Timetabling of Rail Services-A Heuristic Approach

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Accepted 15 Jan 2016, Available online 21 Jan 2016, Vol.6, No.1 (Feb 2016)

Abstract

This paper deals with timetabling process of a suburban rail services. The timetabling of suburban rail services in metropolitan cities is very complex one. As the number of trains are very much and services to the customer is critical parameter, The main focus for the scheduling of trains is to provide best services to the customer in terms of minimization of Total Average Waiting Time (TAWT) of the customer at the stations. A case study of western suburban rail services is taken here to demonstrate, how to achieve the goal with all the available current resources & improve the scheduling system by reducing the TAWT of the customer. As there are so many constraints when schedule is to be prepared, we have developed an algorithm to decide the best sequence and starting time of the train during peak hour* and constraint propagation system to decide the running time of the train between stations with arrival and departure time at each station. The technique described here will be very much useful when resources are limited and requirements are heavy for the trains’ services. The fundamental issue covered here is to schedule the routing of the trains, and the algorithm described here may help to schedule other kind of transportation services too.

Keywords: TAWT, Waiting time, Rail, Mumbai, Scheduling of commuter services

1. Introduction

In these days transportation becomes a crucial part of the system. Especially rail transport and its scheduling playing a vital role in the transportation system. Railroads are major mode of transportation for both freight and passenger traffic. There are many aspects of rail transit system, which have positive effects on the communities that they serve.

- The efficiency of rail travel is high compared to roads considering capacity utilization & energy consumption.
- Traveling speed in rail is more than that of road and is less affected by congestion.
- Effect of pollution is due to train is insignificant as compared to vehicles on the road.
- It improves accessibility of places within metropolitan city.

The market share of transport activities enjoyed by the rail industry has been threatened due to competition from other modes of transport. Because of the complex decision making environment faced by railroads, the need is felt for rationalized planning methods to ensure the efficient acquisition and allocation of the resources. The railroads now benefit from powerful management information system. Technologically, the possibility of using analytical models to support decision-making is becoming more attractive. Rail transportation occurs on networks of links corresponding to physical track sections and nodes representing stations.

Traditionally, transportation planners have sought to consider the issue of timetabling as an optimization problem, with the aim of finding the Best possible timetable, under a set of operating and other constraints. The criterion of Best would typically be the maximization of number of services offered, or maximizing the utilization of coaches, or minimization of customer waiting time at each station. This leads to the formulation of complex optimization problems, which are extremely difficult to solve for any reasonable sized commuter network. Since the objectives themselves are not very sharply prioritized and often change from time to time, it is more meaningful to provide the decision maker with a flexible tool which will take care of all the hard constraints and yet leave enough room to build in the trade-off and qualitative properties of the desired solution.

This paper deals with the objective of minimization of TAWT of the customer at each station. The generation of timetable is done by CLP (Constraint Logic Programming) a technique with a representation and fairly complete set of constraints for a given sequence of trains. The process of timetabling is divided in to six modules
1. Generation of sequence
2. Departure time of the train at the origin
3. Calculation of departure time at each station
4. Resolution of clashes if it occurs
5. Rake linking
6. Calculation of waiting time of the customer

The organization of paper is as follows. In section 2 shows the overview of a typical commuter rail, followed by the Western Suburban Railway Commuter System of Mumbai and constraint description. Section 3 shows an algorithm to find out the best possible sequence in the order to minimize the TAWT of the customer. Section 4 shows the conclusion and results

2. Overview of a Commuter Railway System of Mumbai

As the suburban rail system is an important mode of transportation in Mumbai, the scheduling of trains should be made very effective and efficient. Per day there are 942 trains running in up & down direction in Western Suburban Railway. The fundamental issues covered are analysis of the existing timetable, traffic patterns, constraint description etc., to generate the timetable.

The objective is to generate a schedule of the trains such that it reduces the TAWT of the customers, satisfying all the constraints considered here. As waiting time is mainly dependent upon the nature of sequence of patterns [section 2.2], an algorithm is designed here, which gives an effective schedule with less TAWT of the customers than the current schedule followed up by the western suburban railways of Mumbai.

2.1 About Western Suburban Railway in Mumbai

The W.S.R. covers a distance of 60 kms., catering to 27 stations. These are:

- Churchgate - Marine Lines - Charni Road - Grant Road

The W.R. provides services from 4.00 a.m. to 1.00 a.m.

