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Research Article

Balancing of Stephenson's Mechanisms

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Abstract

This paper deals with the problem of shaking force and shaking moment balancing of high speed planar mechanisms. The design equations and techniques for the complete shaking force and shaking moment balancing of two types of six-bar linkages due to both linear and rotary inertia but without considering external loads are developed. Shaking force is balanced by the method of redistribution of mass and shaking moment by the geared inertia counterweight and planetary-gear-train inertia counter- weight. These geared inertia counter weights are very useful for balancing of multi-bar linkages. The proposed techniques produce better results than that of the previous techniques.

Keywords: *Shaking force, shaking moment, dynamic balancing, vibration, and Stephenson's mechanisms.*

1. Introduction

A mechanical system with unbalanced shaking force and shaking moment transmits considerable vibration to the frame and foundation of the machine. Different approaches and solutions (G.G.Lowen *et al* 1983 and V.Arakelian *et al* 2005) devoted to this problem have been developed and documented for various planar mechanisms. The dynamic balancing of a mechanism is generally achieved by two steps. First step is the cancellation or reduction of the shaking force and the second the cancellation or reduction of the shaking moment. Generally the cancellation of the shaking force transmitted to the frame can be achieved by adding counterweights in order to make the total centre of mass of moving links stationary by additional structures (V.Vander Wijk *et al* 2012) or by elastic components (G.Alici*et al* 2003).

The shaking moment was balanced by the following methods (i) Balancing by counter-rotations (R.S.Berkof 1973, J.L.Heede 2004, and V.VanderWijk *et al* 2012) (ii) Balancing by adding four-bar linkages (C.M.Gosselin *et al* 2004, Q. Jiang *et al* 2010) (iii) Balancing by adding an inertia fly wheel rotating with a prescribed angular velocity (V.Arakelian *et al*.2008, V.Van Der Wijk *et al* 2010)

The present paper is the extension work of authors (V.Arakelian *et al*.1999, GaoFeng 1989, GaoFeng 1990). The results obtained are better than the previous method results. The paper is organized as follows: section 1 deals with introduction, section 2 presents articulation dyad. Section 3 deals with Asymmetric link with three rotational pairs. Dynamic balancing of Stephenson's mechanisms is given in section 4. Numerical examples and results are discussed in section 5. Conclusions are given in section 6.

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2. Articulation dyad

A. Complete shaking force and shaking moment balancing of an articulation dyad

An open kinematic chain of two binary links and one joint is called a dyad.When two links are articulated by a joint so that movement is possible that arrangement of links is known as articulation dyad.

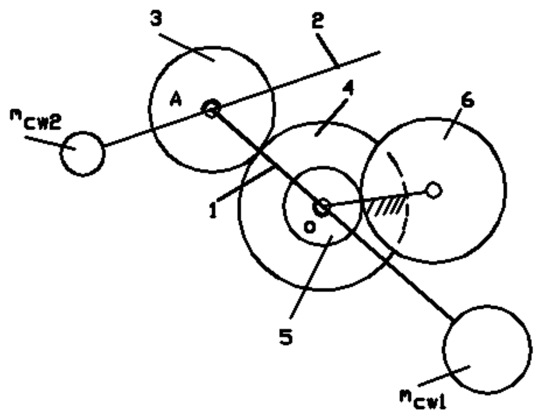

Fig.1 Complete shaking force and shaking moment balancing of an articulation dyad

The familiar scheme of complete shaking force and shaking moment balancing of an articulation dyad [10]-[13] is shown in Fig.1. A counterweight is attached to link 2 which permits the displacement of the center of mass of link 2 to joint A. Then, by means of a counter weight with mass m_{cw_1} [Fig.1] a complete balancing of shaking force is achieved. A complete shaking moment balance is obtained by four gear inertia counter weights 3-6, one of them is of the planetary type and mounted on link 2.

B. Complete shaking force and shaking moment balancing of an articulation dyad by gear inertia counterweights mounted on the base

The scheme used in the present work [Fig.2] is distinguished from the earlier scheme by the fact that gear 3 is mounted on the base and is linked kinematically with link 2 through link $1'$.

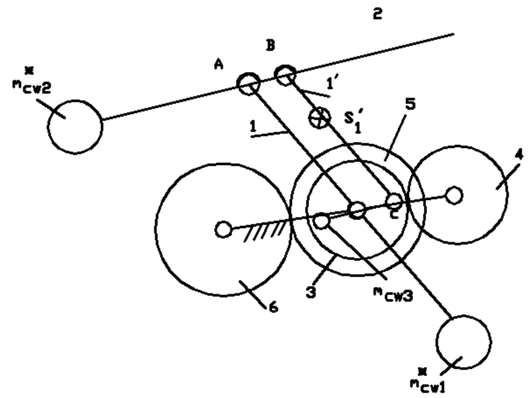

Fig.2 Complete shaking force and shaking moment balancing of an articulation dyad by gear inertia counterweights mounted on the base

To prove the advantages of such a balancing, the application of the new system with the mass of link $1'$ not taken into account is considered. In this case (compared to the usual method Fig.1), the mass of the counter weight of link 1 will be reduced by an amount

$$\delta m_{cw_1} = {m_3 l_{OA}}\Big/{r_{cw_1}} \quad (1)$$

where,

m_3 is the mass of gear 3, l_{OA} is the distance between the centers of hinges O and A, r_{cw_1} is the rotation radius of the center of mass of the counter weight.

