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Research Article

Energy-based Control of an Underactuated Crane System with a Flexible Cable and Large Swing Angle

Mohammad H. Fatehi[†*], Mohammad Eghtesad[‡] and Roya Amjadifard[†]

[†] Department of Electrical Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran.
[‡] School of Mechanical Engineering, Shiraz University, Shiraz, Iran.
[†] Engineering Faculty, Kharazmi University, Tehran, Iran

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Abstract

In this paper, an effective energy based control technique for an overhead crane system with a flexible cable with large swing angle is developed. The studied crane system is categorized as a multi-degree underactuated system whose characteristics can initiate challenges in control design. Thus, simultaneously moving the trolley/payload and suppressing the payload swing and cable vibration is difficult. Due to such a difficulty, to overcome the complexities of the control problem, a corresponding energy-based control strategy will be proposed using linearized model and controlled Lagrangian method. The control objective is moving the payload to the desired position and at the same time, reducing the payload swing and suppressing the cable transverse vibrations. The controller guarantees both tracking of the desired payload position and active damping of payload swing and cable vibration. The simulation results are presented to demonstrate the dynamic behavior and effectiveness of the control system for an illustrative example of the crane systems with flexible cable moving a lightweight payload.

Keywords: *Overhead crane system, Flexible cable, Swing and vibration suppression, Controlled Lagrangian, Underactuated mechanical system.*

1. Introduction

The crane systems are extensively employed in a variety of applications in industries such as land and onshore/offshore construction sites, transportation industry, etc. The most common operation of a crane is point-to-point carrying of a suspended load horizontally, by means of cables and a support mechanism. The cables possess an inherent flexibility and can only develop tension; they do not offer resistance to bending moments or compressive forces. Such natural characteristics certainly cause deflection in transversal direction of the cable and payload swing in crane systems. The suspended load in crane systems is always subject to swings happen by unskilled operators or by disturbances typically induced by motor drive transients, wind, and collision with objects so that it can cause lengthy transportation activities; even the swings may possibly become large and reduce the safety of crane systems.

Abdel-Rahman and Nayfeh presented a detailed review of the challenges in modeling and control of the crane systems (Abdel-Rahman *et al.*, 2003). In most dynamical models, the effects of flexibility and weight of the suspended cable have been ignored and the cable has been considered as a mass-less rigid-link or as a rigid-link including a point-mass (Collado *et al.*, 2000), (Fang *et al.*, 2003), (Fang *et al.*, 2012), (Lee, 1998), (Ma *et al.*, 2008), (Ma *et al.*, 2010), (Park *et al.*, 2008), (Sun and Fang, 2012, 2014), (Sun *et al.*, 2010), (Sun *et al.*, 2011), (Sun *et al.*, 2012), (Yesildirek, 2011).

In these studies, the load swing has been assumed as the major dynamic motion in a crane system. Although such assumptions are usual, they are not genuine for many applications. In certain cases, especially when payload is lightweight and more importantly when cable is long, the effect of flexibility has to be taken into account. In these cases, the tension force is more dependent on the cable weight and also it will be varying along the cable. Under such conditions, the tension force, especially at the end of the cable is low so that the cable weight may have more dynamical effects and transverse vibrations of the cable can take over the behavior of the crane (Starossek, 1994). Thus, to utilize the crane systems in a particular application, in addition to the payload swing, the cable vibration should be suppressed within a given period of time for safety issues; therefore, development of an effective suppression control system is indispensable. In order to achieve these objectives, a more accurate model with more details including the dynamic interconnection of the cable and the payload is a requisite.

*Corresponding author: **Mohammad H. Fatehi**

A few studies have addressed the effects of cable flexibility and weight in crane systems with flexible cables (Alli and Singh, 1999), (D'Andréa-Novel *et al.*, 1992), (D'Andréa-Novel *et al.*, 1994), (D'Andréa-Novel and Coron, 2000, 2002), (Formal'sky, 1999), (Joshi and Rahn, 1995), (Moustafa *et al.*, 2005), (Moustafa *et al.*, 2009), (Rahn *et al.*, 1999), (Thull *et al.*, 2006). They have presented planar models for overhead cranes and assumed that the cable is perfectly flexible and inextensible. Also, they have assumed that the cable slop is small along the cable and also disregarded the swing angle; in other words, in these studies, in a crane system, the major dynamic motion of cable has been assumed to be the cable vibrations. However, this is valid only for slow movements of the trolley or support mechanism and near the end of the traveling. These models might not be accurate when considering certain unavoidable physical and environmental conditions. The angular rotation of cable swing in most applications especially when the high traveling speed is required becomes large. This issue has been disregarded in nearly all of the previous studies on the crane system with a flexible cable (Alli and Singh, 1999), (D'Andréa-Novel *et al.*, 1994), (D'Andréa-Novel and Coron, 2000, 2002), (Formal'sky, 1999), (Joshi and Rahn, 1995), (Moustafa *et al.*, 2005), (Moustafa *et al.*, 2009), (Rahn *et al.*, 1999), (Thull *et al.*, 2006). In an infrequent study, an approximated model was found by introducing the effect of small swing angle just in the trolley dynamics in a crane system with flexible cable (D'Andréa-Novel *et al.*, 1992). In a recent study by the authors, a more accurate dynamical model for an overhead crane system with flexible cable was developed where the swing rotation of the payload and cable was not restricted to small angles and large swing rotations were considered and then a simple linear controller was applied to move trolley/payload and suppress both cable vibrations and payload/cable swing (Fatehi *et al.*, 2014). This system is categorized as a multi-degree underactuated system whose characteristics can impose serious challenges when applying control methods.

