International Journal of Current Engineering and Technology
E-ISSN 2277 – 4106, P-ISSN 2347 – 5161

Available at http://inpressco.com/category/ijcet

Research Article

Comparison between numerical and analytical solution for the motion of viscous fluid with heat and mass transfer through porous medium over a vertical infinite permeable plate in the presence of induced magnetic field

Nabil T. El-dabe[†], Ahmed Y. Ghaly[†], Raafat R. Rizkallah[†], Karem M. Ewis[‡] and Ameen S. AL-bareda[†*]

[†]Department of Mathematics, Faculty of Education, Ain Shams University, Cairo, Egypt,
[‡]Department of Engineering Mathematics and Physics, Faculty of Engineering, El-Fayoum University, El-Fayoum 63514, Egypt

Accepted 20 May 2015, Available online 03 June 2015, **Vol.5, No.3 (June 2015)**

Abstract

The goal of this work is to introduce the comparison between numerical and analytical solution for the motion of viscous fluid with heat and mass transfer through porous medium over a vertical infinite permeable plate in the presence of induced magnetic field. The system is stressed by an external magnetic filed, the induced magnetic filed is considered. The viscous dissipation and heat generation are taken in consideration. The system of non-linear coupled equation which arises from momentum, energy, concentration and Maxwell's are solved numerically by using finite difference technique and analytically by using differential transform method. The solutions are obtained as a functions of the physical problem parameters, then the effects of these parameters on these solutions are illustrated numerically, analytically and graphically. Furthermore, comparisons of the numerical results with the analytical results are performed and showed that the results have high accuracy and are found to be an good agreement

Keywords: *Finite difference method (FDM), Differential transform method (DTM), induced magnetic field, heat generation, Soret and Dufour Effects.*

1. Introduction

The study of forced and free convection flow and heat transfer for electrically conducting fluids past a semi-infinite porous plate under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as geophysics, astrophysics, boundary layer control in the field of aerodynamics. Engineers employ MHD principle, in the design of heat exchangers pumps and flow meters, in space vehicle propulsion, thermal protection, braking, control and re-entry, in creating novel power generating systems etc. From the technological point of view, MHD free-convection flows have great significance for the applications in the fields of Stellar and Planetary magnetospheres, Aeronautics, Chemical engineering, and Electronics. The effect of magnetic field on free convection flow of electrically conducting fluid past a plate studied by many investigators such as (Soundalgekar 1965) (Chamkha and Camille 2000), (Sudhakar reddy *et al*. 2011), (Cramer and Pai 1973) and (Ravikumar *et al*. 2012). Numerical treatment of non Darcian effect on pulsatile MHD power-law fluid flow with heat transfer in a porous medium between two rotating cylinders are investigated by (Abd Elnaby *et al*. 2008). Numerical study of viscous dissipation effect on free convection heat and mass transfer of MHD non-Newtonian fluid flow through a porous medium was studied by (Eldabe *et al*. 2012).

The induced magnetic field is arisen due to a strong magnetic field. Since the problems with induced magnetic field play a decisive role in a number of industrial applications such as fiber or granular insulation, liquid-metals, electrolytes, ionized gases as well as the geothermal systems. (Chaudhary and Sharma 2006) have studied the induced magnetic field effect on a combined heat and mass transfer steady flow over a vertical plate with constant heat and mass fluxes. Numerical solutions of the same problem in case of two dimensional flows have been calculated by (Alam *et al*. 2008). Recently, the induced magnetic field effects on mixed convective transient heat and mass transfer flow with constant heat and mass fluxes have been investigated by (Haque and Alam 2009). The hydrodynamic free convective flow of an optically thin gray gas in the presence of radiation, when the induced magnetic field is taken into account was studied by (Raptis *et al*. 2003). (Sahin 2010) Studied the induced magnetic field with radiation fluid over a porous

*Corresponding author: **Ameen S. AL-bareda**

vertical plate. Thermo-diffusion and chemical reaction effects on a steady mixed convective heat and mass transfer flow with induced magnetic field are studied by (Sravanthi *et al*. 2013). (Sahin and Abdul Batin 2010) have studied analytical model of MHD mixed convective radiation fluid with viscous dissipative heat. (Zueco and Ahmed 2010) discussed the combined heat and mass transfer by mixed convection MHD flow along a porous plate with chemical reaction in presence of heat source.

The main objective of the present work is Comparison between numerical and analytical method for solving the problem of the motion of viscous fluid with heat and mass transfer through porous medium over a vertical infinite permeable plate in the presence of induced magnetic field. The induced magnetic _led is taken in consideration with thermal diffusion and diffusion -therom effects. The governing equations of motion are solved numerically by using (FDM) and analytically by using (DTM). The expressions of velocity, temperature, induced magnetic field and concentration are obtained as a functions of the problem parameters. The effects of various parameters of the problem have been computed and discussed in detail through some figures. Comparisons with analytical solution by using (DTM) of present work are performed and showed that the present results apply with results of analytical solution and are found to be a good agreement.