It is obvious that the moment of inertia of the links is correspondingly reduced. If the gear inertias are made in the form of heavy rims in order to obtain a large moment of inertia, the moments of inertia of the gear inertia counter weights may be presented as

$$I = \frac{m_i D_i^2}{4} \quad (i=3\ldots 6).$$

Consequently, the mass of gear 6 will be reduced by an amount

$$\delta m_6 = 4(m_3 l_{OA}^2 + \delta m_{cw_1} r_{cw_1}^2)\frac{T_6}{D_6^2 T_5} \quad (2)$$

Where,

T_5 *and* T_6 are the numbers of teeth of the corresponding gears. Thus, the total mass of the system will be reduced by an amount

$$\delta m = \delta m_{cw_1} + \delta m_6 \quad (3)$$

Here the complete shaking force and shaking moment balancing of the articulation dyad with the mass and inertia of link $1'$ taken into account is considered. For this purpose initially, statically replace mass m_1' of link $1'$ by two point masses m_B and m_c at the centers of the hinges B and C

$$m_B = m_{1'} l_{CS_{1'}} / l_{BC}$$
$$m_C = m_{1'} l_{BS_{1'}} / l_{BC} \quad (4)$$

Where, l_{BC} is the length of link 1, $l_{CS_{1'}}$ and $l_{BS_1'}$ are the distances between the centers of joints C and B and the center of mass S_1' of link $1'$, respectively.

After such an arrangement of masses the moment of inertia of link $1'$ will be equal to

$$I_{S_1'}^* = I_{S_1'} - m_{1'} l_{BS_1'} l_{CS_1'} \quad (5)$$

where,

$I_{S_1'}$ is the moment of inertia of link $1'$ about the center of mass S_1' of the link. Thus a new dynamic model of the system is obtained, where the link $1'$ is represented by two point masses m_B, m_C and has a moment of inertia $I_{S_1'}^*$.

This fact allows for an easy determination of the parameters of the balancing elements as follows:

$$m_{CW_2} = (m_2 l_{AS_2} + m_B l_{AB}) / r_{CW_2} \quad (6)$$

where,

m_2 is the mass of link 2, l_{AB} is the distance between the centers of the hinges A and B, l_{AS_2} is the distance of the center of hinge A from the center mass of S_2 of link 2, r_{CW_2} is the rotation radius of the center of mass of the counterweight with respect to A, and

$$m_{CW_1} = [(m_2 + m_{CW_2} + m_B) l_{OA} + m_1 l_{OS_1}] / r_{CW_1} \quad (7)$$

where, m_1 is the mass of link 1, l_{OS_1} is the distance of the joint center O from the center of mass S_1 of link 1.

$$m_{CW_3} = m_C l_{OC} / r_{CW_3} \quad (8)$$

where, $l_{OC} = l_{AB}$, r_{CW_3} is the rotation radius of the center of mass of the counterweight. Taking into account the mass of link $1'$ brings about the correction in Eq.(3) in this case,

$$\delta m = \delta m_{CW_1} + \delta m_6 - \delta m_1' \tag{9}$$

where,

$\delta m_1'$ is the value deciding the change in the distribution of the masses of the system links resulting from the addition of link $1'$.

3. Asymmetric link with three rotational pairs

A link with three nodes is called ternary link, where nodes are points for attachment to other links. In the earlier research by Gao Feng relating to balancing of linkages with a dynamic substitution of the masses of the link by three rotational pairs shown in Fig.3 two replacement points A and B are considered. This results in the need to increase the mass of the counter weight. However, such a solution may be avoided by considering the problem of dynamic substitution of link masses by three point masses. Usually the center of mass of such an asymmetric link is located inside a triangle formed by these points.

