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Research Article

Free Vibration Analysis of Beam Structure Utilized Boundary Element Method and Finite Element Method

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Abstract

All real structures, when subjected to loads or displacements, behave dynamically. One of the most important problems accounted in structural engineering is the vibration analysis of beams subjected to static loads. The boundary element formulation for the free vibration analysis of beam structures characterized behavior utilizes in this work to find the natural frequencies and the influence function of the concrete bridge with slant-legged rigid frame which modeled using three types of beam (T-section, I-section , and box-section) for simply supported end conditions using boundary element method. The results obtained by boundary element method are compared by Exact solution and Finite element method for the simply supported beam and the results have that the converging with another techniques.

Keywords: *Free Vibration, Boundary Element Method, Beam Structure, Simply Support*

Introduction

The beam structure as (Slant legged bridge) is one of the most widely used highway bridges. This type eliminates the need for concrete piers and position the supports away from lower roadway. The response of a bridge under a moving (force) vehicle is a complex phenomenon because of the interaction between bridge and the vehicle. Bridge structures that have long service years or long spans are frequently subjected to heavier loadings than their design loads are greatly affected by heavy traffic – induced vibrations.

Tong Lo Wang and **Dongzhou Huang** investigated the dynamic response of a slant legged rigid frame bridge to one or two tracks (side by side)passing over the rough bridge deck. The bridge was modeled as a space bar system. In the free vibration study, each of the longitudinal girders were divided into fifty two elements and each of the leg into five elements. Since, the Mechanical behavior of slant legged rigid frame bridge is same as arch bridge, the first mode shape is anti-symmetrical mode. The second and fourth modes are vertical bending, lateral bending and torsional vibration modes. The third and fifth modes are symmetric vertical bending modes.

Maisel *et al*. conducted studies on concrete box-girders to examine amongst others torsional behaviour. Single cell rectangular box-girders with different geometrical proportions and loaded with an eccentric point-load at mid span were examined. The maximum longitudinal stress at the bottom line was increased through warping and distortion of the girder. To calculate the Eigen frequencies of a beam, its differential equation is expressed by finite difference formulations and the eigenvalues are computed. **Petersen** gives examples for the use of this method for Eigen frequency calculations of beams with constant cross section. Those calculations are rather simple, but accurate even with regard to elastic supports and only few points for the numerical calculation. A collection of examples for the expression of the differential equations of beams with variable cross-sections and varying boundary conditions is found. The Årsta Bridge is comparable with the continuous beam in this paper or, if only one span is considered, with a simple beam and the necessary boundary conditions.

Huang studied impact loading and dynamic behavior of half-through arch bridges and proposed a method for estimating the dynamic response of this type of arch bridge. In his study, both bridge and vehicle were modeled three-dimensionally. **Lacarbonara and Colone** studied the dynamic responses of arch bridges from high speed trains using the Ritz energy method. Most previous investigations of the dynamic analysis of deck-arch bridges may be deficient in some or all of the following aspects: (1) the stiffness effect of the deck stringers, floor beams, and spandrel columns was neglected; (2) the deck surface

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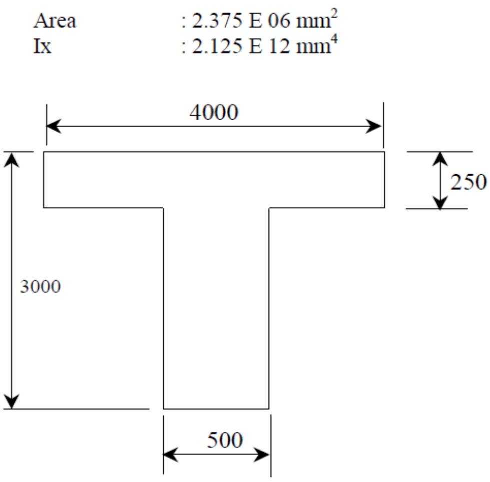
was assumed as smooth; (3) the vehicles were modeled as constant moving forces without considering their mass and spring effect. The dynamic behavior of deck-arch bridges from moving vehicles remains largely uncertain.