2. Mathematical formulation

The equations governing the steady motion of an incompressible viscous electrically conducting fluid in presence of a magnetic field are

$$div\vec{q} = 0 \tag{1}$$

$$\rho(\vec{q}.\nabla)\vec{q} = -\nabla p + \mu\nabla^2\vec{q} + \mu_e\left(\vec{J}\times\vec{H}\right) - \frac{\mu}{K_P}\vec{q} + \rho\vec{g} \tag{2}$$

$$\rho c_p(\vec{q}.\nabla)T' = k\nabla^2 T' + \varphi + \frac{\vec{J}^2}{\sigma} - Q_0 + \frac{\rho D_m k_T}{c_s}\nabla^2 C' \tag{3}$$

$$\nabla\times\left(\vec{q}\times\vec{H}\right) + \eta\nabla^2\vec{H} = 0 \tag{4}$$

$$(\vec{q}.\nabla)C' = D_m\nabla^2 C' - K'_r C + \frac{D_m k_T}{T_m}\nabla^2 T' \tag{5}$$

Where

$$\nabla\cdot\vec{H} = 0, \qquad \nabla\times\vec{H} = \vec{J}$$

where $\vec{q}$ is the velocity vector, p is the pressure, μ is the coefficient of viscosity, g is the gravitational acceleration, μ_e is the magnetic permeability, T' is the temperature, $\vec{H}$ is the magnetic induction vector, $\vec{J}$ is the electric current density, σ is the electrical conductivity, k_p is the Darcy permeability of the medium, k is the thermal conductivity, K_r is the thermal conductivity, c_p is the specific heat at constant pressure, ρ is the density of the fluid, Φ is the viscous dissipation per unit volume, $\eta = \frac{1}{\mu_e\sigma}$ is the magnetic diffusivity, $\vec{J}\times\vec{H}$ is the Lorentz force per unit volume, $\frac{\vec{J}^2}{\sigma}$ is the Joulean heat per unit volume , Q_0 is the heat source parameter, C' is the species concentration, c_s is the concentration susceptibility, k_T is thermal diffusion ratio, T_m is mean fluid temperature and D_m is the coefficient of chemical molecular diffusivity.

Now choose cartesian coordinates (x', y') shown in Fig. (1), $\vec{q}(u',v')$ are the velocity components in x' and y' directions respectively. The following assumptions are implicit in our investigation:

- All the fluid properties except the density in the buoyancy force term are constant.
- The Eckert number E, is small, as appropriate for viscous incompressible regimes.
- The plate is subjected to a constant suction velocity.
- The plate is electrically non conducting.

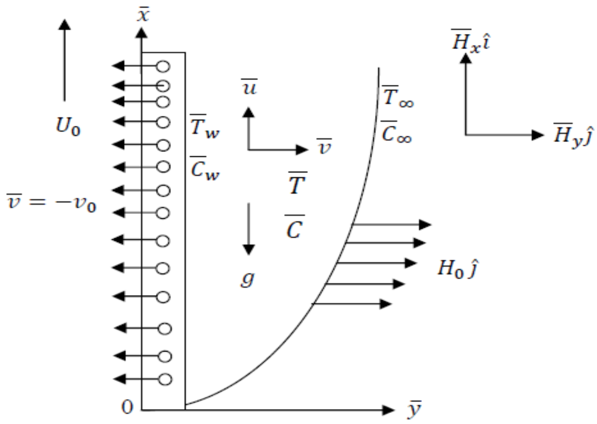

Fig.1 Physical configuration and coordinate

Let $\vec{H} = (H'_x, H'_y, 0)$ be the magnetic induction vector at a point (x', y', z') in the fluid. The x'-axis is taken along the plate in the upward direction, y'-axis is normal to the plate into the fluid region. Since the plate is infinite in length in x'-direction, therefore all the physical quantities except possibly the pressure are assumed to be independent of x'.

With the foregoing assumptions and under the usual boundary layer and Boussinesq approximations, Equations (1 - 5) reduce to:

$$\frac{dv'}{dy'} = 0, \qquad v' = -v_0 \tag{6}$$

$$v'\frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\left(\frac{d^2u'}{dy'^2}\right) + \frac{\mu_e H_0}{\rho}\left(\frac{dH'_x}{dy'}\right) - \left(\frac{\nu}{k_p}\right)u = 0, \tag{7}$$

$$v'\left(\frac{\partial T'}{\partial y'}\right) = \frac{k}{\rho c_p}\left(\frac{d^2T'}{dy'^2}\right) + \frac{\nu}{c_P}\left(\frac{du'}{dy'}\right)^2 + \frac{1}{\sigma\rho c_P}\left(\frac{dH_x'}{dy'}\right)^2 - \frac{Q_0}{\rho c_p}(T' - T'_\infty) + \frac{D_m k_T}{c_s c_p}\left(\frac{d^2C'}{dy'^2}\right) \tag{8}$$

$$v'\left(\frac{\partial H_x'}{\partial y'}\right) = \frac{1}{\sigma\mu_e}\left(\frac{d^2H_x'}{dy'^2}\right) + H_0\left(\frac{du'}{dy'}\right) \tag{9}$$

$$v'\frac{\partial C'}{\partial y'} = D_m\left(\frac{d^2C'}{dy'^2}\right) - K_r(C' - C'_\infty) + \frac{D_m k_T}{T_m}\left(\frac{d^2T'}{dy'^2}\right) \tag{10}$$

$$u' = 0,\ \ T' = T'_w,\ \ H'_x = 0,\ \ C' = C'_w, \quad at\ \ y' = 0 \tag{11}$$

$$u' \to U_0\,,\ \ T' \to T'_\infty,\ \ H'_x \to 0,\ \ C' \to C'_\infty \quad as \quad y' \to \infty$$

Where H_0 is the applied constant magnetic field, ν is the kinematic viscosity of the fluid, U_0 is the uniform velocity, T'_∞ and C'_∞ are the temperature and concentration of the fluid at infinity, β and β^* are the thermal and concentration expansion coefficients respectively.