W.R. provides fast, slow & semislow services to cater the needs of the people. Fast services means it will skip some stations (i.e. the train will stop only at those stations which are dark in station description given above). Slow services means that train will stop at each station.

Semislow service refers to a combination of the both, the slow service and fast service. (i.e. it stops at each station between some major stations like Andheri to Borivali; otherwise it skips stations as a fast services).

The distances between stations and timing of arrival & departure are explained in Western Suburban Railway Time Table. It also includes description of track on which the train is travelling, train times of passing stations, the services committed, number of rakes etc.

Peak Hour Period

It has been identified that office hours are peak hours that are spread from 8.00 am to 11.00 am & 5.30 p.m. to 8.30 p.m. (one is for up direction & other is for down direction respectively). During these periods traffic is very heavy and demand satisfaction is critical.

- Track Description

Usually western railway operates the through track, which may be of fast services or slow services. The services are staggered smoothly throughout the service time.

2.2 Analysis of Existing Time Table

The purpose of studying the existing time table is to identify the nature of patterns, headway times, effect of the long distance trains, periodicity, demand, turnaround time, running time between stations, frequency of trains. It also gives information about other operational parameters & the constraints, which have to be considered in the time tabling process.

2.2.1 Pattern Identification

The study revealed that there are a small number of patterns, which covers the stations so as to cater to the needs of the commuters. These patterns are standardized; there will not be any deviation in these patterns. For fast trains the patterns are characterized as under:

Here some patterns are shown for the fast trains.

<table>
<thead>
<tr>
<th>Pattern no.</th>
<th>Destination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>VR Fast</td>
</tr>
<tr>
<td>2</td>
<td>VR Slow</td>
</tr>
<tr>
<td>3</td>
<td>BVI Fast</td>
</tr>
<tr>
<td>4</td>
<td>BVI Slow</td>
</tr>
<tr>
<td>5</td>
<td>ADH Slow</td>
</tr>
<tr>
<td>6</td>
<td>BA Slow</td>
</tr>
<tr>
<td>11</td>
<td>DA-BV Fast</td>
</tr>
<tr>
<td>16</td>
<td>DA-VR Fast</td>
</tr>
</tbody>
</table>

Similarly, the same patterns for the slow trains are as follows. The train stops at all the stations.

<table>
<thead>
<tr>
<th>Pattern no.</th>
<th>Destination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ADH Slow</td>
</tr>
<tr>
<td>2</td>
<td>BA Slow</td>
</tr>
<tr>
<td>3</td>
<td>BVI Slow</td>
</tr>
<tr>
<td>4</td>
<td>BVI Fast</td>
</tr>
<tr>
<td>11</td>
<td>DA-BV Fast</td>
</tr>
<tr>
<td>16</td>
<td>DA-VR Fast</td>
</tr>
</tbody>
</table>
2.3 Constraint Description

There are a number of different constraints, which have to be considered during construction of timetable of a western suburban rail system.

1. Headway: This imposes the time restrictions between consecutive two trains on the same track. The minimum headway is 3 minutes for W.S.R. The headway varies when a long distance train passes through the same track because it takes more time for acceleration and deceleration.

2. Slack: A slack has to be provided in regular intervals. So as to have a cushioning effect during some time intervals for example, when the long distance train passing on the through track an allowance should be added to the local train passing on the local train since it affects its speed.

3. Turnaround Time: The standard turnaround time is taken to be four to eight minutes but this time can be varied so as to regulate the incoming trains in the starting terminal, for example the trains at Virar should be turnaround depending on the needs of the Churchgate Terminal, so that the balance in incoming and outgoing train can be maintained.

4. Linking Constraint: This imposes a constraint such as the fast trains of down direction are linked as other outgoing fast/slow train in up direction, and same way slow trains of down direction are linked as other outgoing slow/fast train in up direction or vice versa.

5. Conflict: This includes the rules such as, if a train is a semislow service, then the conflicts arise when the train switch over from the fast track to slow track or some long route train occupies the fast track then there should not be any fast train with the same timing at the different stations. The necessary measures to be taken during the track switching over process.

6. Stabling: The constraint gives the stabling capacity of each terminal. The stabling capacity of each a terminal is that terminal can hold the trains if it is not going to run (beyond turnaround time limit) for some time period.