The organization of this paper is presents a brief presentation of the governing equations and boundary conditions for the vibration analysis of beam structure by boundary element formulation. This formulation includes the boundary integral representation of the displacement and its normal derivative accompanied by all possible boundary conditions, and then this study investigates the dynamic behavior of concrete T-beam, I-beam, and box-beam bridge sections model of an actual sections design implemented within the context of the finite element method and BEM results extracted from applying the influence function by using the fundamental solution show a general trend for closer values to the exact solution than those calculated by FE modeling. Also the research results shown that analysis of T-frame bridges may be conveniently performed using the model.

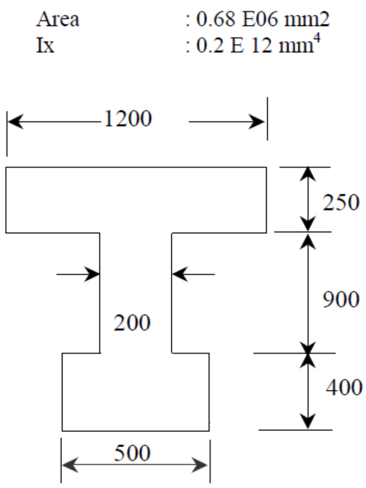
Description of the bridge sections

The structure considered in this case study is a T-beam , I-beam, and box-beam concrete bridge with slant-legged rigid frame. The simply supported span has a length of 48 m for both sections, the bridge section widths is shown in figure (2) which shows a cross-sections view of the frame. The concrete properties used in the analysis are:

Density of Concrete=2380kg/m^3, Poisson's Ratio=0.2, Modulus of Elasticity of Concrete=22.39×10^3 Mpa (Based on the ACI formula $E_c=4730*(f_c)^{1/2}$ for normal – weight concrete).



(a) Tee Section



(b) I - Section (Unsymmetrical)

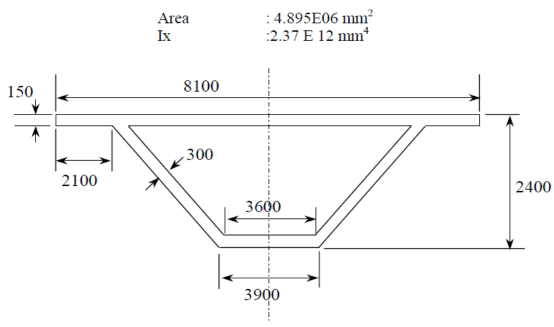


Fig.2: Cross sections area for the study (a) Tee section, (b) I –section Unsymmetrical, and (c) Box-section

Mathematical model

The concrete T-beam, I-beam, and box-beam Bridge sections behavior in static and dynamic (free vibration) analysis is governed by the following boundary element equations, which applies to beams. But first we want to provide the necessary background material for readers who are not familiar with beam analysis.

The deflection $w(x)$ of a beam with a constant stiffness El satisfies the differential equation

$$E\,I\,w^{IV}(x) = p$$

To this operator belong two integral identities

p: $\widehat{w}, w \in C^4\,[0, l]$ X $C^2[0, l]$,

$$q{:}\; G(\widehat{w},w)=\int_0^l EI\,\widehat{W}^{IV}\,w\,dx + [\,\hat{Q}\,w - \widehat{M}\acute{w}]_0^l - \int_0^l \frac{\widehat{M}\,M}{E\,I}dx=0 \quad (1)$$

and

p: $\widehat{w}, w \in C^4[0,l]$,

$$q{:}\; B(\widehat{w}, w) = \int_0^l EI\,\widehat{W}^{IV}\,w\,dx + [\hat{Q}w - \widehat{M}\acute{w} - \widehat{M}\acute{w} + \widehat{W}'\,M - \widehat{w}\,Q]_0^l - \int_0^l \widehat{w}\;EIw^{IV}dx = 0 \quad (2)$$