3. Method of solutions

To solve system of equations (6-11), we shall introduce the following non-dimensional parameters:

$$u = \frac{u'}{U_0},\ \ y = \frac{y'v_0}{\nu},\ \ \theta = \left(\frac{T' - T_\infty}{T'_w - T'_\infty}\right),\ \ \emptyset = \left(\frac{C' - C'_\infty}{C'_w - C'_\infty}\right) \tag{12}$$

$G_r = \frac{g\beta\nu(T'_w - T'_\infty)}{U_0 v^2_0}$ is Grashof number, $G_m = \frac{g\beta^*\nu(C'_w - C'_\infty)}{U_0 v^2_0}$ is the mass Groshof number, $M = \frac{H_0}{v_0}\sqrt{\frac{\mu_e}{\rho}}$ is the magnetic parameter, $D_a = \frac{v^2}{k_p v^2_0}$ is the Darcy number, $E = \frac{U^2_0}{c_p(T'_w - T'_\infty)}$ is the Eckert number, $P_r = \frac{\nu\rho c_p}{k}$ is the Prandtl number, $P_{rm} = \sigma\nu\mu_e$ is the magnetic Prandtl number, $B = \frac{H'_x}{U_0}\sqrt{\frac{\mu_e}{\rho}}$ is the induced magnetic field, $S_c = \frac{\nu}{D_m}$ is the Schmidt number, $K_r = \frac{\nu k'_r}{v^2_0}$ is the Chemical reaction parameter, $Q_0 = \frac{\nu Q_0}{\rho c_p v^2_0}$ is the heat generation/absorption parameter, $D_u = \frac{D_m k_T(C'_w - C'_\infty)}{c_s c_p \nu(T'_w - T'_\infty)}$ is the Dufour number and $S_0 = \frac{D_m k_T(T'_w - T'_\infty)}{T_m \nu(C'_w - C'_\infty)}$ is the Soret number.

Substituting these non dimensional quantities into equations (6-10) we have the following dimensionless equations:

$$\frac{d^2u}{dy^2} + \frac{du}{dy} + M\left(\frac{dB}{dy}\right) + G_r\theta + G_m\emptyset - D_a u = 0 \tag{13}$$

$$\frac{d^2\theta}{dy^2} + P_r\left(\frac{d\theta}{dy}\right) - P_rQ\theta + P_rE\left(\frac{du}{dy}\right)^2 + \left(\frac{P_rE}{P_{rm}}\right)\left(\frac{dB}{dy}\right)^2 + P_rD_u\left(\frac{d^2\emptyset}{dy^2}\right) = 0 \tag{14}$$

$$\frac{d^2B}{dy^2} + P_{rm}\left(\frac{dB}{dy}\right) + P_{rm}M\left(\frac{du}{dy}\right) = 0 \tag{15}$$

$$\frac{d^2\emptyset}{dy^2} + S_c\left(\frac{d\emptyset}{dy}\right) - S_cK_r\emptyset + S_cS_0\left(\frac{d^2\theta}{dy^2}\right) = 0 \tag{16}$$

Also, the subjected boundary conditions in dimensionless form take the following form:

$$u = 0, \quad B = 0, \quad \theta = 1, \quad \emptyset = 1, \quad at\ \ y = 0 \tag{17}$$

$$u \to 1\,,\ \ B \to 0, \quad \theta \to 0, \quad \emptyset \to 0 \quad as \quad y \to \infty$$

3.1 The differential transform method (DTM)

The differential transformation of an analytical function f(t) for one variable is defined as (Zhon 1986).

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=t_0} \tag{18}$$

Where f(t) is the original function and F(k) is transformed function which is called the T-function.
The differential inverse transformation of F(k) is defined as:

$$f(t) = \sum_{k=0}^{\infty} F(k)(t - t_0)^k \tag{19}$$

Combining Eqs. (18) and (19), we obtain

$$f(t) = \sum_{k=0}^{\infty} \frac{(t - t_0)^k}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=t_0} \tag{20}$$

From Eqs. (18-20), it can be seen that the differential transformation method is derived from Taylor's series expansion, but the method does not calculate the derivatives representatively. However, the relative derivatives are calculated by an iterative way which is described by the transformed equations of the original function. For implementation purposes, the function f(t) is expressed by a finite series and Eq. (19) can be written as

$$f(t) \approx \sum_{k=0}^{N} F(k)(t - t_0)^k \tag{21}$$

By Eq. (18) , the following theorems can be deduced:

- **Theorem 1**. If $f(t) = u(t) \pm v(t)$ then $F(k) = U(k) \pm V(k)$.

- **Theorem 2**. If $f(t) = c$ then $F(k) = \delta(k)$.

- **Theorem 3.** If $f(t) = \alpha\, u(t)$ then $F(k) = \alpha U(k)$.
- **Theorem 4.** If $f(t) = t^m$ then

$$F(k) = \delta(k-m) = \begin{cases} 1 & k = m \\ 0 & otherwise \end{cases}.$$

- **Theorem 5.** If $f(t) = \frac{du(t)}{dt}$ then $F(k) = (k+1)U(k+1)$.
- **Theorem 6.** If $f(t) = \frac{d^n u(t)}{dt^n}$ then $F(k) = \frac{(k+1)!}{k!}U(k+n)$.
- **Theorem 7.** If $f(t) = u(t)\cdot v(t)$ then $F(k) = \sum_{r=0}^{k} U(r)V(k-r)$.
- **Theorem 8.** If $f(t) = \frac{du(t)}{dt}\,\frac{du(t)}{dt}$ then

$$F(k) = \sum_{r=0}^{k}(r+1)(k-r+1)U(r+1)U(k-r+1).$$

3.2 Basic Concepts of the Multi-Step differential transform method (MDTM)

When the DTM is used for solving differential equations with the boundary condition at infinity or problems that have highly non-linear behavior, the obtained results were found to be incorrect (when the boundary-layer variable go to infinity, the obtained series solutions are divergent). Besides that, power series are not useful for large values of the independent variable.

To overcome this shortcoming, the MDTM that has been developed for the analytical solution of the differential equations is presented in this section. For this purpose, the following non-linear initial-value problem is considered:

$$u\left(t, f, f', \dots, f^{(p)}\right) = 0 \qquad (22)$$

subject to the initial conditions $f^{(k)}(0) = c_k$, for k = 0; 1,2,…,p-1.