7. Demand Constraint: The railway commits the numbers of services to the destinations during a particular time period to satisfy the needs of the customers. On the other way we can say that there are some amount of trains should be run within a particular time period. This forms a vital constraint in the time tabling process.

8. Traffic & Signalling allowance: Allowances are additional time requirement in addition to slack, to provide cushion effect from any uncertainty. This may keep schedule unchanged, consistent and reliable.

3. Sequence Generation

First of all here are some useful parameters defined which will be used in the algorithm proposed ahead.

Sequence of patterns: As there are fair number of trains with particular pattern during peak hour period, the order of the trains that are going to run within that time period is called sequence of pattern.

Arrival rate of customer: $\lambda_k$ is the number of customers arriving at particular station k per unit of time period (per minute) with uniform rate.

Average waiting time of the customers: The average total number of customers waiting for some time period, to get in to the train at particular station k i.e. the time of waiting of the customers between every two consecutive trains halt at the station k.

Average total waiting time of the customers: It is a total sum of average waiting time of customers over total number of stations.

3.1 Minimisation of waiting time of the customer

As defined earlier rate of customers arriving at station k is $\lambda_k$. now the total number of customers arriving at the station can be calculated as below.

Total number of customers at station k = $\sum \lambda_k \ast (t_{i+1,k} - t_{ik})$ …… 3.1.1

As the arrival rate of customer is uniform.(see figure)

From the figure above it can be seen that the total number of customers are waiting for $t_{i+1,k} - t_{ik}$ time period at station k. But each customer may not have to wait for the same time period so, average time period
is to be taken into consideration for which customers are waiting at particular station \( k \) (that is \( 1/2 \) of \( t_{i+1,k} - t_{ik} \)).

The average waiting time at station \( k \) for the customers can be defined as under:

Average waiting time of the customers at station \( k = \) total number of customers arrived at station \( k \) * average time period of which they are waiting.

\[
W_k = \sum_{i} 1/2 \gamma_k (t_{i+1,k} - t_{ik})^2 \quad \forall \ i
\]

TAWT of the customers

\[
W = \sum_{k} \sum_{i} 1/2 \gamma_k (t_{i+1,k} - t_{ik})^2 \quad \forall \ k \ \forall \ i
\]

Let’s examine the two sequences of patterns, for average waiting time at station \( k \).

There are total 3 train stops at station \( k \), and its departure times are \( t_{ik}, t_{i+1,k} \) and \( t_{i+2,k} \)

Case I

The trains are halting at station \( k \) by approximately equal time duration (see figure, \( a=b \)).

Case II

The trains are halting at station \( k \) by unequal time period (see figure, \( a \neq b \)).

If \( t_{i+1,k} = t_{ik} + a \) and \( t_{i+2,k} = t_{ik} + b \)

So, \( t_{i+2,k} = t_{ik} + a + b \)

here \( a, b \) are fixed as total time period is remain same.

Average waiting time of customers at \( k \),

\[
W_k = 1/2 \gamma_k (t_{i+1,k} - t_{ik})^2 \quad \forall \ i
\]

\[
= 1/2 \gamma_k (t_{i+1,k} - t_{ik})^2 + (t_{i+2,k} - t_{i+1,k})^2
\]

\[
= 1/2 \gamma_k (a^2 + b^2)
\]

The average waiting can be minimized if \( a = b \)

From the above example it can be evaluated that the TAWT at station \( k \) for the case I is less than the waiting time for the case II. More over customers will be more satisfied because of average rush in case I, while in II case during the time interval of \( t_{i+2,k} - t_{i+1,k} \) large number of customers are gathered at station \( k \) and due to heavy rush they will be unhappy. As a result if trains are distributed by same time interval, then the TAWT will be less as well as more customers satisfaction can be achieved. For the different set of sequence of patterns the TAWT of the customers will be different. For example if trains having same type of pattern are assigned together along a sequence then TAWT will be higher. On the other case if trains having same type of pattern are uniformly distributed along a sequence then the TAWT will be lower.