An underactuated system is a system that has fewer independent control actuators than the number of degrees of freedom to be controlled. In recent years, a great attempt has been made in applying the energy-based techniques to the control of underactuated systems (Bloch *et al.*, 2001), (Bloch *et al.*, 2006), (Bo and Hayakawa, 2004), (Chang, 2012), (Dong *et al.*, 2008), (Gao *et al.*, 2009), (Hu *et al.*, 2007), (Liu and Yu, 2013), (Ng *et al.*, 2013), (Ortega *et al.*, 2001), (Sun and Fang, 2012). The main advantage of these methods is that the highly coupled underactuated system dynamics can be analyzed via the system energy with reasonable control performance. Chung and Hauser proposed a nonlinear controller to regulate the swinging energy of the pendulum for a cart and pendulum system (Chung and Hauser, 1995). In some other studies, it has been tried to use the passivity and energy shaping methods to control overhead cranes (Chang, 2012), (Collado *et al.*, 2000), (Fang *et al.*, 2003), (Sun and Fang, 2012). The well-known and extensively studied underactuated systems have one or more actuated variables and only one un-actuated variable such as the following benchmarks: inverted pendulum system, TORA, Pendubot, Acrobot (Fantoni *et al.*, 2000), (Gao *et al.*, 2009), (Lozano *et al.*, 2000), (Tadmor, 2001). These studies reveal that controlling a one-degree underactuated system is very complicated and control design for multi-degrees underactuated system is much more difficult.

The purpose of this study is to present a control design for a crane system with a flexible cable which it is categorized as a multi-degree underactuated. The energy-based techniques are useful to deal with the difficulty of underactuated problem. In the current study, an energy-based method is used to design a controller for the flexible cable crane system applying the controlled Lagrangian procedure and passivity characteristic. The control objective is to generate a driving force for moving the trolley and the payload to the desired final position and at the same time to reduce the payload swing and to suppress the cable transverse vibrations. To achieve two control objectives, swing angle regulation and payload and trolley tracking simultaneously, a potential and kinetic energy shaping based on controlled Lagrangian method will be used. To overcome the complexities of the control problem in using controlled Lagrangian method for the studied multi-degrees underactuated system, system is linearized at its equilibrium point. Then the controlled Lagrangian method is applied for the linearized system to design its controller. First, the matching conditions for the crane system are derived. The matching conditions are solved and associate controllers are obtained. The controller guarantees both moving of payload to desired point and active damping of payload swing and cable vibration. To demonstrate the effectiveness of the proposed control system, the numerical simulations are performed using commercial software.

This paper is organized as follows: The system description is presented in Section 2 which contains the enhanced dynamic equations of motion developed by the authors according to (Fatehi *et al.*, 2014). In Section 3, an energy-based controller is design for crane system using controlled Lagrangian method and in Section 4, the stability analysis of overall control system is presented. To demonstrate the dynamic behavior and effectiveness of the control system for an illustrative example of the crane systems with flexible cable moving a lightweight payload, the simulations are performed in Section 5. Finally, in Section 6, the conclusions are drawn.

2. System Description and Dynamic Equations

An overhead crane system is composed of a support mechanism as a trolley and a flexible cable tied to the suspended payload. The swing motion of the payload and transverse vibrations of cable in this kind of crane system, can be described in plane using two coordinate frames i.e., XZ and $\hat{X}\hat{Z}$, see Fig.1. As shown, three kinds of motion are considered, i.e. crane traveling, swing angle and transverse vibrations which are described as x, θ, and $v(\hat{z},t)$, respectively. It is assumed that every point on the cable has two degrees of freedom; one is the transverse deflection $v(\hat{z},t)$ around the $\hat{Z}$-axis and the other is the swing angle, θ, which is not assumed to be small in this study. Let x and F_x be the trolley position and trolley driving force, respectively.

The parameters M_t, m_p, ρ, ℓ and g are the total mass of the trolley, payload mass, mass per unit length of cable, cable length and gravitational acceleration, respectively. The payload is considered as a point mass and the motion of the trolley on the rail is assumed to be frictionless. The cable is assumed to be inextensible and the transverse deflection is small. To achieve an ODE model describing the transverse deflection of a cable with finite degrees of freedom (modes), the Rayleigh-Ritz discretization method (Meirovitch, 2001) can be used in which the spatial function $v(\hat{z},t)$ is approximated as the finite sum of shape functions $\phi_j(\hat{z})$ multiplied by the time-dependent generalized coordinates $\mu_j(t)$:

$$v(\hat{z},t) = \sum_{j=1}^{m} \mu_j(t)\phi_j(\hat{z}) = P^T(\hat{z})\mu(t) \tag{1}$$

where, $P(\hat{z})$ and $\mu(t)$ are:

$$\begin{aligned} P(\hat{z}) &= [\phi_1(\hat{z}) \quad \phi_2(\hat{z}) \quad \cdots \quad \phi_m(\hat{z})]^T \\ \mu(t) &= [\mu_1(t) \quad \mu_2(t) \quad \cdots \quad \mu_m(t)]^T \end{aligned} \tag{2}$$

