Research Article

Timetabling of Rail Commuter Services using Constraint Logic Programming

Trivedi H.S.†*
†Shantilal Shah Engg. College, Sidsar, Bhavnagar, India

Accepted 02 Jan 2016, Available online 07 Jan 2016, Vol.6, No.1 (Feb 2016)

Abstract

Commuter railway services in cities such as Mumbai are characterized by a large number of services admitting short headways of a few minutes between successive services on the same track. So the timetabling of commuter services in railway operations has certain unique features that make it a very challenging task. This paper aims to demonstrate the viable use of constraint logic programming (CLP), to generate reasonable timetables under a very general framework of constraining conditions. CLP is a powerful programming paradigm which combines the ease of specification and modeling of logical relationships provided by PROLOG, with the general framework of constrained systems specified by linear inequalities. Traditionally, transportation planners have sought to consider the use of timetabling as an optimization problem, with the aim of finding the “Best” possible timetable, under a set of operating and other constraints. The criterion of “Best” would typically be the maximization of number of services offered, or maximizing the utilization of coaches or minimizing the number of coaches required to operate a certain fixed number of services, etc. This leads to the formulation of complex optimization problems, which are extremely difficult to solve.

Keywords: Constraint Logic Programming, PROLOG, Rail Scheduling, Mumbai

1. Introduction

It deals with a concrete example of applying CLP(R), a Constraint Logic Programming tool, as a technique for supporting the timetabling of commuter rail services with a representative & fairly complete set of constraints.

The constraints considered include

Headway

This imposes the time restrictions between consecutive two trains on the same track. The minimum headway is 3 minutes for W.S.R. The headway varies when a long distance train passes through the same track because it takes more time for acceleration and deceleration.

Slack

A slack has to be provided in regular intervals. So as to have a cushioning effect during some time intervals for example, when the long distance train passing on the through track an allowance should be added to the local train passing on the local train since it affects its speed.

Demand

The number of services to the destinations is committed by the railway during a particular time period to satisfy the needs of the customers. On the other way we can say that there are some amount of trains should be run within a particular time period. This forms a vital constraint in the time tabling process.

The above constraints can be formulated with one of the objective like maximization of number of services offered during specified time period as below using CLP(R).

2. Modelling using CLP(R)

CLP(R) is shown to be a valuable modeling and computational tool for the problem of timetabling in view of its general utility, which arises from its logic programming capability and particularly valuable facet of its constraint specification and propagation. The general suitability is because of the elegance and ease of implementation of both algebraic and logical constraint verification. CLP(R) allows very efficient searches of decision spaces just as Prolog does.

The particular applicability of CLP(R) in the timetabling context comes about for the following reason. The process of timetabling follows a pattern of
tentatively specifying a set of variables in some order. This order is typically not known beforehand. The relationships between decision variables are not specified as assignments, but as constraints.

If some $S_{ij}$ and some $F_{ij}$ values are specified, the timetable would perhaps desire to constraint other values appropriately, to obtain time windows in which to schedule other services. The CLP representation enables this to be done for any partial set of specified variables.

For example, with the above system of constraints, with $S_{11}$, $S_{12}$, $S_{21}$, $S_{22}$, $F_{11}$, $F_{12}$, $ST_1$ and $FT_1$ specified an appropriate bound on the value of $ST_2$ can be obtained.

$$S = \text{for slow} \quad i = \text{No of train} $$

$$F = \text{for fast} \quad j = \text{No of Pattern **(Churchgate-Borivali or Churchgate-Virar etc)}$$

The advantage is that these multiple scenarios can be evaluated with these the same representation of the constraints linking the $S$, $F$, $ST$, and $FT$ variables.

3. Constraint representation

We have experimented with two ways of representing the commuter timetabling problem using CLP(R) Since they have their merits, both these styles have been combined in the actual experimentation. Using prolog language we can formulate different constraints as below.

**Scheme 1:**

Prolog permits the specification of clauses with attributes and allows the user to query based on matching of these clauses or simple conditions among the various attributes. For example, the clause

$$S_{local} (\text{track_no}, \text{train_no}, \text{time_at_stn}, \text{station_name}, \text{service_no})$$

could be constructed to represent a train (numbered train_no) on the slow track (i.e. track track_no) at a particular station (station name) at time (time_at_stn). The service number service_no is number specifying the time wise ordering of all trains on that track.

A simple comparison or linear conditions on the attributes could be checked for feasibility of a given set of clauses.

These clauses have to be derived from the values of the primary decision variables $S_{ij}$, $ST_i$, $F_{ij}$ and $FT_j$. Depending on how the CLP(R) programme is used, these decision variables could be specified either in the program or outside. In either case, generating these clauses is straightforward, given the running times between stations and the standard patterns followed by trains on a track.

The advantage of this scheme is that the clauses can be generated from a variety of other considerations and can be specified as constraints, without needing to explicitly model those decisions. This provides the flexibility to specify occasional movements (not following the regular periodic frame work) temporary unavailability of track at some stations at some time, as well as the conflict between fast and slow trains on the slow track without disrupting the long term operating conditions.

