Image Denoising using Fast Bilateral Filter

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Abstract

Image de-noising is the process of mapping a noisy image to a noise free image. The major problem of image denoising algorithms is that edges are vanished after the de-noising process. The bilateral filter is a non linear filter that does spatial averaging without smoothing edges. It consists of a spatial kernel as well as a range kernel to limit averaging to the neighbourhood pixels having similar intensity. The range kernel operates by acting on the pixel intensities. This makes the averaging process non linear and computationally intensive, mainly when the spatial filter is large. Those algorithms whose execution time is independent of size and shape of filter are commonly known as constant time or O(1)algorithm. Here we will see how averaging algorithms can be leveraged for realizing the bilateral filter in constant time, by using trigonometric range kernels.

Keywords: Bilateral Filter, Image Denoising, Constant time filter, Spatial domain filters

1. Introduction

Digital images play a very significant part, both in applications such as magnetic resonance imaging, computer tomography as well as in the field of science and technology such as geographical information system and astronomy. The data collected by image sensors and other devices are generally contaminated by noise. Noise can also be introduced due to transmission errors and compression. Hence denoising is often a necessary and first step to be performed before image data is analysed and processed. An efficient de-noising technique must be applied to compensate for such data corruption. Image de-noising remains as a significant challenge for researchers because de-noising process removes noise but introduces artefacts and also causes blurring.

2. Bilateral Filter

A bilateral filter is a non-linear, edge-preserving and noise-reducing smoothing filter for images. The intensity value at each pixel in an image is replaced by a weighted average of intensity values from nearby pixels. This weight can be based on a Gaussian distribution. Crucially, the weights depend not only on Euclidean distance of pixels, but also on radiometric differences (e.g. range differences, such as color intensity, depth distance, etc.). This preserves sharp edges by systematically looping through each pixel and adjusting weights to the adjacent pixels accordingly. Its ability to decompose an image into different scales without causing haloes after alteration has made it global in computational photography applications such as tone mapping, style transfer, relighting, and denoising.

Bilateral filtering algorithm is a non-linear and non-iterative image de-noising method in spatial domain which makes use of the spatial information and the intensity information between a point and its neighbours to smooth the noisy images while preserving edges well. The bilateral filter is preferred for one unique reason: It reduces noise while preserving details. The bilateral filter embodies the idea of a grouping of domain and range filtering. The domain filter averages the nearby pixel values and thus acts as a low-pass filter. The range filter stands for the non-linear component and plays a significant part in edge preserving. This component allows averaging of similar pixel values only, despite their position in the filter window. If the value of a pixel in the filter window diverges from the value of the pixel being filtered by a certain amount, the pixel is skipped.

The bilateral filtering of an image \( f(x) \) is given by,

\[
\tilde{f}(x) = \eta^{-1} \int w(x, y) \phi(f(x), f(y)) f(y) dy
\]

where

\[
\eta = \int w(x, y) \phi(f(x), f(y)) dy
\]

Here \( x \) denotes the pixel of interest and \( y \) denotes the neighboring pixel. The term \( w(x, y) \) denotes the

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geometric proximity between \( x \) and \( y \). The function \( \phi(f(x), f(y)) \) measures intensity values of the pixels \( x \) and its neighbor \( y \). Here \( \eta \) indicates the normalizing factor used to preserve local mean.

We have \( w(x,y)=w(x-y) \) since it is translation invariant. The range filter depends on intensity difference \( \phi(f(x), f(y))=\phi(f(x) \cdot f(y)) \)

Thus the filter is given by

\[
\tilde{f}(x) = \eta^{-1} \int w(y)\phi(f(x - y) - f(x))f(x - y)dy
\]

Where

\[
\eta = \int w(y)\phi(f(x - y) - f(x))dy
\]

Here \( w(x) \) represents the spatial kernel and \( \phi(s) \) denotes the range kernel. The local support of spatial kernel that consists of the neighborhood over which the averaging takes place is indicated using the symbol \( \Omega \). The spatial and range kernel are usually gaussian.

For the bilateral filter in (1), the difference \( f(x-y) \)-\( f(x) \) is close to zero in homogenous regions, and hence \( \phi(f(x-y)-f(x))=1 \). In this case, (1) simply results in the averaging of pixels in the neighborhood of the pixel of interest. On the other hand, if the pixel of interest is in the surrounding area of an edge, \( \phi(f(x-y)-f(x)) \) is large when \( x-y \) belongs to the same side of the edge as \( x \) and is small when \( x-y \) is on the other side of the edge. As a result, the averaging is limited to neighborhood pixels that are on the same side of the edge as the pixel of interest. This is the basic idea that allows one to carry out smoothing while preserving edges at the same time.