To choose the shape functions $\phi_j(\hat{z})$, the boundary conditions must be satisfied. A useful choice for shape functions to achieve high precision is comparison functions which can satisfy both the geometric and natural boundary conditions, but the comparison functions are often unavailable, as in the studied system. One appropriate way is to use a different class of shape functions, so-called quasi-comparison functions as a linear combination of admissible functions. In order to form the quasi-comparison functions, two admissible functions are used, one satisfying the boundary conditions, $w(0,t) = w(\ell,t) = 0$ and the other satisfying $w(0,t) = w_{\hat{z}}(\ell,t) = 0$. However neither of the conditions are consistent with the actual situation; rather the following quasi-comparison function can be used:

$$\phi_j(\hat{z}) = \lambda sin\left(\frac{2j\pi}{2\ell}\hat{z}\right) + (1-\lambda) sin\left(\frac{(2j-1)\pi}{2\ell}\hat{z}\right) \tag{3}$$

Where $j = 1,2,3,\cdots$ and $0 < \lambda < 1$ can be assumed as an arbitrary constant weight. The dynamic equations of motion of the studied crane system are derived in a matrix form as, (Fatehi *et al.*, 2014):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = W.u \tag{4}$$

where, $W = [w_1^T \quad 0 \quad \cdots \quad 0]^T$ and $w_1^T = [1 \quad 0]$, $u = F_x$ is trolley driving force and $q = (x, \theta, \mu)^T \in R^n$ in which x and θ are crane traveling and swing angle, and $\mu = (\mu_1, \mu_2, \cdots, \mu_m) \epsilon R^m$ is the vector of the generalized coordinates of cable, and $n = m + 2$. $M(q)$ is the total inertial matrix, the second term represents the Coriolis and centripetal forces and $G(q)$ is the potential force due to stiffness and gravitation effects in the system. The symmetric inertia matrix, $M(q)$, can be assembled and simplified as:

$$M(q) = \begin{bmatrix} a_0 & a_1\cos(\theta) - \sin(\theta)\, a_2\mu & \cos(\theta) a_2 \\ \times & a_4 + \mu^T a_3 \mu & a_5 \\ \times & \times & a_3 \end{bmatrix} \tag{5}$$

Also, matrix $C(q,\dot{q})$ and vector $G(q)$ can be assembled as:

$$C(q,\dot{q}) = \begin{bmatrix} 0 & c_{12} & -a_2 sin(\theta)\dot{\theta} \\ 0 & \mu^T a_3 \dot{\mu} & \dot{\theta}\mu^T a_3 \\ 0 & -\dot{\theta} a_3 \mu & 0 \end{bmatrix} \tag{6}$$

$$G(q) = \begin{pmatrix} 0 \\ g_1 sin(\theta) + sin(\theta)\, \mu^T g_2 \mu + g_3 cos(\theta)\, \mu \\ -2cos(\theta) g_2 \mu + g_3^T sin(\theta) \end{pmatrix} \tag{7}$$

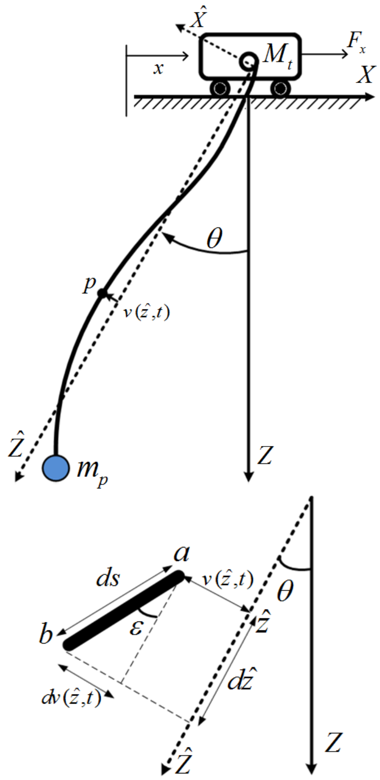

Fig.1 Coordinate frames and the schematic of the overhead crane system and, an infinitesimal element ds at general point p.

where, $c_{12} = -a_1\, sin(\theta)\,\dot{\theta} - a_2\mu\, cos(\theta)\,\dot{\theta} - sin(\theta)\, a_2\dot{\mu}$. The vector $G(q)$ can be rewritten as $G(q) = K(q)q$ where,

$$K(q) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{sin(\theta)}{2\theta}(2g_1 + \mu^T g_2 \mu) & \frac{1}{2} sin(\theta)\, \mu^T g_2 + g_3\, cos(\theta) \\ 0 & \frac{sin(\theta)}{\theta} g_3^T & -2\, cos(\theta)\, g_2 \end{bmatrix} \tag{8}$$

and the respective parameters are:

$$a_0 = M_t + m_p + \rho g \ell \qquad a_1 = m_p \ell + \tfrac{1}{2}\rho \ell^2$$
$$a_2 = \rho \int_0^{\ell} P^T d\hat{z} \qquad a_3 = \rho \int_0^{\ell} (P^T P) d\hat{z}$$
$$a_4 = m_p \ell^2 + \tfrac{1}{3}\rho \ell^3 \qquad a_5 = \rho \int_0^{\ell} \hat{z} P^T d\hat{z}$$
$$g_1 = \left(m_p \ell + \tfrac{1}{2}\rho \ell^2\right) g \tag{9}$$
$$g_2 = \tfrac{1}{2}\rho g \int_0^{\ell} \hat{z} \left(\tfrac{\partial}{\partial \hat{z}} P\right)\left(\tfrac{\partial}{\partial \hat{z}} P\right)^T . d\hat{z}$$
$$-\tfrac{1}{2}(m_p g + \rho g \ell) \int_0^{\ell} \left(\tfrac{\partial}{\partial \hat{z}} P\right) . \left(\tfrac{\partial}{\partial \hat{z}} P\right)^T . d\hat{z}$$
$$g_3 = mg P^T(\ell) + \rho g \int_0^{\ell} P^T d\hat{z}$$