Let [0, T] be the interval over which we want to find the solution of the initial-value problem (22). In actual applications of the DTM, the approximate solution of the initial value problem (22) can be expressed by the following finite series:

$$f(t) = \sum_{n=0}^{N} a_n (t-t_0)^n, \qquad t \in [0, T] \qquad (23)$$

The multi-step approach introduces a new idea for constructing the approximate solution. Assume that the interval [0, T] is divided into M subintervals [tm-1, tm], m=0,1,2,....,M of equal step size h=(T / M) by using the nodes t = mh . The main ideas of the MDTM are as follows. First, we apply the DTM to Eq. (23) over the interval [0, t_1], we will obtain the following approximate solution:

$$f_1(t) = \sum_{n=0}^{N} a_{1n} (t-t_0)^n, \qquad t \in [0, t_1] \qquad (24)$$

using the initial conditions $f_1^{(k)}(0) = c_k$. For $m \geq 2$ and at each subinterval $[t_m, t_{m-1}]$ we will use the initial conditions $f_m^{(k)}(t_{m-1}) = f_{m-1}^{(k)}(t_{m-1})$ and apply the DTM to Eq. (23) over the interval $[t_m, t_{m-1}]$, where t_0 in Eq. (19) is replaced by t_{m-1}. The process is repeated and generates a sequence of approximate solutions fm(t), m=1,2,...,M , for the solution f(t):

$$f_m(t) = \sum_{n=0}^{N} a_{mn} (t-t_{m-1})^n, \quad t \in [t_m, t_{m-1}] \qquad (25)$$

Where $N = K\cdot M$. In fact, the MDTM assumes the following solution:

$$f(t) = \begin{cases} f_1(t), & t \in [0, t_1] \\ f_2(t), & t \in [t_1, t_2] \\ & . \\ & . \\ & . \\ f_M(t), & t \in [t_{M-1}, t_M] \end{cases} \qquad (26)$$

The new algorithm, MDTM, is simple for computational performance for all values of h . It is easily observed that if the step size h = T, then the MDTM reduces to the classical DTM. As we will see in the next section, the main advantage of the new algorithm is that the obtained series solution converges for wide time regions and can approximate non-chaotic or chaotic solutions.

3.3 Analytical solutions by using the MDTM

By applying the MDTM to Eqs. (13-16), gives the following recursive relations in each sub-domain $[t_i, t_{i+1}]$, i=0,1,...,N-1.

$$\begin{aligned}&(k+1)(k+2)U[k+2] + (k+1)U[k+1] \\ &+ M(k+1)B[k+1] + G_r\theta[k] + G_m\emptyset[k] - D_aU[k] \\ &= 0,\end{aligned} \qquad (27)$$

$$\begin{aligned}&(k+1)(k+2)\theta[k+2] + P_r(k+1)\theta[k+1] \\ &+ P_rE\sum_{r=0}^{k}(r+1)(k-r+1)U[r+1]U[k-r+1] \\ &+ \frac{EP_r}{P_{rm}}\sum_{r=0}^{k}(r+1)(k-r+1)B[r+1]B[k-r+1] \\ &- P_rQ\theta[k] + P_rD_u(k+1)(k+2)\emptyset[k+2] \\ &= 0,\end{aligned} \qquad (28)$$

$$\begin{aligned}&(k+1)(k+2)B[k+2] + P_{rm}(k+1)B[k+1] \\ &+P_{rm}M(k+1)U[k+1] = 0\end{aligned} \qquad (29)$$

$$\begin{aligned}&(k+1)(k+2)\emptyset[k+2] + S_c(k+1)\emptyset[k+1] \\ &- S_cK_r\emptyset[k] + S_cS_0(k+1)(k+2)\theta[k+2] \\ &= 0,\end{aligned} \qquad (30)$$

where U[k], θ[k], B[k] and ∅[k] are the differential transform of u(y), θ(y), B(y)and ∅(y).

The differential transformed boundary conditions in Eq. (17) to:

$$U[0]=0,\quad B[0]=0,\quad \theta[0]=1,\quad \emptyset[0]=1,$$
$$U[1]=u_1\,,\ B[1]=b_1\,,\ \theta[1]=t_1\,,\ \emptyset[1]=c_1 \qquad (31)$$

where $u_1,\ b_1,\ t_1,\ c_1$ are constants. These constants are computed from the boundary condition.

Moreover, substituting Eq. (31) into Eqs. (27-30) and by using the recursive method, we can calculate other values of $u(k),\theta(k),B(k)and\ \emptyset(k)$. Hence, substituting all $(k),\theta(k),B(k)and\ \emptyset(k)$, into Eq. (21), we obtain series solutions.

The velocity, temperature, induced magnetic field and concentration distributes achieved with the aid of MATHAMATICA application software.

