements

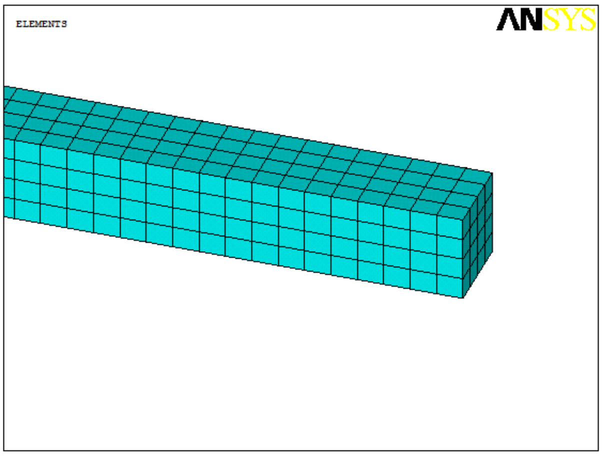

Fig.4 Beam as 3D hexahedral element

Tetrahedral element

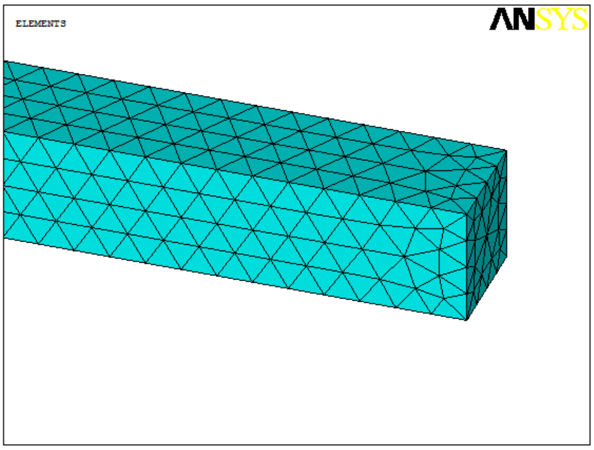

Fig.5 Beam as 3D tetrahedral element

3. Results

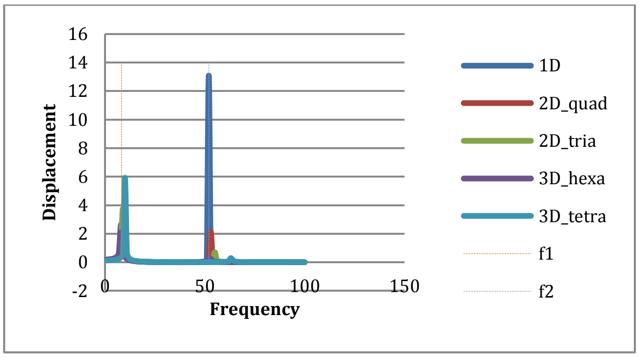

Fig.6 Displacement-frequency plot

The above graph shows displacement of free end of beam in direction of applied harmonic load on the beam with respect to the frequency of the harmonic load from 0-100 Hz.
Peaks in the graph show occurrence of resonance. At these points the frequency matches with the natural frequency. f1 and f2 are the natural frequencies which are calculated analytically.

It can be observed from the above graph that 3D hexahedral elements give result with least deviation from the exact value, while 3D tetrahedral elements give result with highest deviation from analytical natural frequency. Similarly in 2D elements quadrilateral elements give higher accuracy than triangular elements. In this article cantilever beam is analyzed for which 1D element give accurate results.

Conclusion

In this article, the effects of element type on accuracy of finite element models and simulation results were investigated through harmonic analysis. From the results obtained, for harmonic analysis 1D, 2D quadrilateral and 3D hexahedral elements should be used to get accurate results.

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