We say $go(y, x)$ or g1 (y, x), respectively, is a solution of the differential equations

$$E\,I\,g_0{}^{iV}\{y,x)=(\delta_0(y-x) \qquad EIg_1^{iv}(y,x) = \delta_1(y - x)$$

if the shear force or the bending moment, respectively,

$$Q = -EI\frac{d^3}{dy^3}go\,(y,x),\; M=-El\frac{d^2}{dy^2}g_1(y,x)$$

suffers at the source point i a jump discontinuity of magnitude 1.

$$\lim_{Z\to 0}\; \{Q(x+\varepsilon,x) - Q(x-\varepsilon, x)\} = 1,$$
$$\lim_{Z\to 0}\; \{M(\chi+\varepsilon, x) - M(x-\varepsilon,x)\} = 1$$

If go is such a fundamental solution and x an interior point then the identities read

$$G(go, w) = w(x) + [Q_{0w} - M_0w]_0^l - \int_0^l \frac{M_0M}{EI}dy \quad (3)$$
$$B(go,w) = w(x) + [Q_0w - M_0w' + g'_0M - g_0Q]_0^l - \int_0^l g_o\,EI\,w^{lv}\,dy = 0 \quad (4)$$

In the case of the second solution g_1 we only replace the free term. $w(x)$ by $w'(x)$. This concludes the introduction and we shall show in the following how these results are applied to solve beam problems.

The deflection of the beam in Figure (1) can be calculated by forming the 2-scalar product between the bending moment $M_0(y,x)$ and M(y)

$$1X\,w(x)= \int_0^l \frac{M_0(y,x)M(y)}{EI} \quad (5)$$

Or by forming the L2-scalar product between the deflection $G_0(y,x)$ that is caused by a concentrated force $\hat{p}$ =1 acting at x and the constant load p

$$1X\,w(x) = \int_0^l G_0(y,x)p\,(y)dy \quad (6)$$

Equation (5) is based on the *principle of virtual forces*: the external work 1 x $w(x)$ is equal to the virtual internal strain energy, and Eq.(6) on *Betti's principle:* the reciprocal external work of two equilibrium systems is the same .

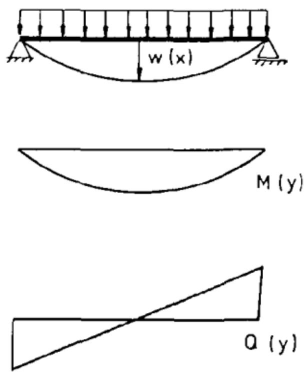


Fig.1 Deflection of beam

$$g_0(y,x) = \frac{1}{6EI}X\begin{cases}\alpha(x)y-(1-x)y^3 & y\le x\\ (y-x)^3+\alpha(x)y-(1-x)y^3 & x\le y\end{cases}$$

$$\alpha(x) = x(1-x)(2-x)$$

Beams then have the same length and both are they in equilibrium so that the reciprocal external work of their exterior forces must be the same:

$w_{1,2}$= 1 X *w(x)* — Qo(0,x) w(0) + Mo(0, *x)w'(0)* + $Q_0(l,x)w(l)$ -$M_0(l,x)w'(l)$ = $\int_0^1$ $go(y,x)p(y)dy$ - $Q(0)g_o(0,x)$ + M(0) g_0' (0, x) + $Q(l)go(l,x)$ - $M(l)g'_0(l, x)$ = $W_{2,1}$

On the left we find the work of the exterior forces of the auxiliary beam and on the right the work of the exterior forces of the real beam. If we put the term 1 x *w(x)* on the left side alone,

1x $w\{x\} = \int_0^l$ *gopdy* + work on the boundary ($W_{2,1}$)
-work on the boundary ($w_{1,2}$)

Then we obtain an influence function for the deflection *w(x).* To calculate the rotation w'(x) we let a concentrated couple M = 1 act on the infinite beam the corresponding deflection is g_1 (y,x) = dg_0/dx and we repeat the formulation of Betti's principle

1 x $w'(x) = \int_0^l$ $g_1\, pdy$ + work on the boundary ($w_{2,1}$)
-work on the boundary ($w_{1,2}$) ,