3.4 Numerical solutions by using the FDM

In the present problem, the set of similar Eqs. (13 - 16) are solved by a finite difference method. The central finite difference formulas for first and second derivative:

$$f'(x_i)=\frac{f(x_{i+1})-f(x_{i-1})}{2\Delta x} \qquad (32)$$

$$f''(x_i)=\frac{f(x_{i-1})-2f(x_i)+f(x_{i+1})}{(\Delta x)^2} \qquad (33)$$

By substituting from Eqs.(32-33) in Eqs.(13-16) we get:

$$\left(\frac{u_{i+1}-2u_i+u_{i-1}}{\Delta^2}\right)+\left(\frac{u_{i+1}-u_{i-1}}{2\Delta}\right)+M\left(\frac{B_{i+1}-B_{i-1}}{2\Delta}\right)+G_r\theta_i+G_m\emptyset_i-D_a u_i=0 \qquad (34)$$

$$\left(\frac{\theta_{i+1}-2\theta_i+\theta_{i-1}}{\Delta^2}\right)+P_r\left(\frac{\theta_{i+1}-\theta_{i-1}}{2\Delta}\right)-P_rQ\theta_i+P_rE\left(\frac{u_{i+1}-u_{i-1}}{2\Delta}\right)^2+\frac{EP_r}{P_{rm}}\left(\frac{B_{i+1}-B_{i-1}}{2\Delta}\right)^2+P_rD_u\left(\frac{\emptyset_{i+1}-2\emptyset_i+\emptyset_{i-1}}{\Delta^2}\right)=0 \qquad (35)$$

$$\left(\frac{B_{i+1}-2B_i+B_{i-1}}{\Delta^2}\right)+P_{rm}\left(\frac{B_{i+1}-B_{i-1}}{2\Delta}\right)+P_{rm}M\left(\frac{u_{i+1}-u_{i-1}}{2\Delta}\right)=0 \qquad (36)$$

$$\left(\frac{\emptyset_{i+1}-2\emptyset_i+\emptyset_{i-1}}{\Delta^2}\right)+S_c\left(\frac{\emptyset_{i+1}-\emptyset_{i-1}}{2\Delta}\right)+S_cK_r\emptyset_i+S_cS_0\left(\frac{\theta_{i+1}-2\theta_i+\theta_{i-1}}{\Delta^2}\right)=0 \qquad (37)$$

Here the subscript i designates the grid points with y coordinate. The velocity, temperature, induced magnetic field and concentration distributes at all interior nodal points computed by successive applications of the above finite difference equations and these are achieved with the aid of MATLAB application software.

4. Results and discussion

The values of the velocity, temperature, induced magnetic field, concentration distributes are computed for different values of physical parameters of the problem like magnetic M, Grashof number G_r, Solutal Grashof number G_m, Darcy numbe D_a, heat source parameter Q , magnetic Prandtl number P_{rm}, Prandtl number P_r, Dufour parameter D_u , chemical reaction K_r and Soret number S_0. The effects of these parameters on the velocity, temperature, induced magnetic field and concentration distributions are solved analytically by using (DTM), numerically by using (FDM). Also graphically and illustrated through asset of (2-14).

Figs 2-5 clear the variations of dimensionless velocity profile u against y under the influence of governing parameters. The effect of M on the velocity field is presented in Fig. 2. It is seen that the velocity increases when M increases. Fig. 3 clears the effect of Gr on the velocity profiles. It is shown that the velocity increases with increasing Gr. The effect of Gm on the velocity profile is illustrated through Fig. 4. It is seen that the velocity increases when Gm increases. Also, the velocity decreases when Da increases, this shown in Fig. 5. The temperature distributions are shown in Figs. 6 -9 for different values of Pr, Q, Du and E. The effect of temperature profiles for different values of Pr is presented in Fig. 6. It is observed that the temperature decreases with the increase of Pr . Fig. 7 present the decreasing result of temperature when Q is increasing. Fig. 8 clears the effect of Du on the temperature, it is seen that the temperature increases when Du increase. Also, the temperature increases with increases of E , which shown in Fig. 9. Fig. 10 clears the effect of M on the induced magnetic field and it is observed that the induced magnetic field decreases with the increase of M. The effect of Prm on the induced magnetic field is presented in Fig.11. It is seen that the induced magnetic field decreases when Prm increases.

Figs 12-14 clears the variations of concentration distribution ∅ against y under the influence of the parameters Sc, Kr and So. It is observe that the concentration decreases when Sc increases, this shown in Fig. 12. Also, the concentration decreases when Kr increases, this shown in Fig. 13. From Fig. 14, we observed that the concentration increases with increasing So.

Tables (1-4) shows comparison between numerical solution by using (FDM) and the analytical solution using (DTM) where $M=0.25,\ D_a=0.05,\ G_r=2,\ G_m=2,\ P_r=0.71,\ E=0.01,\ Q=0.5,\ D_u=0.1,\ P_{rm}=0.1,\ S_c=0.3,\ K_r=0.5$ and $S_o=0.1$. It is observed that this approximate numerical solution is in excellent agreement with the results of the analytical solution of by using (DTM).

Table 1: Comparison between the obtained values of U(y)

y	DTM	MDTM	FDM
0	0.0	0.0	0.0
1	2.62546	2.62546	2.625724
2	2.49006	2.49006	2.490438
3	1.9344	1.9344	1.934431
4	1.38095	1.4887	1.488354
5	-884.896	1.21003	1.209562
6	-1.38516×10^{6}	1.05735	1.056971
7	-6.89893×10^{8}	0.985806	0.985573
8	-1.48948×10^{11}	0.963907	0.963798
9	-1.69976×10^{13}	0.97234	0.972306
10	-1.17373×10^{15}	1.0	1.0

Table 2: Comparison between the obtained values of Θ(y)

y	DTM	MDTM	FDM
0	1.0	1.0	1.0
1	0.369118	0.369118	0.368965
2	0.137656	0.137656	0.138022
3	0.0533971	0.053357	0.053584
4	4.8467	0.021431	0.021407
5	41032.1	0.008905	0.008783
6	6.59624×10^{7}	0.003828	0.003718
7	3.36438×10^{10}	0.001687	0.001622
8	7.41387×10^{12}	0.000732	0.000706
9	8.6125×10^{14}	0.000265	0.00026
10	6.04089×10^{16}	0.0	0.0

