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*Research Article*

# Spectral Efficiency and BER of OFDM Systems with Carrier Frequency Offset (CFO)

**Rashi Sharma†* and Neeraj Shrivastava†**

†Electronics & Communication Engineering Department, Rustamji Institute of Technology, BSF Academy, Tekanpur (M.P), India

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**Abstract**

*Orthogonal frequency division multiplexing (OFDM) is a parallel transmission method, where single data stream is separated into a number of lower rate subcarriers, and each carrier is orthogonal to all other carriers. One of the main problems for OFDM system is Carrier frequency offset (CFO). OFDM is sensitive to CFO, which spoils the orthogonality between the sub-carriers, and cause Inter-carrier interference (ICI) in the OFDM, which corrupts the overall performance of OFDM and creates the frequency difference between the local oscillators of the transmitter and receiver, and the signals transmitted on each carrier are not independent of each other. In this paper, we introduce the analysis for calculating the Spectral Efficiency of OFDM systems with CFO. We further present the bit error rate (BER) performance of OFDM system with various CFOs under additive white gaussian noise (AWGN). This paper presents, the CFO in OFDM and the analysis for calculating the effect of the various CFOs on the performance of the OFDM system in term of BER under AWGN. Finally, in this paper, the results are sufficient to prove the Spectral Efficiency analysis and BER performance of OFDM system with various CFOs in AWGN channel.*

**Keywords:** *Orthogonal frequency division multiplexing (OFDM), Carrier frequency offset (CFO), Intercarrier interference (ICI), Spectral efficiency (SE), Bit error rate (BER), Signal-to-noise ratio (SNR), Additive white Gaussian noise (AWGN).*

## 1. Introduction

Now-a-days, Orthogonal frequency division multiplexing (OFDM) (A. Peled *et al*, 1980; L. J. Cimini, 1985; M. Sandell, 1996; R. Haas, 1996) has received bigger attention. In OFDM, frequency band is divided into a number of subcarriers. These subcarriers are orthogonal to each other. OFDM is commonly used in many communication systems for its ability to increase the data rate and reduce the bandwidth. OFDM is one of the techniques for parallel transmission. These techniques must be able to provide high data rate, allowable Bit error rate (BER), and maximum delay. Important features of OFDM systems include immunity to multipath fading and impulsive noise. OFDM is the method for achieving high Spectral Efficiency in wireless communication systems. Due to its multicarrier feature, OFDM systems are more sensitive than single-carrier systems to frequency synchronization errors (T. Pollet *et al*, 1996). OFDM is a bandwidth efficient signaling scheme, where the orthogonality between the subcarriers should be maintained to a high degree of exactness. And the spectrum of the sub-carriers are overlapping, an accurate frequency synchronization technique is needed. Carrier Frequency Synchronization is the basic functions of an OFDM receiver. CFO plays an important role in OFDM wireless communication. CFO in the OFDM systems due to synchronization errors and doppler shift results in a loss of orthogonality between the subcarriers and degradation in SINR. The weakness of OFDM technique is its sensitivity to frequency offset errors caused by Doppler shifts or transmitter receiver oscillator instabilities. The BER and SNR analysis plays very important roles in understanding and improving the design of OFDM system.

The AWGN channel model is used in studying OFDM. Mainly, in this paper, we consider the analysis for calculating the spectral efficiency of OFDM systems with CFO and furthermore the Bit error rate (BER) performance of OFDM system with various Carrier Frequencies Offset (CFOs) under additive white gaussian noise (AWGN) channel. In OFDM systems, frequency offset destroys orthogonality (Luca Rugini *et al*, 2005) between carriers and introduces Inter Carrier Interference (ICI). The orthogonality of the OFDM relies on the condition that transmitter and receiver operate with exactly the same frequency reference. If this is not the case, the perfect orthogonality of the subcarrier will be lost, which can result to subcarrier leakage, this is known as the ICI. OFDM is highly sensitive to frequency synchronization errors, which

*Corresponding author: **Rashi Sharma**

destroy the orthogonality between the subcarriers, giving increase to ICI. Lack of the synchronization of the local oscillator signal, for down conversion in receiver with carrier signal contained in received signal causes CFO and it can cause to degrade the performance of OFDM, which can create the following factors: Frequency mismatched in the transmitter and the receiver oscillator, doppler effect and ICI.