From the above discussion it implies that the waiting time is dependent on nature of sequence of the patterns, i.e. the waiting time can be calculated if we have a sequence of patterns and departure times of the trains at particular station. One way is to examine all the possible number of sequence of patterns but it will take more time to get the best sequence out of them. So, some initial sequence can be picked up and iterate such that it converges to the some local minimum of TAWT from the set of total possible sequence of patterns. But as the parameters like, departure times at each station, detail information of patterns (where the train is halting) should be known to us initially (which is not available) and the TAWT of the customers varies with the square difference between the departure timings of two consecutive trains (fast/slow), which stops at station \( k \). More over after making a schedule clash free the timings will be changed to some extent. So, it would be feasible to convert it in to linear form with less parameter.

As we know that the \( \gamma_k \) constant the \( W_k \) is dependent on the nature of sequence of patterns, from the two cases described above if the trains halt at particular station at some equal time period then the TAWT will be less. Similarly pattern of trains is nothing but the halting information of the trains at different stations and the service level. So, the pattern should be spread out on the time span as uniform as possible. Here a heuristic is proposed to find out the good feasible sequence. It just looks at the pattern followed by the different trains; the departure times at each station need not to be considered.

3.2 Heuristic Approach

Here are some criteria to generate good feasible sequence.
1. The same patterns should be spread out in the sequence because the waiting time can be reduced to a significant amount.
2. Demand of the customer should be satisfied.
3. rake availability (for the case of rake of 9 & 12 coaches).
4. Other (like weather condition, holiday schedules, track maintenance etc.).

Here the method is proposed keeping first three criteria described above.

Assumptions

1. All the patterns are independent of each other means there is no relation between any pattern considering demand, service level, i.e. particular train like BVI slow and ADH slow has different demand and similarly for all other pattern also.
2. Fast & Slow trains are also treated as independent of each other although their destination & termination points are same i.e. CCG - BVI fast & CCG - BVI slow both are treated as different trains considering their demand, customers, service level.
3. Long route trains are not taken in to the account because it has fixed departure time & customers are also different. (That can be taken in to account later at the time of deciding tentative departure times of the trains).

To describe the procedure we have taken some of trains from period 18:30 – 19:30.

Fast:
Pattern name no. of trains Pattern_no
CCG - BVI 1 3
CCG - VR 3 1

Slow:
Pattern name no. of trains Pattern_no
CCG - BVI 4 4
CCG - ADH 2 5

Number of trains are to be taken as input in decreasing order of their frequencies because from the method suggested below it can be seen that score with respect to particular train is higher at the ends of the sequence. So, if the scores with respect to particular train is arranged in descending order of it’s value then each train has an good opportunity to prove its existence in the sequence of patterns as best as possible (it will be distributed uniformly). Ex. from the table it can be seen that there are higher scores at the ends so if that are arranged in descending order then all the patterns which are clustered together will be distributed alternately, and further iteration gives a uniform distribution of the patterns.

Following sequence of pattern is taken as input.

4 4 4 4 1 1 1 5 5 3

Step 1: Input the sequence of pattern in descending order with respect to the frequency of patterns as above.

Pattern number, X(i) = j
Where i = index of the train, from 1 to n (total number of trains)

j = index of the pattern, from 1 to j (total number of patterns)

Step 2: Compare the different patterns with other patterns in the sequence, For i = 1 to n

Where, n = total no., of trains

for k = 1 to n

if X(i) = X(k)

score = mode( k - i ) * w ( w is the weightage factor that is 1 if the same pattern and 2 if the pattern is different ).

else score = mode(k-i) * 2

for each train i with particular pattern j we will have a score.

Step 3: Find the score of each train i having particular pattern j. Arrange them in descending order. Simultaneously arrange corresponding train with particular pattern to its score.

Step 4: Now another set of sequence is obtained. Again find out the score of a new sequence of pattern with the method suggested above.

Step 5: If the score for the previous sequence of pattern is less than new sequence of pattern generated then go to step 6, otherwise go to step 2 with new order of sequence of patterns.

Step 6: Print the previous sequence of patterns (which can be a local minima of the total possible number of sequences).

Step 7: Any interchanges if required then apply.