Since its beginning, the bilateral filter has found extensive use in several image processing, computer graphics, and computer vision applications. This includes denoising, video abstraction, demosaicing, optical flow estimation, and stereo matching, to name a few.

It is the presence of term \( \phi(f(x-y)-f(x)) \) in (1) that makes the filter nonlinear. In the absence of this term, that is, when \( \phi(s) \) is constant, the filter is simply given by averaging

\[
\tilde{f}(x) = \int w(y)f(x - y)dy
\]

Our present idea is to leverage these fast averaging algorithms by expressing (1) in terms of (3), where the averaging is performed on the image and its simple point wise transforms.

Our observation is that we can do so if the range kernel is of the form

\[
\phi(s) = \cos(gs), -T \leq s \leq T
\]

By plugging (4) into (1), we can write the integral as

\[
\cos(gf(x)) \int w(y)\cos(gf(x-y))f(x-y)dy
\]

\[
+ \sin(gf(x)) \int w(y)\sin(gf(x-y))f(x-y)dy
\]

This is clearly shown to be the linear combination of two spatial averages, performed on images \( \cos(Yf(x))f(x) \) and \( \sin(Yf(x))f(x) \). Similarly, we can write the integral in (2) as

\[
\cos(gf(x)) \int w(y)\cos(gf(x-y))f(x-y)dy
\]

\[
+ \sin(gf(x)) \int w(y)\sin(gf(x-y))f(x-y)dy
\]

In this case, the averaging is on images \( \cos(Yf(x)) \) and \( \sin(Yf(x)) \). This is the trick that allows us to express (1) in terms of linear convolution filters applied to point wise transforms of the image.

3. Proposed Work

Algorithm

Input: Image \( f(x) \)

\( \sigma^2_s \): Variance of spatial kernel
\( \sigma^2_r \): Variance of Range kernel

The range kernel is defined as,

\[
\phi(s) = \cos(gs) \quad N = \cos\left(\frac{\pi s}{2T}\right)
\]

Dynamic Range \([-T, T]\)

Set \( \gamma = \pi / 2T \) and \( \rho = \gamma \sigma_r \)

Calculate the maximum Dynamic Range \( T \)

\[
N = (\gamma \sigma_r)^{-2} = 0.405 \left(\frac{T^2}{\sigma_r^2}\right)
\]

For \( 0 \leq n \leq N \), set up images

\[
h_n(x) = \exp(j\gamma(2n-N)f(x)) / \rho \sqrt{n}
\]

\[
g_n(x) = f(x)h_n(x) \quad \text{and coefficients}
\]

\[
d_n(x) = 2^{-N} (N\mathbb{C}) \exp\left(-j\gamma(2n-N)f(x) / \rho \sqrt{n}\right)
\]

Filter \( h_n(x) \) and \( g_n(x) \) with a variance \( \sigma^2_s \) to get \( \tilde{h}_n(x) \) and \( \tilde{g}_n(x) \)

\[
\tilde{f}(x) = \frac{\sum_{n=0}^{N} d_n(x) \tilde{g}_n(x)}{\sum_{n=0}^{N} d_n(x) \tilde{h}_n(x)}
\]

Return: Filtered Image \( \tilde{f}(x) \)
4. Methodology and Design

Here our aim is to bring the bilateral filter in nonlinear form to the linear form. This is achievable if the range kernel is of the form

\[ \phi(s) = \cos(\gamma s)^N = \cos\left(\frac{\pi s}{2T}\right)^N, \quad -T \leq s \leq T \]

4.1 General Trigonometric kernels

The idea mentioned above is extended to more general trigonometric functions having the form

\[ \phi(s) = a_0 + a_1 \cos(\gamma s) + \ldots + a_N \cos(N\gamma s) \]

Rewriting in terms of complex exponentials, we have

\[ \phi(s) = \sum_{n \leq N} c_n \exp(-jn\gamma s) \]

The coefficients must be real and symmetric.

4.2 Raised Cosines

In order to qualify as a valid range kernel, the range kernel should be nonnegative and monotonic, aside from being symmetric.