## 2. System Model

The transmitted baseband OFDM signal can be written as (Ahmed Almradi *et al*, 2015),

$$s(t)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}a_n e^{j2\pi\frac{n}{T}t}, t\in[-T_G, T] \quad (1)$$

where, N is number of sub-carriers, $T_G$ is length of cyclic prefix (CP), and T is length of OFDM symbol duration. Here $T=N/R$ and R is symbol rate of input data to be transmitted. The time such that R = 1, and T = N. It is assumed $a_n$, n $\in[0, N-1]$ are zero mean complex RVs with

$$E[a_n a^*_m]=\delta(n-m)\varepsilon_n \forall_n$$

where, $a_n$ is the M-ary complex information bearing symbol on n[th] sub-carrier, $\delta(\cdot)$ denotes delta function, $E(\cdot)$ is expectation operator, and $\varepsilon_n$ is energy of transmitted symbol on n[th] sub-carrier. The channel impulse response of time-invariant frequency-selective multipath rayleigh fading channel is

$$h(\tau)=\sum_{l=0}^{L-1}g_l\delta(\tau-\tau_l) \quad (2)$$

where $g_l$ and $\tau_l$ are complex amplitude and propagation delay of $l$[th] path, respectively, $\tau_0<\tau_1<\ldots<\tau_{L-1}$. It is assumed that $g_l$, $l\in[0, L-1]$ are zero mean complex Gaussian random variables with

$$\mathbb{E}\lfloor g_l g^* k\rfloor=\delta(l-k)\forall l$$

where, $\gamma_l$ = $E[|g_l|]^2$ assuming that power is normalized

$$\textstyle\sum_{l=0}^{L=1}\gamma_l=1.$$

Assuming synchronization and normalized CFO of Δ, received signal after removing CP is,

$$r(t)=\frac{1}{\sqrt{N}}e^{j2\pi\frac{\Delta}{T}t}\sum_{l=0}^{L-1}g_l\sum_{n=0}^{N-1}a_n e^{j2\pi\frac{n}{T}(\tau-\tau_l)}+w(t) \quad (3)$$

where $w(t), t\in[0,T]$ is complex AWGN with zero mean and two sided power spectral density $N_0/2$ per dimension. The *N*-point FFT samples at the receiver are

$$y_p=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}r\left(k\frac{T}{N}\right)e^{-j2\pi\frac{k}{N}P}, P=0,1\ldots., N-1$$

$$=\frac{1}{N}\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}\sum_{l=0}^{L-1}g_l a_n e^{j\frac{2\pi}{N}(n-p+\Delta)k}e^{-j2\pi n\frac{\tau_l}{T}}+w_p \quad (4)$$

where, $w_p$ is zero mean complex Gaussian RV with variance $N_0$. OFDM is generated by taking N-point IDFT on information symbol $X(k), 0\le k<N$. OFDM symbol at transmitter side is,

$$x(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X(k)e^{j2\pi kn/N}, n=0,1\ldots.., N-1 \quad (5)$$

where, $j=\sqrt{-1}$

After passing through the channel, the received signal at the baseband is,

$$y(n)=e^{\frac{j2\pi\varepsilon n}{N}}.[h(n)\otimes x(n)]+w(n) \quad (6)$$

where, $h(n)$ is baseband channel impulse response, $\otimes$ denotes convolution, and $w(n)$ is complex AWGN introduced in channel. For demodulation, where $\varepsilon$ is a normalized frequency offset, and is given by $\Delta fNT_s$. $\Delta f$ is frequency difference, and $T_s$ is sub-carrier symbol period. DFT is applied to received signal,

$$Y(n)=\frac{1}{\sqrt{N}}\sum_{l=0}^{N-1}y(n)e^{\frac{j2\pi kn}{N}}, k=0,1\ldots.., N-1 \quad (7)$$

$$=\frac{1}{N}\sum_{l=0}^{N-1}\left[\sum_{n=0}^{N-1}e^{\frac{j2\pi n(l-k+\varepsilon)}{N}}\right]H(l)X(l)+W(k) \quad (8)$$