Table 3: Comparison between the obtained values of B(y)

y	DTM	MDTM	FDM
0	0.0	0.0	0.0
1	-0.01014	-0.01014	-0.01014
2	-0.04017	-0.04017	-0.04018
3	-0.056990	-0.05699	-0.05701
4	-0.060105	-0.05997	-0.05999
5	-1.07106	-0.05413	-0.05414
6	-1486.99	-0.04384	-0.04384
7	-695680	-0.03195	-0.03195
8	-1.41623×10^{8}	-0.02014	-0.02014
9	-1.52901×10^{10}	-0.00933	-0.00933
10	-1.0019×10^{12}	0.0	0.0

Table 4: Comparison between the obtained values of Φ(y)

y	DTM	MDTM	FDM
0	1.0	1.0	1.0
1	0.578899	0.578899	0.578902
2	0.332657	0.332657	0.332647
3	0.190123	0.190124	0.190120
4	-0.0421473	0.108141	0.108143
5	-1277.27	0.061099	0.061103
6	-2.05255×10^{6}	0.034076	0.034079
7	-1.04653×10^{9}	0.018453	0.018454
8	-2.30551×10^{11}	0.009270	0.009270
9	-2.67759×10^{13}	0.003671	0.003671
10	-1.87769×10^{15}	0.0	0.0