These nonlinear coupled ordinary differential equations describe the dynamic motions of the whole crane system with flexible cable. It is clear that q is the configuration variable vector of the system with $q_1 = x$ is the actuated variable and $q_2 = (\theta, \mu)^T$ is the un-actuated variable vector of the system. Since there are three configuration variables to be controlled with only one actuated configuration variable, the flexible cable crane system is an underactuated system. There are some general properties of inertia matrix $M(q)$, note that this matrix is symmetric, and positive definite for all q. Calculating $\dot{M} - 2C$ is a skew-symmetric matrix which has an important property

$$z^T\left[\dot{M}(q) - 2C(q,\dot{q})\right] z = 0 \quad \forall z \tag{10}$$

3. Energy-Based Control Design

The obtained crane model is a complex matrix equation with strong coupled dynamics. The crane degrees of freedom are trolley motion, cable deflection and payload/cable swing where the second and third kinds of degrees are underactuated. Control design for an underactuated system is challenging and attracts many researchers. The well-known and extensively studied one-degree underactuated systems such as the following benchmarks: inverted pendulum system, TORA, Pendubot, Acrobot. The number of actuated degrees is no less than the number of un-actuated degrees in most underactuated systems we deal with. The purpose of this section is to present a control design based on controlled Lagrangian procedure for the flexible cable crane system which it is a multi-underactuated system. The control objective is to move the payload to the desired position and at the same time, to reduce the payload swing and to suppress the cable transverse vibrations. The controller should guarantee both moving of payload to desired point and active damping of payload swing and cable vibration. First the System (4) is linearized at its equilibrium, then the controlled Lagrangian method is used for the linearized system. Suppose that the origin point $(q, \dot{q}) = (0, 0)$ is an equilibrium point of system (4). By linearizing the system (4) at the equilibrium point, a linear underactuated system as following can be obtained:

$$\bar{M}\ddot{q} + \bar{K}q = Wu \tag{11}$$

where $\bar{M}$ and $\bar{K}$ are $n \times n$ positive constant matrices and $W = [1 \quad 0 \quad \cdots \quad 0]^T$. The generalized degrees of freedom are $q = (q_1, q_2) \in R^n$ in which $q_1 = x \in R^1$ and $q_2 = (\theta, \mu)^T \in R^{n-1}$ where, $\mu = (\mu_1, \mu_2, \cdots, \mu_m)$. It is worthwhile noting that the control bundle W with $Rank(W) = 1$ shows that the system is underactuated. Given a desired set-point as $q_d = (q_{1d}, q_{2d})$ in which $q_{2d} = 0$, $q_{1d} = x_d$ where x_d is fixed desired point of payload. Defining errors as $\tilde{q} = q - q_d$ and applying the following control signal as:

$$u = W_0 \bar{K} q_d + u_1 \tag{12}$$

in which, $W_0 = (W^T W)^{-1} W^T$, then, the dynamical model is given by

$$\bar{M}\ddot{\tilde{q}} + \bar{K}\,\tilde{q} = W u_1 \tag{13}$$

The point of interest is $\tilde{q} = 0$, which corresponds to zero tracking error for payload motion null cable vibration and payload swing. The controlled Lagrangian method as explained in (Chang *et al.*, 2002) is a control strategy to find a Lagrangian with center equilibrium point by shaping kinetic and potential energies through solving a matching equation presented as follows and injecting a dissipative force to obtain the stable-focus equilibrium point.

$$w_r^{\perp}\left[\left(C_r - M_r \hat{M}^{-1}\hat{C}\right)\dot{q}_r - F_r^v + M_r \hat{M}^{-1}\hat{F}^v\right] = 0 \tag{14}$$

$$w_r^{\perp}\left[G_r - M_r \hat{M}^{-1}\hat{G} - F_r^q + M_r \hat{M}^{-1}\hat{F}^q\right] = 0 \tag{15}$$

$$\hat{w} = \hat{M} M_r^{-1} w_r \tag{16}$$

where, the subscript $'r'$ and the symbols with $'\wedge'$ denote the original and the new Lagrangian systems, respectively. The matrix $w_r^{\perp}$ is the orthogonal complement of the control bundle matrix $w_r = W$, and M and $G = \frac{\partial U}{\partial q_r}$ are inertia matrix and nonlinear generalized force vector resulting from the potential energy $U(q_r)$, respectively. The matrix C represents the Coriolis and centripetal forces. The forces $F^q(q_r), F^v(q_r, \dot{q}_r)$ are the velocity independent and velocity dependent parts of the resultant external force as $F = F^q + F^v$. In our case of a crane system, fortunately there are not external forces so, $F_r^q = 0$ and $F_r^v = 0$ and there is no need to consider the external forces acting at the second Lagrangian system so, $\hat{F}^q = \hat{F}^v = 0$. In addition to kinetic and potential shaping equations (14) and (15), another equation (16) describing the control bundle of the new Lagrangian system. Using Eq. (14) for system (13), the following equation will be obtained as:

$$W^{\perp}\left[\left(0 - \bar{M}\hat{M}^{-1}\hat{C}\right)\dot{\tilde{q}}\right] = 0 \tag{17}$$