$$where, K=0,1\ldots\ldots., N-1$$

where, H(l) is the transfer function of channel at frequency of l[th] SC, and N is total number of sub-carriers. Let $S(l-k)$ denote ICI-coefficient of SC l on SC k, which is given by,

$$S(l-k)=\frac{1}{N}\sum_{n=0}^{N-1}e^{\frac{j2\pi n}{N}}(l-k+\varepsilon), \quad (9)$$

$$where, k=0,1\ldots.., N-1$$

$S(l-k)$ are complex coefficients for the ICI components in the received signal. After some algebraic management, $S(l-k)$ is reduced to

$$S(l-k)=\frac{\sin\left(\pi\left(l-k+\varepsilon\right)\right)}{N\sin\left(\pi\frac{l-k+\varepsilon}{N}\right)}e^{j\pi(l-k+\varepsilon)\left(1-\frac{1}{N}\right)} \quad (10)$$

$$Y(k)=\sum_{k=0}^{N-1}S(l-k)H(l)X(l)+W(k) \quad (11)$$

$$where, k=0,1.....,N-1$$

Equation (11) can be re-written as,

$$Y(k)=S(0)H(k)X(k)+\sum_{l=0,l\neq k}^{N-1}S(l-k+\varepsilon)H(l)X(l)+W(k), \quad (12)$$

$$where, k=0,1......,N-1$$

From (12), we can see original symbol $\left(X(K)\right)$ suffers at reception from attenuation $\left(\frac{\sin(\pi\varepsilon)}{N\sin(\pi\frac{\varepsilon}{N})}\right)$, phase rotation $\left(e^{j\pi\varepsilon(1-\frac{1}{N})}\right)$, and ICI from other symbols $\left(S(l-k)\right)$. This is due to CFO $(\varepsilon=0)$.

### 3. Carrier Frequency Offset (CFO)

When CFO happens, it causes the receiver signal to be shifted in the frequency $(\delta_f)$.

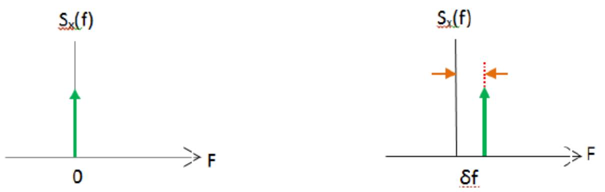


**Fig.1** Frequency Offset $(\delta_f)$

The subcarriers (SCs) will sample at their peak, and this can only occur when there is no frequency offset,

if there is any frequency offset, sampling will be done at the offset point, which is not the peak point. This causes to reduce the amplitude of anticipated subcarriers, which can result to raise the ICI from adjacent subcarriers (SCs).In the presence of CFO, the sampling points in the frequency domain are shifted from optimal points. CFO divided by the subcarrier spacing. CFO can put an extra phase factor in each subcarrier,

$$e^{j2\pi\Delta Ft} \quad (13)$$

Let $f_c$ and $f'_c$ denote the carrier frequencies in transmitter and receiver. Let $f_{offset}$ denote their difference (i.e., $f_{offset}=f_c-f'_c$). This frequency offset is defined (Saeed Mohseni *et al*, 2012) as

$$f_{offset}=f_c-f'_c \quad (14)$$

Doppler frequency $f_d$ is determined by the carrier frequency $f_c$ and the velocity v of receiver as

$$f_d=\frac{v.f_c}{c} \quad (15)$$

$f_d$, c and v are the doppler frequency, the speed of light, and the velocity of the moving receiver (i.e. 100 km/h). The normalized CFO, $\varepsilon$, as ratio of CFO to subcarrier spacing $\Delta f$,

$$\varepsilon=\frac{f_{offset}}{\Delta f} \quad (16)$$

Δf is the subcarrier spacing, the ε has the two portions, one integer $(\varepsilon_i)$ and one fractional $(\varepsilon_f)$,