W(0) = $\int_0^l g_0[0]\ p\ dy$ + work on the boundary - work on the boundary

W(1) = $\int_0^l g_0[l]\ pdy$ +work on the boundary - work on the boundary,

$\dot{w}(0)= \int_0^l g_1$ [0] *pdy +* work on the boundary - work on the boundary,
$\acute{w}(1)= \int_0^1 g_1[l]$ *pdy* + +work on the boundary —-work on the boundary,

To make this clear we, first, make all boundary terms the same positive direction and we then put all the

u_1 = w(0), u_2 = -w'(0), u_3 = *w(l)*, u_4= -$\acute{w}$(l),

on the left side and all the force terms

f_1= -Q(0), $f_2 = -M(0)$, $f_3 = Q(l)$, f_4 = M(l),

on the right side. The resulting four equations can then be written as

$$H_{2\,3}\, u_3 = G_{23}\, f_3 + d_z \tag{7}$$

Hence, the stiffness matrix of a beam formulates a coupling condition between the end displacements and end actions of a beam. In conjunction with the influence function for the deflection

$$w(x)= xw(l) + (1 - x)w(0) + x(l - l)w'(l) + (1/6El)\{[-3(1 - X) -\alpha\ (x)+ 3l^2(1\text{-x})\ M\ (l) + \alpha(x)M(0) +[(l - x)^3 + a(x)l - (1\text{-x})\ l^3\ Q(1) + \int_0^x[\ \alpha(x)y-(1-x)y^3]\ p(y)dy + \int_x^l[\ (y-x)^3 +\ \alpha\ (x)y\text{-}\ (1 - x)y^3]p(y)\ dy, \tag{8}$$

$\alpha\ (x)$= x(1-x) (2-x)

To determine these four unknown terms

$$\frac{EI}{l^3}\begin{bmatrix}12 & -6l & -12 & -6l\\ 0 & 4l^2 & 6l & 6l^2\\ 0 & 0 & 12 & 6l\\ sys & 0 & 0 & 4l^2\end{bmatrix}\begin{bmatrix}w(0)\\ -\acute{w}(0)\\ w(1)\\ -\acute{w}(1)\end{bmatrix} = \begin{bmatrix}-Q(0)\\ -M(0)\\ -Q(1)\\ -M(1)\end{bmatrix} + \begin{bmatrix}p1\\ p2\\ p3\\ p4\end{bmatrix} \tag{9}$$

where

$p_i=\int_0^l p(x)\,\psi i(x)dx$

The terms p_i are the negative end fixing forces.

Dynamical loads cause inertial forces ρ u¨ in a structure. These forces appear on the left-hand side of the differential equation

$Du+\rho\, \ddot{u}=p(x,t)$

Now the differential equation of the vibrating beam

$E\,I\, w^{IV} + \mu\, \ddot{w} = p(x) \cos (wt+\varphi)\ ,\ \mu=\rho\, A$

Become after a separation of the variables

$W(x,t)=w(x) \cos(wt+\varphi)$

A differential equation for the amplitude

$E\,I\, w^{IV}(x) - \mu\, \omega^2\, w(x) =p(x)$

To this differential equation belong the identities

$G(\hat{w},w)=\int_0^l(EI\ \hat{\omega}^{IV}-\mu\,\omega\, 2)w\ dx+[\ \hat{Q}\ w-\hat{M}\acute{w}]_0^l$ - $\int_0^l$ $(EI\hat{\omega}''\omega''-\ \mu\,\omega 2\ \hat{\omega}\,\omega)dx = 0$

The general homogeneous solution is

$w(x)=a_1\cos(\lambda x) +a_2\sin(\lambda x)+a_3\cosh(\lambda x)+a_4\sinh(\lambda x)$

where $\lambda=(\mu\omega^2/EI)^{1/4}$

by an appropriate choice of the integration constants we can obtain four solution ψ_i which correspond to unit end displacements