Where, matrix $\hat{M}$ is the mass matrix of the second Lagrangian system and matrix $W^{\perp}$ is left annihilator of matrix W where $W^{\perp}W = 0$. Since the inertia matrix, $\bar{M}$ which characterizes the system, is independent of q, the simplest form of solution of Eq. (17) is considering an arbitrary constant symmetric matrix for $\hat{M}$ where $\hat{M} > 0$. Assuming matrix $\hat{M}$, the potential shaping equation (15) is then obtained as follows

$$W^{\perp}\left[\bar{K}\tilde{q} - \bar{M}\hat{M}^{-1}\hat{G}\right] = 0 \tag{18}$$

Let $H = \bar{M}\hat{M}^{-1}$ where that is a positive definite matrix by choosing $\hat{M} > 0$, A general solution of this equation can be obtained as:

$$\hat{G} = H^{-1}\,\bar{K}\tilde{q} + \hat{G}_0 = \hat{K}\tilde{q} + \hat{G}_0 \tag{19}$$

where, $\hat{K} = H^{-1}\,\bar{K}$ in which $H = \bar{M}\hat{M}^{-1}$ and vector $\hat{G}_0 = [\hat{g}_1\ \hat{g}_2]^T$ where $\hat{g}_1 \in R^1$ and $\hat{g}_2 \in R^{n-1}$ can be obtained solving following equation.

$$W^{\perp}H\hat{G}_0 = 0 \tag{20}$$

$\hat{G}_0$ is in the null space of matrix $W^{\perp}H$. Let $y = W^{\perp}H$ is a vector where $y \in R^n$, thus $\hat{G}_0$ and y are orthogonal. $\hat{G}_0$ can obtained as following where $\hat{g}_1$ is arbitrary.

$$\hat{G}_0 = \begin{pmatrix} \hat{g}_1 \\ -H_{22}^{-1}H_{21}\hat{g}_1 \end{pmatrix} \tag{21}$$

In order to obtain the control bundle of the matching system, the third matching equation (16) resulting in

$$\widehat{W} = \hat{M}\bar{M}^{-1}W = H^{-1}W \tag{22}$$

The energy shaping control u_{es} is given by:

$$\begin{aligned} u_{es} &= W_0\left(G_r - M_r\hat{M}\hat{G}\right) \\ &= W_0\left(\bar{K}\tilde{q} - \bar{M}\hat{M}^{-1}\hat{G}\right) = -W_0H\hat{G}_0 \end{aligned} \tag{23}$$

Where, $W_0 = (W^TW)^{-1}W^T$, $\hat{G}$ and $\hat{G}_0$ are obtained using Eq. (19) and Eq. (21). The controller design is completed with using another control signal corresponding to the damping injection. To deal with the situation in the presence of Rayleigh dissipative Forces the dissipative force chosen as:

$$u_{diss} = -\gamma\widehat{W}^T\dot{\tilde{q}} = -\gamma(H^{-1}W)^T\dot{\tilde{q}} \tag{24}$$

In which, γ is a positive scalar and $\widehat{W}$ is obtained using Eq. (22). Eventually, the stabilizing control law for the conservative system as presented in Ortega et al. (Ortega *et al.*, 2002) can be used as following in which $H = \bar{M}\hat{M}^{-1}$, $W_0 = (W^TW)^{-1}W^T$ and $\hat{G}_0$ and $\widehat{W}$ are obtained using Eq. (21) and Eq. (22).

$$\begin{aligned} u_1 &= u_{es} + u_{diss} \\ &= -W_0H\hat{G}_0 - \gamma^2\widehat{W}^T\dot{\tilde{q}} = -H_0\hat{g}_1 - \gamma^2\widehat{W}^T\dot{\tilde{q}} \end{aligned} \tag{25}$$

where, $H_0 = H_{11} - H_{12}H_{22}^{-1}H_{21}$. Using Eq. (12), the overall control is determine as

$$u = W_0\bar{K}q_d - H_0\hat{g}_1 - \gamma^2\widehat{W}^T\dot{\tilde{q}} \tag{26}$$

4. Stability Analysis

The controlled Lagrangian system using control signal (26) is

$$\begin{aligned} &M(\tilde{q})\ddot{\tilde{q}} + C\left(\tilde{q},\dot{\tilde{q}}\right)\dot{\tilde{q}} + G(\tilde{q}) = Wu_1 \\ &u_1 = -H_0\hat{g}_1 - \gamma^2\widehat{W}^T\dot{\tilde{q}} \end{aligned} \tag{27}$$

The closed form of above equation can be written as:

$$M(\tilde{q})\ddot{\tilde{q}} + C\left(\tilde{q},\dot{\tilde{q}}\right)\dot{\tilde{q}} + G(\tilde{q}) + WH_0\hat{g}_1 + \gamma R\dot{\tilde{q}} = 0 \tag{28}$$

In which, matrix $R = WW^TH^{-T}$ and $H_0 = H_{11} - H_{12}H_{22}^{-1}H_{21}$ are positive definite by choosing $\hat{M} > 0$. The last term, $\gamma R\dot{\tilde{q}}$ is a Rayleigh dissipative Forces. To analyze the stability of the closed-loop system, the controlled energy of the new Lagrangian system can be considered as a Lyapunov function candidate:

$$V\left(\tilde{q},\dot{\tilde{q}}\right) = E\left(\tilde{q},\dot{\tilde{q}}\right) = T\left(\tilde{q},\dot{\tilde{q}}\right) + U(\tilde{q}) \tag{29}$$