$$\varepsilon=\varepsilon_i+\varepsilon_f \quad (17)$$

where, $\varepsilon_i=\lfloor\varepsilon\rfloor$, with the presence of frequency offset, sampling cannot be exactly done at the center frequencies of the subcarriers therefore amplitude of the desired subcarriers will be decreased. The received signal contains samples from this shifted subcarrier, leading to the Inter Carrier Interference (C. Shin *et al*, 2007) (ICI). $f_\delta=f_{Tx}-F_{Rx}$. $f_\delta$ is the frequency offset. The absolute value of a CFO is $f_\in$ is either an integer multiple or fraction of $\Delta f$. If $f_\in$ is a normalized to the subcarrier spacing $\Delta f$ then normalized CFO of the channel is,

$$\varepsilon=\frac{f_\in}{\Delta f}=\delta+\in \quad (18)$$

where, $\delta$ is an integer and $|\varepsilon|=<0.5$, If CFO occurs then the symbol transmitted on a certain sub carrier k, will shift to another sub carrier, $k_s=k+\delta$.

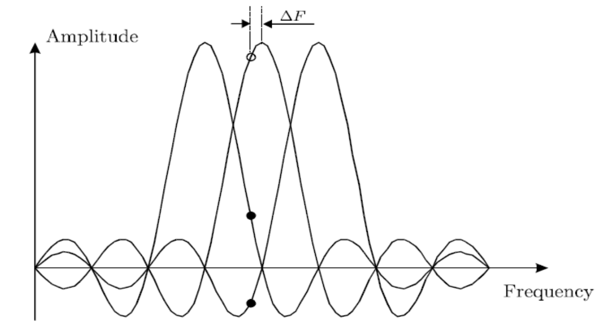


**Fig.2** The impact of frequency offset $\Delta F$ : reduction in signal amplitude (o), and ICI ( ● )

## 4. Spectral Efficiency Analysis

Spectral efficiency is measured in bits/s/Hz. Spectral efficiency of OFDM systems for CFO Δ is,

$$G(\Delta) = \frac{1-\alpha}{N+T_G}\sum_{p=0}^{N-1} R_p(\Delta) \tag{19}$$

where, α is the fraction of time resources allocated to CFO, $4R_p(\Delta)$ is the conditional average mutual information over the p[th] sub-carrier for given $\Delta$.

$$Rp(\Delta) = \mathrm{E}_{H|\Delta}\{\log_2(1+SINR_P)\}$$

$$= \mathrm{E}\left[\log_2\left(1+\frac{|c(p,p)|^2|H_p|^2}{\sum_{n\neq p}|c(n,p)|^2|H_n|^2+\frac{1}{SNR_D}}\right)\right] \tag{20}$$

That is the exact overall Spectral efficiency of OFDM for any given probability density function for CFO, $\Delta$. Here,

$$\log_2\left(1+\frac{SINR_p}{\Gamma}\right)$$

where, $\Gamma$ is SNR gap which denotes amount of extra coding gain needed to achieve shannon capacity. In order to reduce required computational complexity, we rely on following lemma to transform (20),

: For any $x \geq 0$ then,

$$\log(1+x) = \int_0^\infty \frac{1}{z}\left(1-e^{-zx}\right)e^{-z}dz \tag{21}$$

: for any $x, y \geq 0$ then,

$$\log\left(1+\frac{1}{x}\right) = \int_0^\infty \frac{1}{z}\left(e^{-zy}-e^{-z(x+y)}\right)dz \tag{22}$$

The joint moment generating function of these RVs as,

$$M(z_0.z_1,\ldots,z_{N-1}) = E\left[e^{-\sum_n z_n|c(n,p)|^2|H_n|^2}\right]$$

$$= \frac{1}{\left|I+diag(z_0,z_1,\ldots.z_{N-1})\Lambda_p\right|} \tag{23}$$

where, $\sum_n = \sum_{n=0}^{N-1}$, I is N × N identity matrix, $diag(\cdot)$ is the $N \times N$ diagonal matrix, $\Lambda_p = C_p\Lambda$ where,

$$C_p = diag\left(|c(0,p)|^2, |c(1,p)|^2, \ldots., |c(N-1,p)|^2\right)$$

and $\Lambda$ is the complex covariance matrix of the channel vector with entries.

$$R_p(\Delta) = \log_2 e\int_0^\infty \frac{1}{z}\left(M(z,\ldots.,\underset{p}{0},\ldots.,z) - M(z,z,\ldots,z)\right)e^{-z/SNR_D}dz \tag{24}$$