Method

<table>
<thead>
<tr>
<th>Pattern No.</th>
<th>Total</th>
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<tr>
<td>Trn No.</td>
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</table>

To

<table>
<thead>
<tr>
<th>From</th>
<th>Number of Trains</th>
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<tbody>
<tr>
<td>1</td>
<td>2</td>
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<td>3</td>
<td>4</td>
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<td>5</td>
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Number of Trains

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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tbody>
<tr>
<td>From</td>
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Pattern

<table>
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<tr>
<th>Trn No.</th>
<th>Pat No.</th>
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Total

<table>
<thead>
<tr>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>630</td>
</tr>
</tbody>
</table>
Tabulation for the first raw is shown as below

<table>
<thead>
<tr>
<th>Service number</th>
<th>Pattern</th>
<th>Weightage</th>
<th>Score</th>
</tr>
</thead>
<tbody>
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<td>4</td>
<td>1</td>
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<td>9</td>
<td>5</td>
<td>1</td>
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</tr>
<tr>
<td>10</td>
<td>5</td>
<td>1</td>
<td>592</td>
</tr>
</tbody>
</table>

Now arrange the score with respect to each train in to descending order. That will give a new sequence 3 4 5 4 5 4 1 4 1 1 with score 604, and do the same procedure until minimum score is obtained, that is called as local optima from the set of total number of the sequence of patterns. The procedure is shown as below:

Sequence of patterns | Score
---|---
Input | 4 4 4 4 1 1 5 5 3
1\(^{st}\) iteration | 3 4 5 4 5 4 1 1 604
2\(^{nd}\) iteration | 3 1 1 4 5 4 1 5 4 5 92
3\(^{rd}\) iteration | 3 4 1 4 5 1 5 1 4 5 78
4\(^{th}\) iteration | 3 4 4 1 5 1 5 1 4 5 92

From the above sequences it can be seen that at 3\(^{rd}\) iteration good feasible sequence has obtained. Now compare all the sequences with respect to TAWT (using eq 3.1.3).

Sequence of patterns | TAWT
---|---
1\(^{st}\) iteration | 3 4 5 4 5 4 1 1 571 hr:59min
2\(^{nd}\) iteration | 3 1 1 4 5 4 1 5 4 54 971 hr:51min
3\(^{rd}\) iteration | 3 4 1 4 5 1 5 1 4 819 hr:24min
4\(^{th}\) iteration | 3 4 4 1 5 1 5 1 4 853 hr:20min

From the above calculations of TAWT of the customers it implies that the sequence which has lower score also has lower waiting time.

Advantages

1) It gives a local optima from the set of all possible sequences of patterns.
2) The computation time is very less.
3) Any interchanges can be made very easily if required any (i.e. during second phase if any unfeasibility found then patterns can be interchanged in the sequence).

Disadvantage

1) It doesn’t give the sequence of pattern with least score among all possible number of sequence of patterns.
2) Some other approaches like simulated annealing or genetic algorithm may give better solution.

3.3 Waiting time measurement

As described before, reducing the TAWT is the main objective. Here that parameter is taken as a performance measure of the schedule. To decide a goodness of the sequence it needs to be compared the waiting time of the current schedule followed by the western suburban railways with the schedule generated by us. This algorithm should be applied after generating a clash free time table means*. There are two approaches to find out TAWT of the sequence.

* Clash free time table can be obtained by resolving the constraints stated in section 2.3. Initially just assign the departure time to the trains at their origin and calculate the arrival time and departure time of the trains at each station using various data like runtime, headway, slack, track info etc. For more details see Tabulation of Rail Services, by Hemant Trivedi, M.Tech dissertation 1999, IIT Bombay

3.3.1 Approach 1

From the discussion made before it is assumed that the customer arrival rate is uniform and the criteria for them to get in to the train is depends upon level of service.

It means \( \lambda_{kt} = \) Uniform arrival rate of customers who wants to go from station k to t.

As from the schedule, figure out the departure times of each train, the nature of patterns, which gives a halting information about trains at all the stations. Accordingly multiply \( \lambda_{kt} \) to find out total number of customers and then find out waiting time using the same method described before.

The calculation can be carried out as under,

\[
TAWT = \sum \sum \frac{1}{2} \lambda_{kt} \times X_{i+1,k,t} \times (t_{i+1,k} - t_{ik})^2, \quad \forall i, \forall t
\]

\( X_{i+1,k,t} = 1 \) if train i halts at station k and t.

\( = 0 \) otherwise

\( X_{i+1,k,t} \) is dependent upon the nature of a pattern of trains.

As seen from the equation above calculation becomes much complicated. So, an another approach is proposed here, which is same as before explained in section 3.1 before.