In which, $T\left(\tilde{q},\dot{\tilde{q}}\right) = \frac{1}{2}\dot{\tilde{q}}^TM(\tilde{q})\dot{\tilde{q}}$ and $\frac{\partial}{\partial\tilde{q}}U(\tilde{q}) = G(\tilde{q}) + WH_0\hat{g}_1$. In order to achieve stability, the positive condition should be imposed on the controlled energy of the system. Hessian matrix of should be positive definite to hold positive definite conditions of controlled energy. The Hessian Matrix around equilibrium point $(\tilde{q},\dot{\tilde{q}}) = (0,0)$ is defined as follows:

$$\delta^2E|_{(0,0)} = \begin{bmatrix} \left.\dfrac{\partial^2}{\partial\tilde{q}^2}U(\tilde{q})\right|_{\tilde{q}=0} & 0 \\ 0 & M(\tilde{q})|_{\tilde{q}=0} \end{bmatrix} > 0 \tag{30}$$

So, the controlled energy function remains positive near its minimum equilibrium point and it can be used as the Lyapunov candidate. Thus, the time derivative of the controlled energy is negative for $\gamma > 0$ and this will guarantee the stability of the closed-loop system from LaSalle's lemma. Matrix $M(\tilde{q})$ is positive definite and must

$$\left.\frac{\partial^2}{\partial\tilde{q}^2}U(\tilde{q})\right|_{\tilde{q}=0} = \left.\frac{\partial}{\partial\tilde{q}}(G(\tilde{q}) + WH_0\hat{g}_1)\right|_{\tilde{q}=0} > 0 \tag{31}$$

A chosen function for $\hat{g}_1$ can be $\hat{g}_1 = K_0\tilde{q}$ in which K_0 is arbitrary matrix where $K_0 \in R^{1\times n}$ and $(WH_0K_0) \geq 0$. The derivative of $V\left(\tilde{q},\dot{\tilde{q}}\right)$ with respect to time is following by taking (10).

$$\begin{aligned} \dot{V} &= \dot{\tilde{q}}^TM(\tilde{q})\ddot{q} + \tfrac{1}{2}\dot{\tilde{q}}^T\dot{M}(\tilde{q})\dot{\tilde{q}} + \dot{\tilde{q}}^T\left(\frac{\partial}{\partial\tilde{q}}U(\tilde{q})\right) \\ &= \dot{\tilde{q}}^T\left(-C\left(\tilde{q},\dot{\tilde{q}}\right)\dot{\tilde{q}} - G(\tilde{q}) - WH_0\hat{g}_1 - \gamma R\dot{\tilde{q}}\right) \\ &\quad + \tfrac{1}{2}\dot{\tilde{q}}^T\dot{M}(\tilde{q})\dot{\tilde{q}} + \dot{\tilde{q}}^T(G(\tilde{q}) + WH_0\hat{g}_1) \\ &= \tfrac{1}{2}\dot{\tilde{q}}^T\left(\dot{M} - 2C\right)\dot{\tilde{q}} - \gamma\dot{\tilde{q}}^TR\dot{\tilde{q}} = -\gamma\dot{\tilde{q}}^TR\dot{\tilde{q}} < 0 \end{aligned} \tag{32}$$

Therefore, the closed-loop control system is stable with the stability criteria of Lyapunov.

5. Simulation Results

In this section, a simulation is performed to demonstrate the dynamic behavior and effectiveness of the control system for an illustrative example of the crane systems with flexible cable moving a lightweight payload. Since the crane system's degrees of freedom are cable deflection, payload/cable swing angle and trolley movement, time histories of these motions will be plotted to illustrate the behavior of the sample crane system. The sample flexible cable crane system with a lightweight payload is considered with a trolley mass of 10 (kg), cable length of 5 (m) and cable mass per unit length of 0.62 (kg/m), and payload mass of 2 (kg). It is assumed that the crane system is initially at rest. The simulation results are presented when the proposed controller is applied to move the payload to the desired final position and to reduce the payload/cable swing angle and to suppress the cable vibration. Suppose the crane assignment is to move a 2 (kg) payload mass to a final position such that the travel distance is 5 (m), thus, a reference trajectory as Fig.3 is planned for the trolley motion. To determine the transverse deflection of the cable, the first four modes of vibrations are considered. The trolley control force is applied as control law as Eq. (29) to control the crane system. The proper values for the control parameters are chosen as:

$$\widehat{M} = diag(2,10,10,10,10,10), K_0 = [50\ 1\ 1\ 1\ 1\ 1], \gamma = 10$$

The generated control force is shown in Fig.2. To investigate the effectiveness of the proposed controller, time histories of the trolley and the payload motions, the swing angle, the amplitudes of the first four vibration modes and transverse deflections of the cable are shown in Fig.3 and Fig.5 to Fig.7, respectively. The transverse deflections of cable are demonstrated considering four distinct points along the cable with the same distances. The transverse deflections of these distinct points are shown in Fig.7. In Fig.5 the payload swing angle is displayed and Fig.3 and Fig.4 present the horizontal displacements and velocity of the trolley and the payload along X-axis. It can be seen that the closed loop control system reasonably reduces the payload/cable swing angle and suppresses the cable vibrations. The maximum magnitude of the payload swing angle has been reduced to about ±6 degrees. Moreover, the horizontal travelling motions of the trolley and the payload to their desired positions have much smoother behavior; moreover, they are with zero steady state error and a settling time of about 8 Seconds with a minimal overshoot.