To sum up (19) and (24) give exact conditional Spectral efficiency of OFDM for given CFO. Conditional average mutual information over p[th] subcarrier for given $\Delta$ can be upper bounded,

$$R_p(\Delta) \leq \log_2\left(1+\mathrm{E}\{SINR_p\}\right)$$

$$\leq \log_2\left(\frac{SNR_D+1}{SNR_D\left(1-\left|\frac{\sin(\pi\Delta)}{N\sin(\frac{\pi}{N}\Delta)}\right|^2+1\right)}\right)$$

The SE in general over any PDF for CFO, Δ. The overall average mutual information over p[th] sub-carrier is conditional average mutual information over p[th] subcarrier for given CFO Δ,

$$R_p = \mathrm{E}_\Delta\left[\mathrm{E}_{H|\Delta}\{\log_2(1+SINR_P)\}\right] \tag{26}$$

$$R_p = \log_2 e\int_0^\infty \frac{1}{z}\mathrm{E}_\Delta\left(M(z,\ldots.,\underset{p}{0},\ldots.,z) - M(z,z,\ldots.,z)\right)e^{-z/SNR_D}dz \tag{27}$$

This is a simplified overall average mutual information over the p[th] sub-carrier. Each sub-carrier is decoded independently, the overall spectral efficiency of the OFDM system with CFO,

$$G = \frac{1-\alpha}{N+T_G}\sum_{P=0}^{N-1} R_p \tag{28}$$

Eq. (27) and (28) give exact overall Spectral efficiency of OFDM systems for any given probability density function for CFO, $\Delta$

$$R_p(\Delta) = \log_2 e \sum_{i=1}^{I} \frac{\alpha_i}{\beta_i}\left(M(\lambda_i,\ldots,\underset{p}{0}\ldots,\lambda_i) - M(\lambda_i,\lambda_i,\ldots,\lambda_i)\right) + R_N \quad (29)$$

where, $\lambda_i = SNR_D \beta_i, \alpha_i$ and $\beta_i$ are the i[th] and weight, remainder $R_N$ is sufficiently small for $I \geq 15$.

## 5. The Bit Error Rate (BER) Analysis

The BER and SNR analysis plays an important and urgent role in understanding and improving the design of OFDM system. BER is inversely related to SNR. The bit error rate (BER) analysis of OFDM in the presence of CFO has recently attracted main research attention (P. C. Weeraddana *et al*, 2008) due to increased popularity of OFDM and its high sensitivity to CFO.

### *5.1 Analysis of Signal to noise ratio (SNR)*

SNR is known as SNR per bit used to compare the BER performance of different digital modulation schemes without taking bandwidth into report. The function $c(n, p)$ is given by,

$$c(n, p) = \frac{1}{N} \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(n-p+\Delta)k}$$

$$= \frac{\sin c(n-p+\Delta)}{\sin c\left(\frac{1}{N}(n-p+\Delta)\right)} e^{j\pi(1-\frac{1}{N})(n-p+\Delta)} \quad (30)$$

It is value nil that in the ICI free case $\Delta = 0$, and

$c(n, p) = \delta(n-p)$.

Assuming one equalizer and perfect channel state information at the receiver, the p[th] decision variable,

$$\hat{a}_p = a_p + \frac{\sum_{\substack{n=0\\n\neq p}}^{N-1} a_n c(n, p) H_n + w_p}{c(p, p) H_p} \quad (31)$$

The N-dimensional random vector $H = [H_0, H_1, \ldots 1H_{N-1}]^T$ is a proper complex Gaussian random vector having a probability density function given as,

$$f_H(x) = \frac{1}{\pi^N |\Lambda|} \exp\left(-x^{\dagger} \Lambda^{-1} x\right) \quad (32)$$

where, † denotes the complex conjugate transposition. The SINR over the p[th] sub-carrier for a given realization of H and $\Delta$ becomes a RV given as

$$SINR_p = \frac{|c(p, p)|^2 |H_p|^2}{\sum_{n\neq p} |c(n, p)|^2 |H_n|^2 + \frac{1}{SNR_D}}, SNR_D = \frac{N}{N + T_G + T_E} \quad (33)$$

$T_E$ is effective per OFDM block training symbols length used to approximate CFO, and $SNR(SNR = \frac{E_b}{N_0})$ is the SNR in AWGN channel in absence of the ICI.