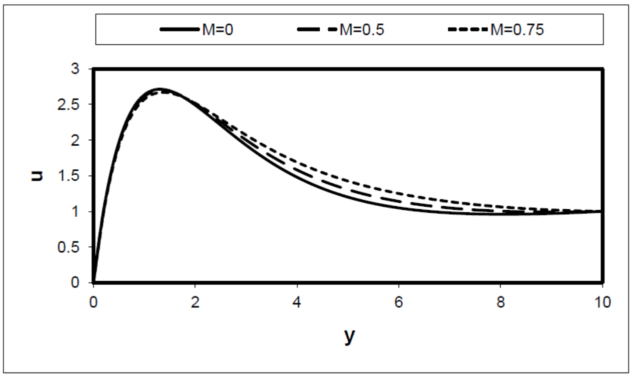

Fig.2 Velocity profiles for different values of M

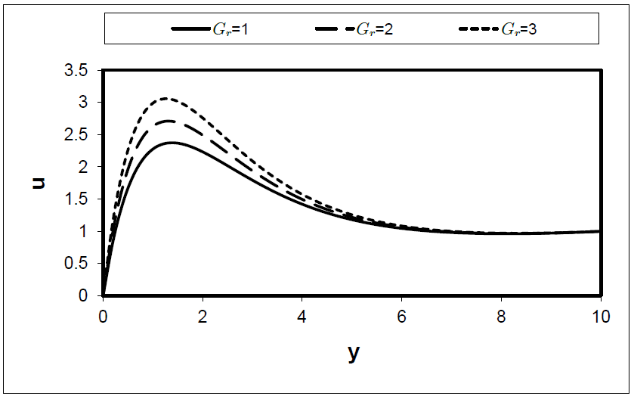

Fig.3 Velocity profiles for different values of Gr

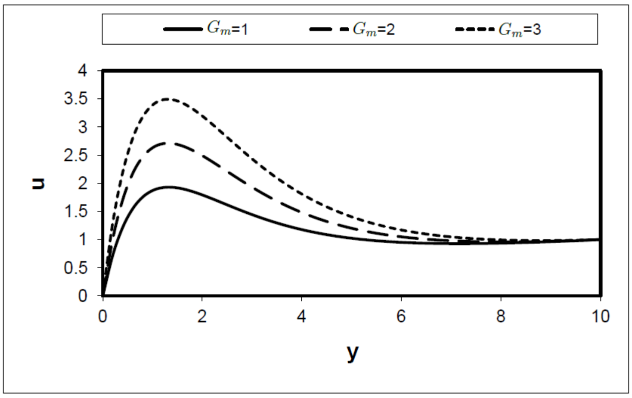

Fig.4 Velocity profiles for different values of Gm

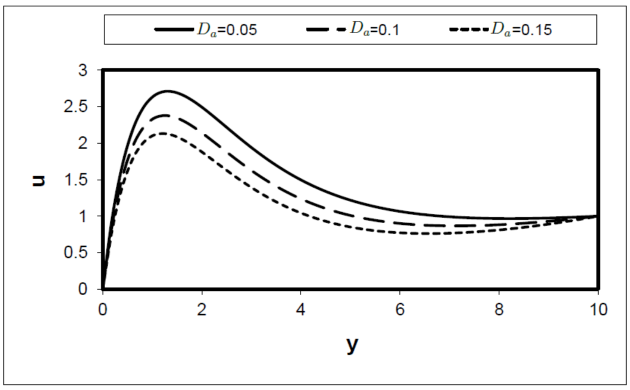

Fig.5 Velocity profiles for different values of Da

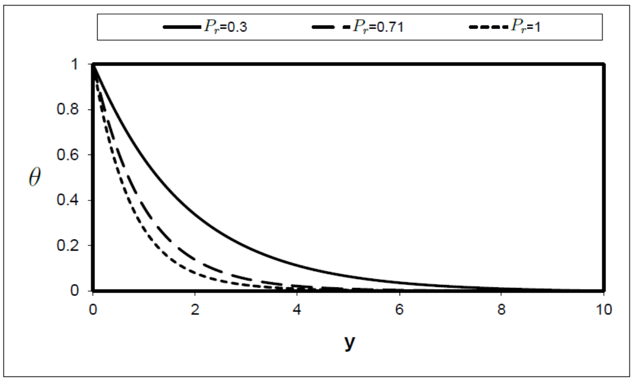

Fig.6 Temperature profiles for different values of Pr

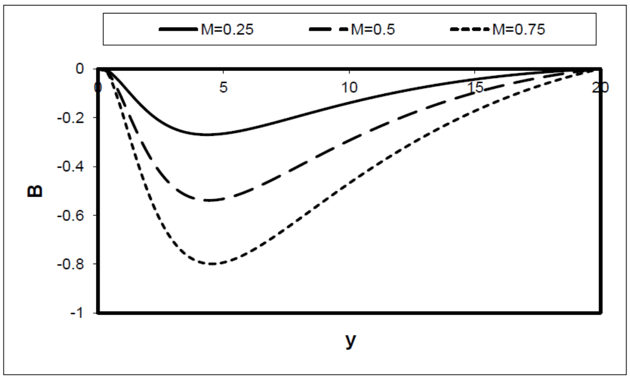

Fig.10 Induced magnetic field for different values of M

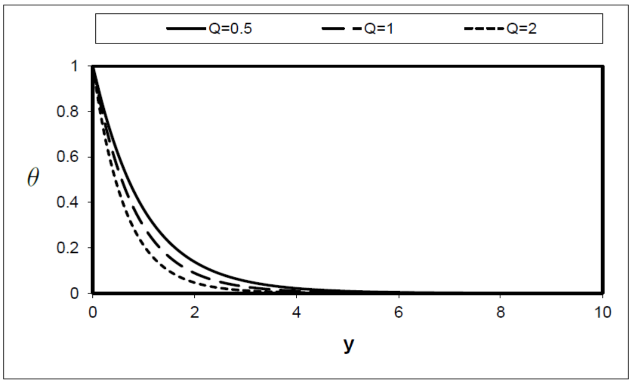

Fig.7 Temperature profiles for different values of Q

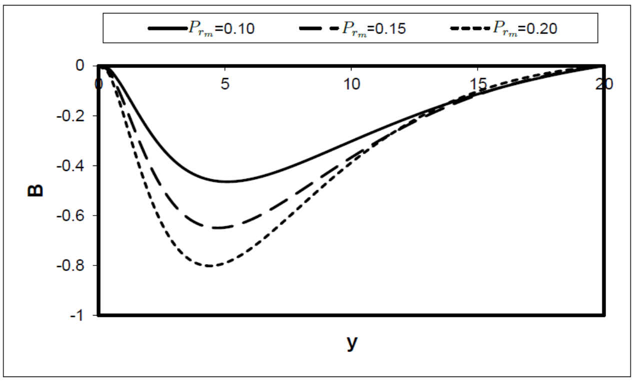

Fig.11 Induced magnetic field for different values of Prm

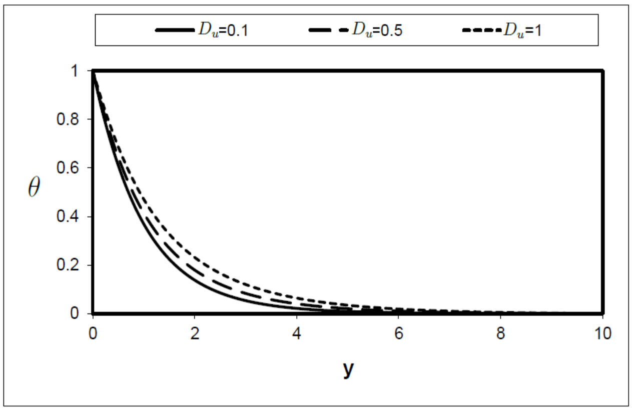

Fig.8 Temperature profiles for different values of Du

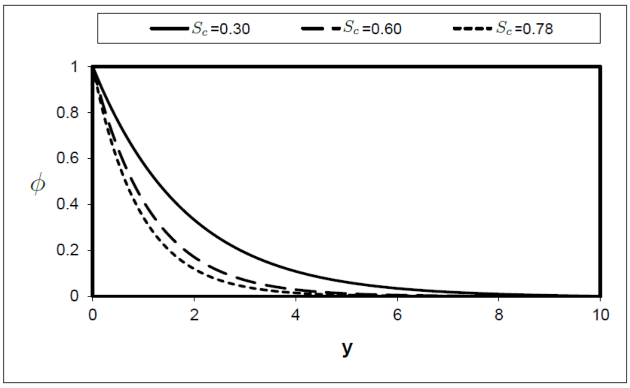

Fig.12 Concentration profiles for different values of Sc

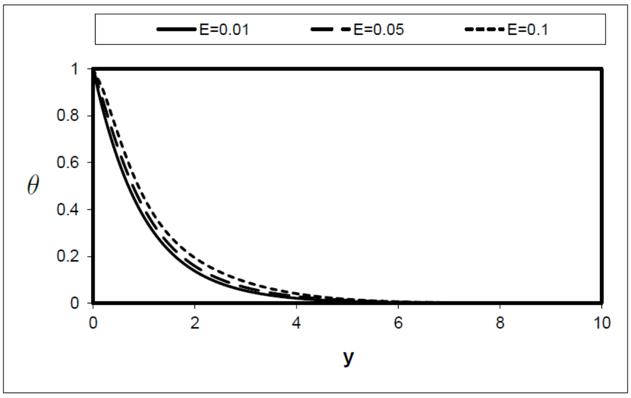

Fig.9 Temperature profiles for different values of E

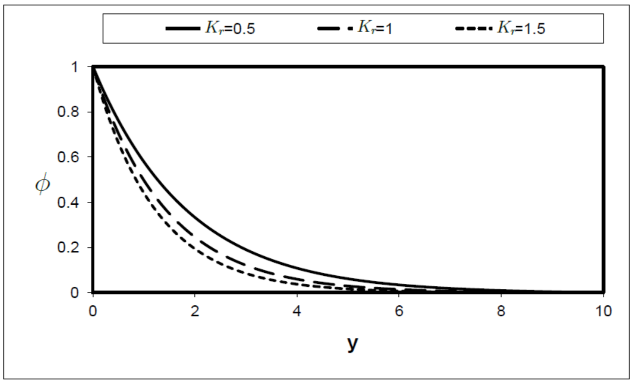

Fig.13 Concentration profiles for different values of Kr

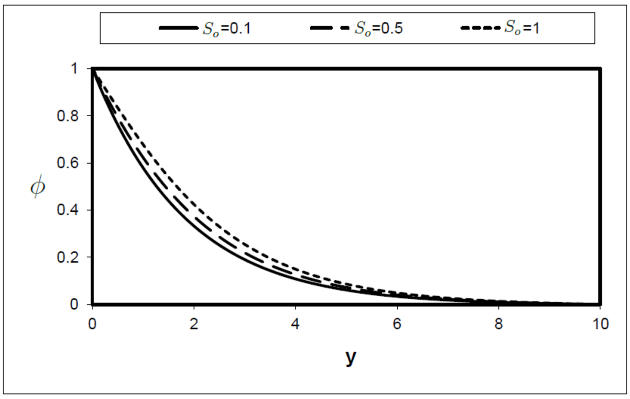

Fig.14 Concentration profiles for different values of So

Conclusions

In this work we have obtained a numerical and analytical solution of the problem of the motion of viscous fluid with heat and mass transfer through porous medium over a vertical infinite permeable plate in the presence of induced magnetic field. The effects of thermal diffusion and diffusion thermo with induced magnetic field is taken in consideration. The transformed system of non-linear, coupled, ordinary differential equations governing the problem were solved numerically by using (FDM) and analytically by using (DTM). The figures and tables clearly show that the results by using (FDM) are in good agreement with the results of the analytical solution by using (DTM). The important results of the study concluded as follows:

1) The velocity distribution increases with the increase of M, Gr and Gm , while it decreases with the increase of the Da.
2) The temperature distribution increases with the increase of Du and E, while it decreases with the increases of Pr and Q.
3) The induced magnetic field distribution decreases with the increase of the magnetic parameter M and magnetic Prandtl number Prm.
4) The concentration distribution increases with increase of S0, while it decreases with the increase of Kr and Sc.

References

V. M. Soundalgekar (1965), Hydro magnetic flow near an accelerated plate in the presence of a magnetic field, Applied scientific research, 12 (1), pp. 151-156.

A. J. Chamka and I. Camille (2000), Effects of heat generation /absorption and thermophoresis on hydro magnetic flow with heat and mass transfer flow over a at surface. International Journal of Numerical Methods for heat fluid flow, 10, pp. 432-448.