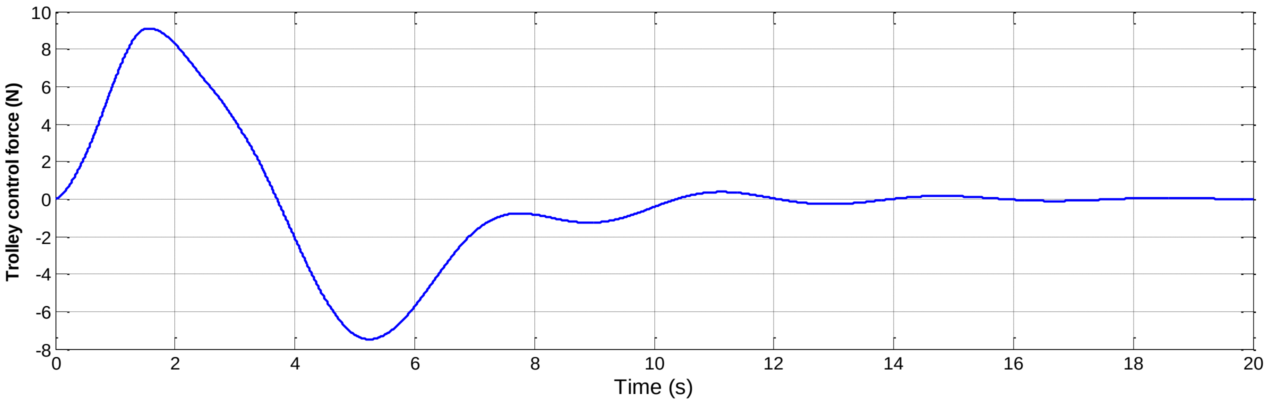

Fig.2 Time history of the applied trolley control force for moving the trolley and payload

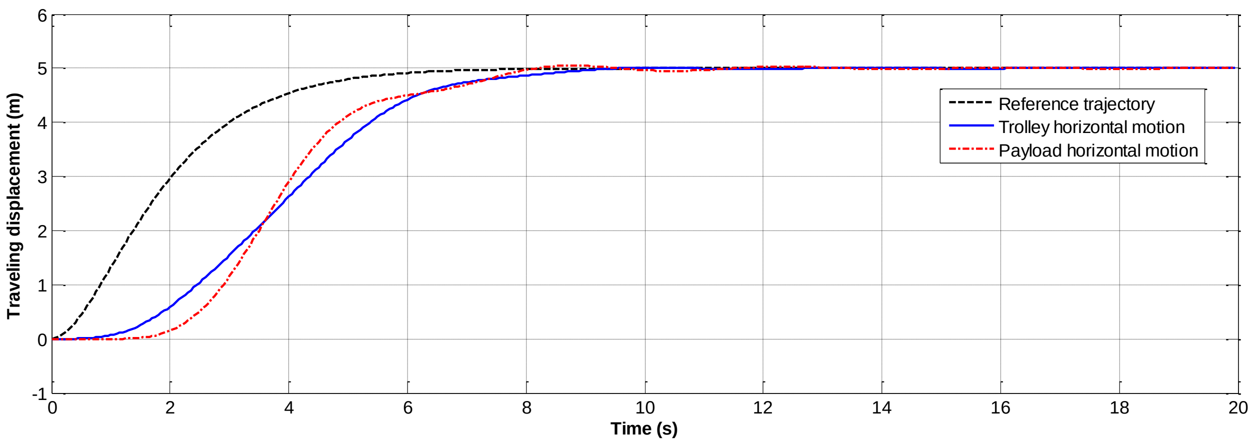

Fig.3 Time histories of reference trajectory and the trolley and payload horizontal travelling

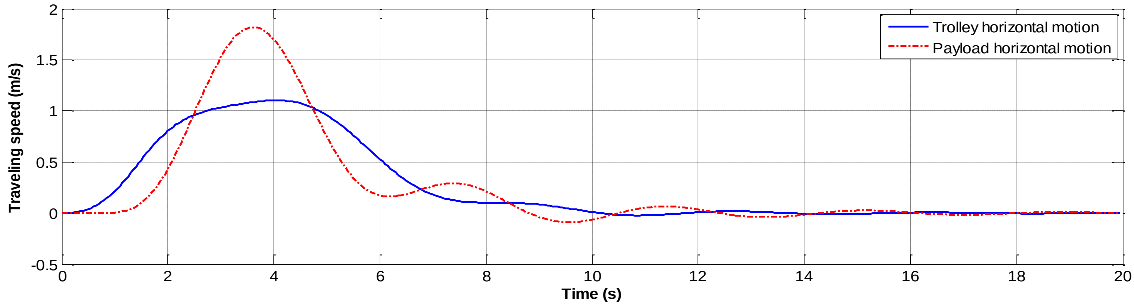

Fig.4 Time histories of the trolley and payload velocity

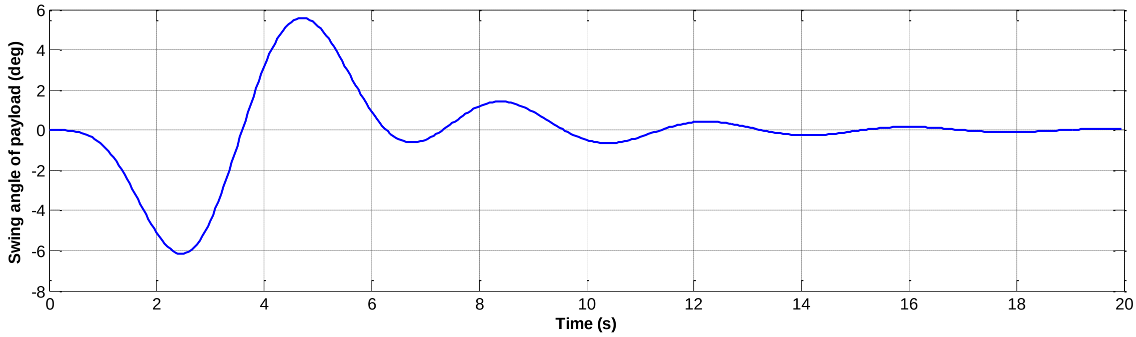

Fig.5 Time history of the swing angle during the traveling of the trolley and payload