$SNR_D = \frac{\varepsilon}{N_0}$ Assuming that,

$$\varepsilon_1 = \varepsilon_2 = \ldots = \varepsilon_{N-1} = \varepsilon$$

$$E\left[\log_2\left(1 + \frac{|c(p, p|^2 |H_p|^2}{\sum_{n\neq p} |c(n, p)|^2 |H_n|^2 + \frac{1}{SNR_D}}\right)\right] = \log_2 e \int_0^{\infty} \frac{1}{z}\left(e^{-z\sum_{n\neq p}|c(n,p)^2|H_n||^2} - e^{-z\sum_n |c(n,p)^2|H_n||^2}\right) e^{-z/SNR_D} dz \quad (34)$$

This SINR is in terms of CFO and variance of $\sigma^2$, this is without timing jitter $(\xi)$. The equation of SINR can be defined by,

$$SINR(\varepsilon, \sigma^2{}_u) \geq \frac{\gamma\{\sin c^2(\pi\varepsilon)\}}{1 + \gamma\left[0.5947(\sin \pi\varepsilon^2) + \left\{\frac{\sigma_u{}^2}{2N} \sin c^2(\pi\varepsilon) \sum_{r=1}^{N-1} \frac{1}{\sin^2}(\frac{\pi\varepsilon}{N})\right\}\right]} \quad (35)$$

### *5.2 Analysis of Bit-Error Rate (BER)*

We consider BER performance in the presence of CFO for OFDM in AWGN channel. Suppose,

$$X_1(0), \ldots, X_1(M-1)$$

are independent with zero mean and variance,

$\mathrm{E}\left\{|X_1(n)|^2\right\} = \sigma^2 x$ And,

$$\mathrm{E}\left\{|X1(n)|^2\right\} = E\left\{\sum_{l=0}^{L-1} |h_1|^2\right\} = E\underline{h} \quad (36)$$

for all subcarriers noted by $n, 0 \leq n \leq M-1$. Define,

$$\mathrm{I}_{m,n} = \frac{1}{M} \sum_{k=0}^{M-1} X_1(n) H(n) e^{j2\pi\frac{k(n-m+\varepsilon)}{M}} \quad (37)$$

$$\mathrm{E}\left\{\sum_{n=0}^{M-1} |\mathrm{I}_{m,n}|^2\right\} = \mathrm{E}\left\{\sum_{n=0}^{M-1} |\mathrm{I}_{m,n}|\right\} = \sigma^2 x E\underline{h}$$

For each subcarrier m. For the AWGN channel,

$\mathrm{E}\underline{h} = 1$ And,

$$\mathrm{E}\left\{\left|ICI_1(m)^2\right|\right\} = E\left\{\sum_{n=0,n\neq m}^{M-1}\left|\mathrm{I}_{m,n}\right|^2\right\}$$

$$= \sigma^2 x\left(1 - E\left\{\left|\mathrm{I}_{m,n}\right|^2\right\}\right)$$

$$= \sigma^2 x\left(1 - \frac{\sin^2 \pi\varepsilon}{M^2 \sin^2 \frac{\pi\varepsilon}{M}}\right) \tag{38}$$

If the CFO is small enough, i.e., $\varepsilon << 1$, (38) becomes

$$\mathrm{E}\left\{ICI_1(m)^2\right\} \approx \frac{\sigma^2 x(\pi\varepsilon)^2}{3} \tag{39}$$

Eq. (38) becomes more correct than Eq. (39) when 5 increase. The $\mathrm{ICI}_1$, (m) can be approximated as a Gaussian distributed random variable. The Gaussian approximation for ICI in OFDM is very simple, it leads to the BERs with acceptable correctness. By this Gaussian approximation critical BER with the BPSK OFDM can be gained as,

$$BER \approx Q\left(\sqrt{\frac{\frac{\sigma^2{}_X \sin^2 \pi\varepsilon}{M^2 \sin^2 \frac{\pi\varepsilon}{M}} \cos^2 \frac{\pi\varepsilon(M-1)}{M}}{\sigma^2{}_x\left(1 - \frac{\sin^2 \pi\varepsilon}{M^2 \sin^2 \frac{\pi\varepsilon}{M}}\right) + \sigma^2 n}}\right) \tag{40}$$

## 6. Simulation Results

We present the Spectral efficiency curve is shown in figure 3.