$\psi_1(0)=1\ ,\ \psi'_1(0)= \psi_1(l)= \psi'_1(l)=0$

The energy producers of these functions constitute the elements of the stiffness matrix k. this matrix formulates a coupling condition between the end displacement u_i and end actions f_i of a smooth amplitude $w \in C^4\ [0,l]$

$$Ku=f+p \tag{10}$$

Where $k_{ij}=E(\psi_i,\psi_l)= \int_0^l(EI\ \psi_i\ \ \psi_l - \mu\omega^2\ \psi_i\ \psi_l)\ dx$
$p_i=\int_0^l p(x)\,\psi i(x)dx \qquad p(x) = EI\ w^{IV} - \mu\ \omega^2\ w$

By a simple rearrangement of equation (10) we can derive the transfer matrix and, therefore, the matrix-displacement method is complete.

Table 1: Effect number of elements for beam with the parameter β, (Natural frequency $w_n = \beta \sqrt[4]{\frac{EI}{ML^4}}$)

Mode	No. of Elements (FEM , BEM)	Exact Solution	FEM Solution	BEM Solution	Error %	
					$\frac{Exact - BEM}{Exact}$	$\frac{Exact - FEM}{Exact}$
1	256 , 35	3.141592	3.131481	3.141571	0.000	0.321
2		6.283185	6.262578	6.282979	0.003	0.328
3		9.424777	9.374246	9.404565	0.214	0.536
4		12.56637	12.49578	12.53428	0.255	0.561
5		15.70796	14.98759	14.99688	4.525	4.586

Results and Discussion

In this section, the results of simply support beam were presented. Table (1) presents the effect number of elements on the values of non-dimensional natural frequencies between the methods (FEM and BEM) comparing with Exact solution of beam Some types of error are increase or decrease with increasing or decreasing the number of elements and it is noted that, 35elements of BEM gave a convergence in the results (with Exact) and then they will be used to discretize the beam system.

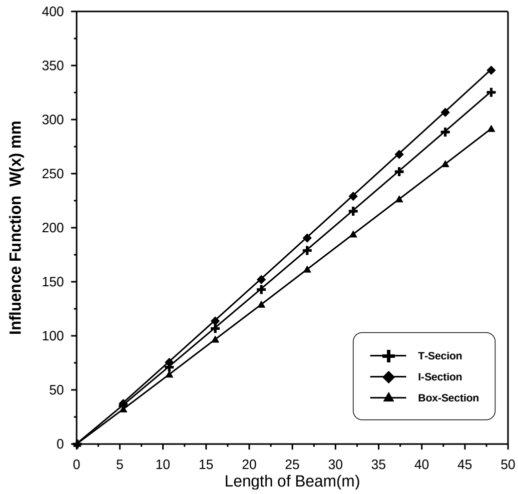


Fig.4: The Influence Function of Three Types of Beams

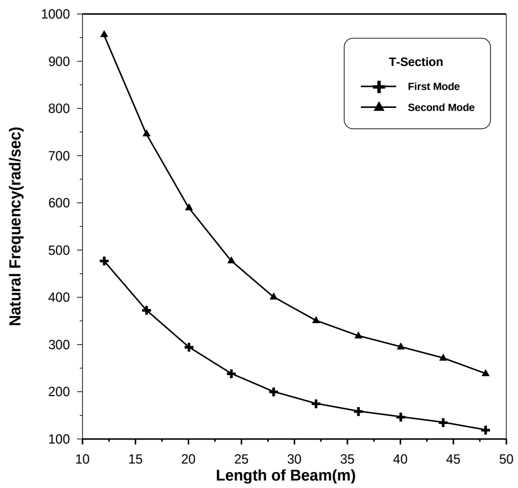


Fig.5: The 1st & 2nd mode natural frequency of T-beam

The properties of a bridge, which are used for the calculations of beam structure analysis for three types of cross sections (I, T, Box section) as shown in figures. Fig (4) shown the influence function of beam with the length where the relation between them its proportional with increase the length of the beam lead to increase the influence function of beam, equation (8) represent the influence function as a function of load support and moment reactions depend on the condition of ends beam.