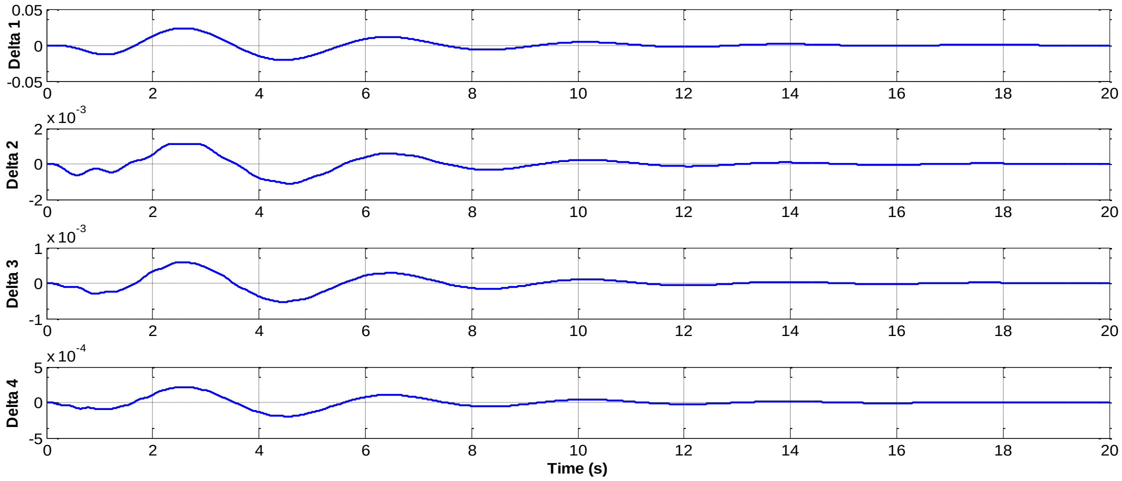

Fig.6 Time histories of the amplitudes of the first four vibration modes of the cable

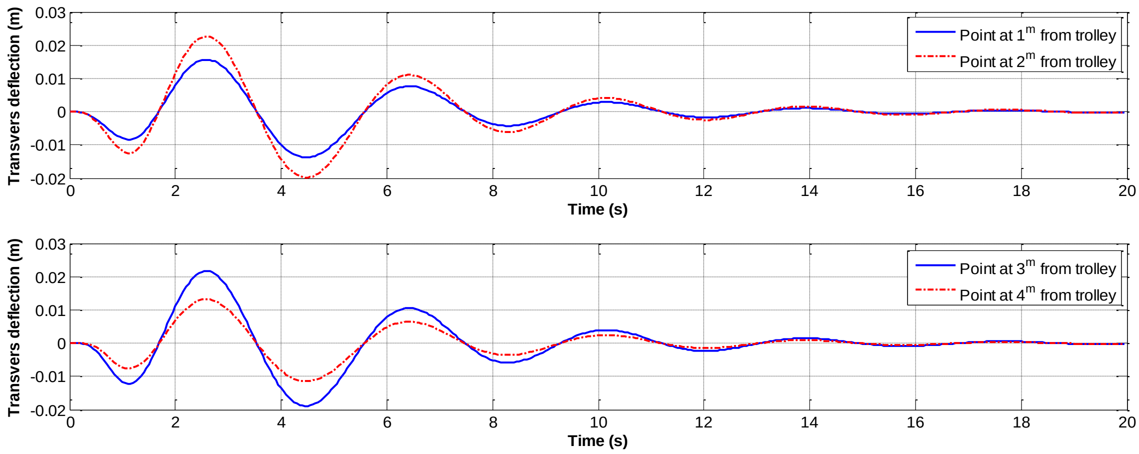

Fig7 Time histories of transverse deflections of the four selected points along the cable

5. Discussion and Conclusion

In this study, an energy-based control design using controlled Lagrangian method was presented for a crane system with a flexible cable which it is categorized as a multi-degree underactuated system. The control objectives was simultaneously moving the trolley/payload and suppressing the payload swing and cable vibration. The underactuated characteristics can put challenges in control design procedure because the number of control inputs of the system is smaller than the system's degrees of freedom and so, it is difficult to directly apply traditional nonlinear control methods to design a suitable controller. Due to such a difficulty, to overcome the complexities of the control problem, a corresponding energy-based control strategy was proposed using linearized model and controlled Lagrangian method. The controller guarantees both tracking of the desired payload position and active damping of payload swing and cable vibration. The simulation was performed to demonstrate the dynamic behavior and effectiveness of the control system for an illustrative example of the crane systems with flexible cable moving a lightweight payload. The simulation results demonstrated that the proposed control system was able to yield smooth trolley/payload motions with zero steady state error, a small settling time and a minimal overshoot.

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