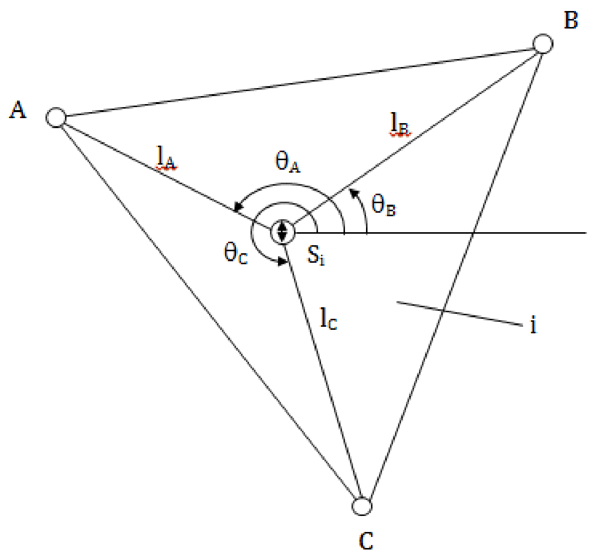

Fig.3 Dynamic substitution of the masses of the link by three rotational pairs

The conditions for dynamic substitution of masses are the following:

$$\begin{bmatrix} 1 & 1 & 1 \\ l_A e^{i\theta_A} & l_B e^{i\theta_B} & l_C e^{i\theta_C} \\ l_A^2 & l_B^2 & l_C^2 \end{bmatrix} \begin{bmatrix} m_A \\ m_B \\ m_C \end{bmatrix} = \begin{bmatrix} m_i \\ 0 \\ I_{S_i} \end{bmatrix}$$

Where,

m_A, m_B, and, m_C are point masses,

l_A, l_B and l_C are the moduli of radius vectors of corresponding points,

θ_A, θ_B and θ_C are angular positions of radius vectors;

m_i is the mass of link,

I_{S_i} is the moment of inertia of the link about an axis through S_i (axial moment of inertia of link).

From this system of equations the masses are obtained

$$m_A = D_A/D_i\,; m_B = D_B/D_i\,; m_C = D_C/D_i \tag{10}$$

where, D_A, D_B, D_C and D_i are determinants of the third order obtained from the above system of equations.

4. Dynamic Balancing of Stephenson's mechanisms

4.1 Stephenson's mechanism with two fixed points (Mechanism with high degree of complexity)

The Stephenson's mechanism with two fixed points shown in Fig.4 is obtained when one of the binary links in the basic Stephenson's chain is fixed. This is a mechanism with high degree of complexity as more than one radii of path curvature of motion transfer points C and D are not known. This mechanism is used in the reversing gear of steam engines. The balanced Stephenson's mechanism with two fixed points is shown in fig.4

4.1.1 Shaking force balancing of the mechanism

Link 5 is replaced by dynamic substitution of link masses by three point masses m_{C5}, m_{D5} and m_{E5}; The conditions for dynamic substitution of masses are the following:

$$\begin{bmatrix} 1 & 1 & 1 \\ l_C e^{i\theta_C} & l_D e^{i\theta_D} & l_E e^{i\theta_E} \\ l_C^2 & l_D^2 & l_E^2 \end{bmatrix} \begin{bmatrix} m_{C5} \\ m_{D5} \\ m_{E5} \end{bmatrix} = \begin{bmatrix} m_5 \\ 0 \\ I_{S_5} \end{bmatrix}$$

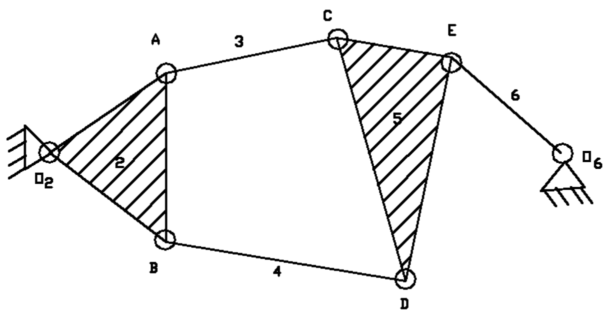

Fig. 4 Stephenson's mechanism with two fixed points

Where,

m_{C5}, m_{D5} and m_{E5} are point masses;

l_C, l_D, l_E are the moduli of radius vectors of corresponding points.

θ_C, θ_D and θ_E are the angular positions of radius vectors.

m_5 is the mass of link 5.

I_{S_5} is the moment of inertia of link 5 about an axis through S_5 (axial moment of inertia of link 5)

$$m_{C5} = D_C / D_5 ; m_{D5} = D_D / D_5 ; m_{E5} = D_E / D_5 \quad (11)$$

Where D_C, D_D and D_5, D_E are determinants of the third order obtained from the above system of equations.