3.3.2 Approach 2

The patterns of all the trains are more or like similar and covering almost all stations (specifically slow trains). More over some trains are subset of others i.e. ADH -Slow is a subset of BVI - Slow and BVI - Slow is a subset of VR - Slow customers doesn’t have a specific choice to get in to the train.

An other important factor is, customers are well aware of the timings of trains so they don’t wish to wait for long time i.e. customers who wants to go to Virar are coming around the timing of such Virar trains similarly for other trains also.
So, a uniform arrival rate of customers \( \lambda_k \) (arrival rate of customer at station \( k \) per minute) is taken in to consideration that gets in to the train no matter where the train is going. Although it may not give us exact waiting time but by keeping some higher \( \lambda_k \) closer measurement of average total waiting time can be obtained. Another reason to keep some higher \( \lambda_k \) is that some customers are not get in to the train because of heavy rush and unawareness of the western suburban railway system (specially visitors).

Now, the calculation of the waiting time can be done as under

\[
TAWT \, W = \sum \left[ \sum \left( \frac{1}{2} \left( t_{i+1,k} - t_{ik} \right)^2 + \lambda_k \right) \right] \] 
\[
X_{i+1,k} = 1 \text{ If train } i \text{ halts at station } j.
\]
\[
X_{i+1,k} = 0 \text{ otherwise.
}\]

This is a simpler way to find out TAWT of customers for whole schedule. As the detail of pattern of the trains is available, the following steps lead to find out the TAWT of the customers. The procedure is to be used for both, the current schedule followed up by the western suburban railways and the proposed schedule generated by us for the purpose of comparison.

**Step 1:** As an input the departure time \( t_{ik} \) of the trains in increasing order at each station is taken here from the schedule, no matter whether it is fast or slow. If train does not stop at particular station then enter the same time \( t_{ik} \) of previous train.

And customer arrival rate at station \( k, \lambda_k \).

**Step 2:** For each station \( k \), compare the timing of all the trains \( (1 \text{ to } n) \) which stops at that station.

for \( k = 1 \text{ to } K \)
for \( i = 1 \text{ to } n \)

\[
W_k = \sum \left( \frac{1}{2} \left( t_{i+1,k} - t_{ik} \right)^2 + \lambda_k \right)
\]

find \( \sum W_k \), \( \forall n \)

the average waiting time for all the customers at station \( k \).

**Step 3:** Summarize the average waiting time of all stations, that will give the TAWT of the customers for the given schedule.

Compare the waiting time of current schedule followed by W.S.R. and the schedule generated by the algorithm. If waiting time of the schedule generated by us is less than the current schedule followed up by W.S.R., it implies that the schedule generated here is better than the current one. Otherwise change in the sequence of patterns, followed by making clash free schedule and again find out the TAWT of the schedule until a better schedule is generated.

**Conclusion**

As timetabling of rail services for the western suburban railways has been taken as a problem, the system has been analysed as better as possible to design a new schedule for the peak hour period. Solution of the constraints like headway, slack, demand, conflicts, platform occupancy, rake linking & turn around time, stabling, rake availability are not described here.*

In this paper it is assumed that enough rakes are available and stabled at Churchgate. So one can generate a schedule incorporating both the criteria (rake linking and stabling) and make it more reliable & consistent. Although if an algorithm is used for long time period them the rakes will be dispersed among all the terminal from where train starts & ends (partially solves both the constraints).

The algorithm designed to find out a good feasible sequence will work for large amount of data also, but it will not give the optimal schedule. So one can try for generation of optimal schedule. The tentative departure time calculation is mainly based on demand of the trains (i.e. here all the trains are scheduled in as minimum as possible time period). As the average waiting time of the customer is the main objective, it has been reduced by 2536 hr : 23 min for the peak hour period. As seen from the implementation details the average waiting time of the customers up to Borivali for the proposed schedule by us is less than the current schedule*, but this is not the case from Borivali to Virar. To reduce the average waiting time of the customers from Borivali to Virar it needs to run some more trains from Borivali to Virar as that track is not occupied heavily.

In future some other tools like CHIP can be used to generate schedule more effectively as it has powerful in build constraint solving capacity. Also, the constraints which are not considered can be incorporated to make schedule more realistic.

**References**


Western Railway Working Time Table No. 61 Bombay Division Suburban Section-1995.

Western railway timetable for long route trains (1997).