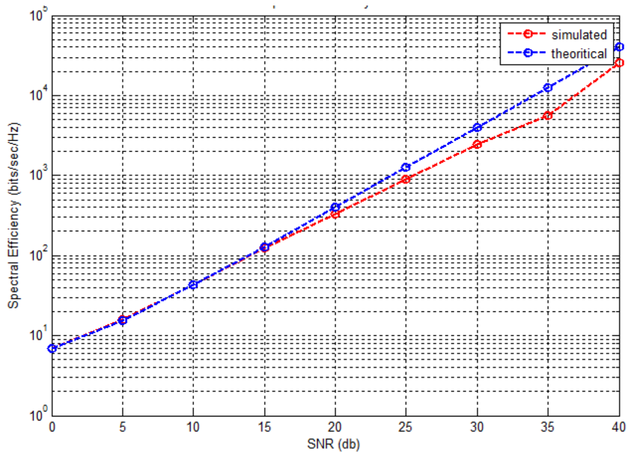


**Fig.3** Simulated and theoretical curve of Spectral Efficiency against SNR for OFDM

Both the simulated and theoretical Spectral efficiency curves are displayed in figure 3. The plotting is done to calculate the performance on the basis of such as Spectral efficiency Vs SNR. The Spectral efficiency is defined by the maximum information bits for a given bandwidth in a given communication system. From figure 3 we can see the difference and little improvement between simulated and theoretical Spectral efficiency of OFDM with CFO and the higher number of subcarriers, N or SNR, higher the Spectral efficiency. The simulation results show the simulated Spectral efficiency is in good agreement for OFDM system in the presence of CFO. Further, we present the BER analysis.

BER is inversely related to SNR that is high BER causes low SNR and BER is getting reduced as SNR increases. The performance of BER of OFDM systems with various CFO is compared in terms of BER versus SNR pilots under AWGN channel, and by finding their BER for different values of SNR. We start with a derivation of approximated ICI, which allows us to do the derivations of new BER terms for OFDM systems with various CFO in AWGN channel.

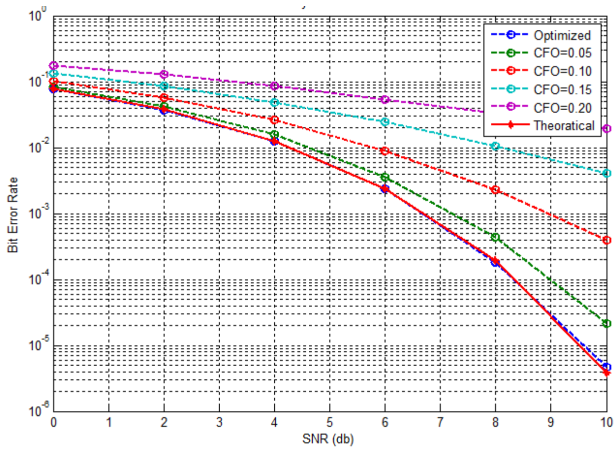


**Fig.4** Simulated and theoretical BERs performance of OFDM in AWGN under various CFOs

Figure 4 shows BER sensitivity Vs CFO under AWGN channel.

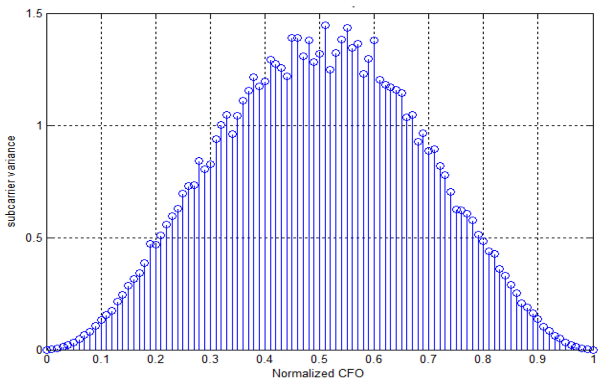


**Fig.5** The estimation of ICI density Vs the normalized CFO in OFDM under AWGN channel.

In the simulation results, we are taking values of Carrier frequency offset (CFO) and observe the impact of these CFOs on the performance of OFDM spectrum, and pilot the performance graph. Finally, i am optimizing and find out 5 (five) values that fits and sustainable performance of OFDM. And found one of them closest to theoretical performance of OFDM system. CFOs are, 0 KHz, 0.05 KHz, 0.10 KHz, 0.15 KHz, and 0.20 KHz. Results show a close match between simulated and theoretical results.