![Fig. 1 Family of raised cosines, over the dynamic range -T≤s≤T, as N goes from 1 to 5 (outer to inner curves)](image)

The properties of symmetry, nonnegativity, and monotonicity are concurrently enjoyed by the family of raised cosines of the form

\[ \phi(s) = \cos(\gamma s), -T \leq s \leq T \]

We have

\[ \cos(x) = \frac{e^{jx} + e^{-jx}}{2} \]

And applying the binomial theorem we see that

\[ \phi(s) = \sum_{n=0}^{N} 2^{-N} N C_n \exp\left( j(2n - N)\gamma s \right) \]

Since \( \phi(s) \) has a total of \( (N+1) \) terms, this gives a total of \( 2(N+1) \) auxiliary images. The central term \( n=N/2 \) is constant when \( N \) is even, and we have one less auxiliary image to process in this case.

4.3 Gaussian Kernel Approximation

Figure below shows raised cosines of order \( N=1 \) to \( N=5 \). As \( N \) increases \( \phi(s) \) becomes more gaussian like, over the half period \([-\Pi, \Pi]\).

![Fig. 2 Raised cosine approximation of gaussian kernel](image)

\[ \lim_{N \to \infty} \left[ \cos\left(\frac{\gamma s}{\sqrt{n}}\right) \right]^N = \exp\left(\frac{-\gamma^2 s^2}{2}\right) \]

(5)

It is clear that raised cosine offers better approximation than its polynomial counterpart.

4.4 Control of width of range kernel

The approximation in (5) also suggests a means of controlling the variance of the raised cosine, namely, by controlling the variance of the target Gaussian. The target Gaussian (with normalization) has a fixed variance of \( \gamma^{-2} \). This can be increased by simply rescaling the argument of the cosine in (5) by some \( \rho > 1 \). In particular, for sufficiently large \( N \)

\[ \left[ \cos\left(\frac{\gamma s}{\rho \sqrt{n}}\right) \right]^N \approx \exp\left(\frac{-s^2}{2 \rho^2 \gamma^2}\right) \]

5. Experimental Results

The Lena image of size 256 x 256 in png format was chosen as the test image, initially. Additive white gaussian noise with sigma=20 was added in to the original image. The noisy image was filtered using the proposed algorithm and the output obtained was plotted.
Table 1 shows the maximum dynamic range obtained for six standard test images, lena, barbara, boats, checker and house. The maximum value of dynamic range of a grayscale image is 255. From the table, it is clear that the maximum dynamic range obtained is much lower than the worst case estimate (T=255).

The de-noising results obtained using the proposed algorithm for various standard test images were compared both visually and in terms of psnr values.

Table 2 Comparison of psnr values

<table>
<thead>
<tr>
<th>SLNo</th>
<th>Test Image</th>
<th>Noisy Image</th>
<th>SBF</th>
<th>FBF</th>
<th>Proposed Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Checker</td>
<td>22.15</td>
<td>26.70</td>
<td>28.85</td>
<td>35.19</td>
</tr>
<tr>
<td>2</td>
<td>Lena</td>
<td>22.11</td>
<td>25.52</td>
<td>27.08</td>
<td>31.48</td>
</tr>
<tr>
<td>3</td>
<td>Barbara</td>
<td>22.13</td>
<td>24.83</td>
<td>26.97</td>
<td>29.54</td>
</tr>
<tr>
<td>4</td>
<td>Boats</td>
<td>22.16</td>
<td>25.21</td>
<td>26.67</td>
<td>30.57</td>
</tr>
<tr>
<td>5</td>
<td>House</td>
<td>22.12</td>
<td>26.17</td>
<td>28.18</td>
<td>33.29</td>
</tr>
<tr>
<td>6</td>
<td>Peppers</td>
<td>22.12</td>
<td>25.36</td>
<td>27.02</td>
<td>31.56</td>
</tr>
</tbody>
</table>

Table 2 shows the comparison of the peak signal to noise ratio (psnr), obtained for six standard images. Columns 3, 4 and 5 represents the psnr obtained for standard bilateral filter (SBF), Fast bilateral filter (FBF) and the proposed method respectively for the six standard images. The psnr value obtained for checker image using standard bilateral filtering is 26.70 dB and with fast bilateral filtering is 28.85 dB. The proposed method resulted in the rise of psnr value to 35.19 dB, a rise of about 6 dB. Thus from the above table, it is clear that the quality of de-noised image can be improved by using this algorithm.

Conclusion

The enhancement of noisy image is an essential task in image processing. The objective of any de-noising algorithm is to de-noise the image in minimum time, preserving the edge information as well as the image details. Thus edge preserving bilateral filter was realized with the raised cosine approximation of range kernel of the bilateral filter. It is clear that raised cosines of large order provide close approximations of the Gaussian kernel. The de-noising results obtained using this algorithm was compared both visually and in terms of psnr values. Along with the enhancement in quality, the reduction in computational complexity is another advantage of this method. This algorithm can also be extended for removing noise from color images.

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