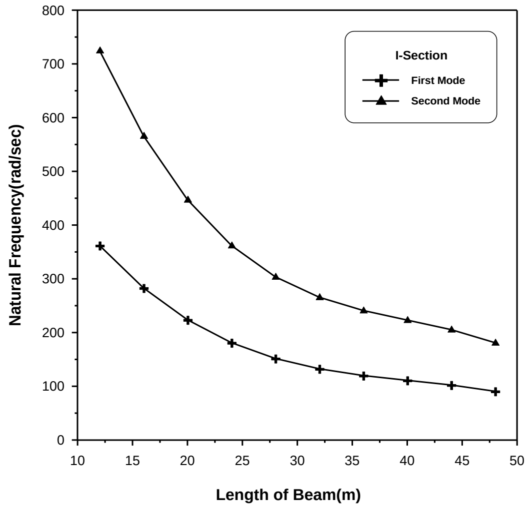


Fig.6: The 1st & 2nd mode natural frequency of I-beam

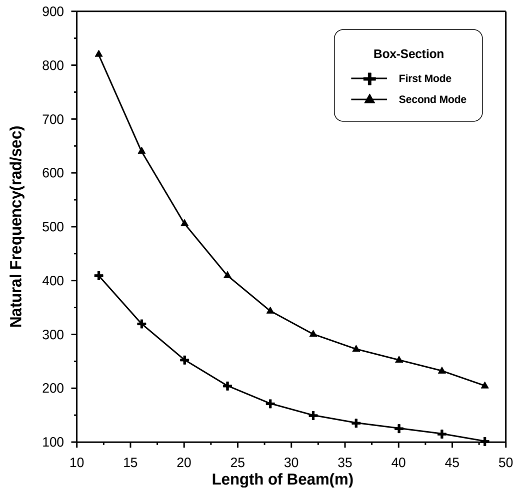


Fig.7: The 1st & 2nd mode natural frequency of box-beam

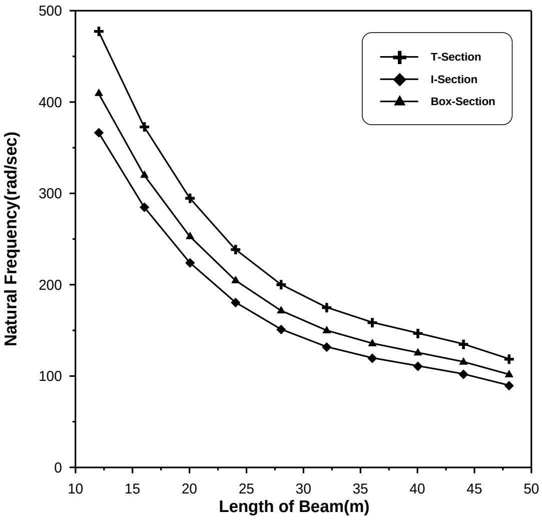


Fig.8: Natural frequency of 1st mode for the three types of sections

Figs.(5-8) present effect the length on the natural frequency of simply supported beam structure . The figures are remarked as data points for easier comparison. As a general view, it was noted that the frequency is decreased with increasing the length of beam. For the first and second modes for same type of beam and vibration mode is symmetric vertical bending. The T-section clearly has greater value of frequency from another type.

Conclusions

This paper mainly focused on the boundary element solution for natural frequency for three type of concrete bridge with slant-legged rigid sections. The following conclusions are drawn

1) The dynamic behaviors of a rigid T-section, I-section, Box-section Bridge were investigated by boundary element method. Based on the comparison study on exact results, one may obtain more accurate designs using the boundary element solutions.
2) The cross section area of the beam structure has important role in design and dynamic behavior.
3) The T-section beam structure given the large value of the natural frequency.
4) The fundamental natural frequency of T & I-beam could be numerically specified and it was almost identical to the theoretical solution of a simply supported beam
5) The Boundary Element Method appears good agreement when comparison with Exact and Finite Element Method.

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