From figure 5 we can see, the density of CFO and the maximum power in the center. In this simulation results, 1024 total number of data subcarriers is used. Channel is AWGN. The AWGN channel block adds white Gaussian noise to a real and complex input signal. When the input signal is real, this block adds real Gaussian noise and produces a real output signal.

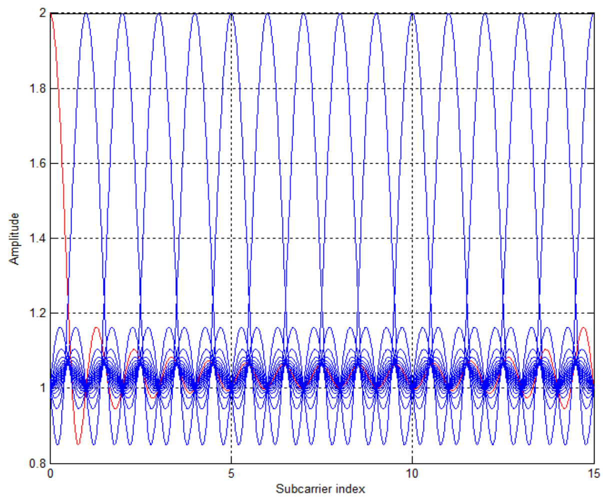


**Fig.6** Sensitivity of 16- Consecutive OFDM Subcarriers with CFO

Figure 6 shows the sensitivity of Orthogonal subcarriers with Carrier frequency offset (CFO). In this simulation results, we take the CFO, $\varepsilon = 0$, and calculate the effect of the CFO. The simulations results are presented in this paper are performed in MATLAB. MATLAB is used as complete tool today and used in research and development for communication system.

## Conclusions

1) In this paper, the Spectral Efficiency analysis with Carrier Frequency Offset (CFO) is presented.
2) This paper presents, the Bit-error rate (BER) performance degradation of OFDM systems with various carrier frequencies offset (CFO) in AWGN channel.
3) The simulation results show, that the simulated spectral efficiency is in good settlement with the theoretical spectral efficiency of OFDM system with CFO.
4) In this paper, we have offered, the CFO and simulated the influence of various CFOs on the performance of the OFDM system in term of BER under AWGN channel.
5) The graphical results verify that the optimized BER of OFDM system with various CFOs is close to the theoretical BER of OFDM systems.
6) The simulations results are presented in this paper are performed in MATLAB 7.10.0 (R2010a).

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## Biography

**Rashi Sharma** received her B.Tech. Degree in Electronics & Communication Engineering from Shri Rawatpura Sarkar Institute of Technology & Science, Datia (M.P.) in 2013, which is affiliated from RGPV University, Bhopal (M.P). She is currently pursuing her M.Tech. Degree (final semester) in department of Electronics & Communication

Engineering from (RJIT) Rustamji Institute of Technology, BSF Academy, Tekanpur (M.P), which is affiliated from RGPV University, Bhopal (M.P.), India. She has carried out this work for the completion of their final year thesis. She has published papers in journals. Her fields of interest are OFDM Systems, Wireless Communication, Data Communication & Networking and MATLAB.

**Dr. Neeraj Shrivastava** received the Ph.D. degree in Electronics and Communication Engineering. He has published many papers in national and international journals and conferences. His teaching and research interest include Advanced Digital Communication, OFDM Systems, developing signal processing for MIMO Wireless Communication Systems. Currently, He is the H.O.D. (Head of Department) at the department of Electronics & Communication Engineering in Rustamji Institute of Technology, BSF Academy, Tekanpur (M.P), which is affiliated from RGPV University, Bhopal (M.P.), India.