The limitation of this approach is that this representation is useful primarily as a feasibility check and is not flexible enough to allow different partial specifications of the schedule and derive good alternatives for the remaining choices. This is the heart of the matter and leads us to scheme 2. A detailed example of this time is given below:

**Scheme 2**

The second approach is to construct the linear system of inequalities and equalities directly, using the decision variables. There is considerable scope for modeling the various constraints using the choice of variables that we have made. We can include logical constraints between variables and explicit integrality (usually 0-1 constraints) but the basic search is efficient only when the associated linear programming relaxation is effective.

In our problem, the specification of the linear system takes the form of groups of constraints for headway, slack, demand, periodicity etc. The example given below makes it clear.

4. Examples and computations

**Example_1**

This example follows Scheme 2 described above. It considers two patterns each for both fast trains and slow trains, and three trains for each pattern. Thus $S_{11}$ represents the fact that the first slow train follows the first pattern and $FT_1$ represents the time of departure of the first fast train. $FT_1$ to $FT_3$ represents the departure time of the fast trains and $ST_1$ to $ST_3$ represent the departure time of the slow trains. For convenience, the subscripts are omitted in the actual code.

The program considers the headway constraints, slack constraints and demand constraints.

The user has the following choices.

- To have one or more unknown in the schedule.
- To have one of more unknown in the departure times.
- To have both the combinations.
The general form of the Query is

\[ \text{Solve } (F_{11}, F_{12}, F_{21}, F_{22}, F_{31}, F_{32}, S_{11}, S_{12}, S_{21}, S_{22}, S_{31}, S_{32}, FT_1, FT_2, FT_3, ST_1, ST_2, ST_3) \]

We now consider examples where queries are asked to the system according to one of the considerations above.

Constraint Representation for the Timetabling Problem:

\[ \text{Solve } (F_{11}, F_{12}, F_{21}, F_{22}, F_{31}, F_{32}, S_{11}, S_{12}, S_{21}, S_{22}, S_{31}, S_{32}, FT_1, FT_2, FT_3, ST_1, ST_2, ST_3): \]

- Headway (FT_1, FT_2, FT_3, ST_1, ST_2, ST_3):
  \[ \begin{align*}
  &\text{FT}_2 - \text{FT}_1 \geq 3 \\
  &\text{ST}_2 - \text{ST}_1 \geq 3
  \end{align*} \]

/* The above constraints specify headway of at least 3 minutes between successive trains on the same track. */

- demand (F_{11}, F_{12}, F_{21}, F_{22}, F_{31}, F_{32}, S_{11}, S_{12}, S_{21}, S_{22}, S_{31}, S_{32}, FT_1, FT_2, FT_3, ST_1, ST_2, ST_3):
  \[ \begin{align*}
  &F_{11} \geq 0, \quad F_{11} \leq 1 \quad F_{21} \geq 0, \quad F_{21} \leq 1 \quad F_{31} \geq 0, \quad F_{31} \leq 1 \\
  &F_{12} \geq 0, \quad F_{12} \leq 1 \quad F_{22} \geq 0, \quad F_{22} \leq 1 \quad F_{32} \geq 0, \quad F_{32} \leq 1 \\
  &S_{11} \geq 0, \quad S_{11} \leq 1 \quad S_{21} \geq 0, \quad S_{21} \leq 1 \quad S_{31} \geq 0, \quad S_{31} \leq 1 \\
  &S_{12} \geq 0, \quad S_{12} \leq 1 \quad S_{22} \geq 0, \quad S_{22} \leq 1 \quad S_{32} \geq 0, \quad S_{32} \leq 1
  \end{align*} \]

/* The previous constraints contain the linear relaxation of the 0-1 indicator variables for the pattern decisions. */

- \[ F_{11} + F_{21} + F_{31} = 1 \] /* Exactly 1 fast train of pattern 1 */

- \[ F_{12} + F_{22} + F_{32} = 2 \] /* Exactly 2 fast trains of pattern 2 */

- \[ S_{11} + S_{21} + S_{31} = 2 \] /* Exactly 2 slow trains of pattern 1 */

- \[ S_{12} + S_{22} + S_{32} = 1 \] /* Exactly 1 slow trains of pattern 2 */

- \[ F_{11} + F_{12} = 1 \] /* These constraints stipulate that a train can either be of pattern 1 or pattern 2 */

- \[ F_{21} + F_{22} = 1 \]

- \[ F_{31} + F_{32} = 1 \]

- \[ S_{11} + S_{12} = 1 \]

- \[ S_{21} + S_{22} = 1 \]

- \[ S_{31} + S_{32} = 1 \]

As timetabling of rail services for the western suburban railways has been taken as a problem, the system has been analyzed as better as possible to design a new schedule for the peak hour period. Constraints like headway, slack, demand, conflicts, platform occupancy, rake linking & turnaround time considered. But some constraints are not taken in to consideration like stabling, rake availability.

Here it is assumed that enough rakes are available and stabled at Chuchgate. So one can generate a schedule incorporating both the criteria (rake linking and stabling) and make it more reliable & consistent. Although if an algorithm is used for long time period then the rakes will be dispersed among all the terminal from where train starts & ends (partially solves both the constraints).

**References**