T. Sudhakar Reddy, M. C. Raju and S. V. K. Varma (2011), Unsteady MHD free convection Oscillatory Couette flow through a porous medium with periodic wall temperature in presence of chemical reaction and thermal radiation, International Journal of Science and advances technologies, 1 (10), pp. 51-58.

K. R. Cramer and S. I. Pai (1973), Magneto fluid dynamics for engineers and applied physics, Mc Graw Hill Book Co., New York, pp. 204-237.

V. Ravikumar, M. C. Raju and G. S. S. Raju (2012), MHD three dimensional Couette flow past a porous plate with heat transfer, IOSR Jour. Maths., 1 (3) , pp. 3-9.

M. A. Abd Elnaby, N.T. Eldabe and H. A. Asfour (2008), Numerical treatment of non-Darcian effect on pulsatile MHD power-law fluid flow with heat transfer in a porous medium between two rotating cylinders, J .Mech .Cont. Math. Sci., 3, pp. 286-298.

N. T. Eldabe, S. N. Sallam and M. Y. Abou-zeid (2012), Numerical study of viscous dissipation effect on free convection heat and mass transfer of MHD non-Newtonian fluid flow through a porous medium, Journal of the Egyptian Mathematical Society, 20, pp. 139-151.

R. C. Chaudhary and B. K. Sharma (2006), Combined heat and mass transfer by laminar mixed convection flow from a vertical surface with induced magnetic field, J. Appl Phys, 99 , pp.034901-10 .

M. M. Alam, I. M. Rafiqul and F. Rahman (2008), Steady heat and mass transfer by mixed convection flow from a vertical porous plate with induced magnetic field, constant heat and mass fluxes, Tham. Int. J. of Sci. and Tech., 13(4) ,pp.1-13.

M. M. Haque and M. M. Alam (2009), Transient heat and mass transfer by mixed convection flow from a vertical porous plate with induced magnetic field, constant heat and mass fluxes, AMSE Journal, 78(4),54.

A. A. Raptis, C. Perdikis and A. Leontitsis (2003), Effects of radiation in an optically thin gray gas flowing past a vertical infinite plate in the presence of a magnetic field, Heat and mass Transfer, 39 , pp. 771-773.

A. Sahin (2010), Induced magnetic field with radiation fluid over a porous vertical plate: analytical study, Journal of Naval Architecture and Marine Engineering, 7, pp. 83-94.

C. S. Sravanthi, A. Leela Ratnam and N. Bhaskar Reddy (2013), Thermo-diffusion and chemical reaction effects on a steady mixed convective heat and mass transfer flow with induced magnetic field, International Journal of Innovative Research in Science, Engineering and Technology, 2 , pp. 4415-4424.

A. Sahin and Abdul Batin (2010), Analytical model of MHD mixed convective radiation fluid with viscous dissipative heat, International Journal of Engineering Science and Technology, 2(9) , pp. 4902-4911.

J. Zueco and S. Ahmed (2010), Combined heat and mass transfer by mixed convection MHD flow along a porous plate with chemical reaction in presence of heat source, Applied Mathematics and Mechanics, 31(10), pp. 1217-1230.

J. K. Zhou (1986), Differential transformation and its application for electrical circuits, Huazhong Univ. Press, Wuhan, China.