Link 6 is dynamically replaced by two point masses m_{E6} and m_{P6} and attached a counterweight m_{CW_6}. For link 6 to be dynamically replaced by two point masses the condition to be satisfied is $k_6^2 = l_{ES_6} l_{P_6S_6}$, Where, k_6 is the radius of gyration of link 6 about its center of mass l_{ES_6} is arbitrarily fixed and $l_{P_6S_6}$ is obtained from the above condition

$$m_{CW_6} = {(m_6 l_{O_6S_6} + m_{E5} l_{O_6E})}/{r_{CW_6}}$$

$$m_{E6} = {m_6 l_{P_6S_6}}/{(l_{ES_6} + l_{P_6S_6})}$$

$$m_{P6} = {m_6 l_{ES_6}}/{(l_{ES_6} + l_{P_6S_6})}$$

Where $r_{CW_6} = l_{P_6S_6} - l_{O_6S_6}$ is the radius of rotation of counterweight m_{CW_6}

Now, link 2 is dynamically replaced by three point masses m_{O_2}, m_{A2}, m_{B2}

$$\begin{bmatrix} 1 & 1 & 1 \\ l_{O_2} e^{i\theta_{O2}} & l_{A2} e^{i\theta_{A2}} & l_{B2} e^{i\theta_{B2}} \\ l_{O_2}^2 & l_{A2}^2 & l_{B2}^2 \end{bmatrix} \begin{bmatrix} m_{O2} \\ m_{A2} \\ m_{B2} \end{bmatrix} = \begin{bmatrix} m_2 \\ 0 \\ I_{S_2} \end{bmatrix}$$

where, m_{O_2}, m_{A2}, m_{B2} are the point masses,

l_{A2}, l_{B2}, l_{O_2} are the moduli of radius vectors of the corresponding points, $\theta_{O_2}, \theta_{A2}, \theta_{B2}$ are angular positions of radius vectors, m_2 is the mass of link 2. I_{S_2} is the mass moment of inertia of link2 about its center of mass

$$m_{O_2} = D_{O_2} / D_2 ; m_{A2} = D_{A2} / D_2 ; m_{B2} = D_{B2} / D_2 ; \quad (12)$$

where,

$D_{O_2}, D_{A2}, D_{B2}, D_2$ are determinants of the third order obtained from the above system of equations.

Now link 4 is dynamically replaced by two point masses m_{D4}, m_{P4} and linked kinematically with gear inertia counterweight 9 by link $4'$ and attached a counter weight m_{CW_4} against link 4. For link 4 to be dynamically replaced by two point masses the condition to be satisfied is

$$k_4^2 = l_{DS_4} l_{P_4S_4}$$

where,

k_4 is the radius of gyration of link 4 about its center of mass,

l_{DS_4} is arbitrarily fixed and

$l_{P_4S_4}$ is obtained from the above condition.

$$m_{D4} = {m_4 l_{P_4S_4}}/{(l_{P_4S_4} + l_{DS_4})}$$

$$m_{P4} = {m_4 l_{DS_4}}/{(l_{P_4S_4} + l_{DS_4})}$$

Link $4'$ is statically replaced by two point masses m_F, m_G

$$m_F = {m_4' l_{GS_4'}'}/{l_{FG}}$$

$$m_G = {m_4' l_{FS_4'}'}/{l_{FG}}$$

where,

$l_{FG} = l_{O_2B}$

After static replacement of masses the moment of inertia of link $4'$ is equal to $I_{S_{4'}}^* = I_{S_4}' - m_4' l_{FS_{4'}}' l_{GS_4}'$

where,

$I_{S_4}'^*$ is the changed moment of inertia of link $4'$

The counterweight attached against mass m_G is equal to

$$m_{CW_9} = {m_G l_{O_2G}}/{r_{CW_9}}$$

$$m_{CW_4} = {(m_4 l_{BS_4} + m_{D5} l_{BD} + m_F l_{BF})}/{r_{CW_4}}$$

$$m_{B_{CW}} = {(m_4 + m_{D5} + m_{CW_4} + m_F + m_{B2}) l_{O_2B}}/{r_{B_{CW}}} \quad (13)$$

where $m_{B_{CW}}$ is the counterweight attached against point B $r_{CW_4} = (l_{P_4S_4} - l_{BS_4})$ is the radius of rotation of counterweight m_{CW_4}

Now link 3 is dynamically replaced by two point masses m_{C3}, m_{P3}.

For link 3 to be dynamically replaced by two point masses the condition to be satisfied is $k_3^2 = l_{CS_3} l_{P_3S_3}$

where, k_3 is the radius of gyration of link 3 about its center of mass, l_{CS_3} is arbitrarily fixed and $l_{P_3S_3}$ is obtained from the above condition.

Now gear 11 is mounted on the base and linked kinematically with link $3'$ and link $3'$ is statically replaced by two point masses m_H, m_J

$$m_H = {m'_3 l'_{JS_3}} \big/ {l_{HJ}}$$

$$m_j = {m'_3 l'_{HS_3}} \big/ {l_{HJ}}$$

where $l_{HJ} = l_{AO_2}$

After static substitution of masses the moment of inertia of link $3'$

$$I'^{*}_{S_3} = I'_{S_3} - m'_3 l'_{HS_3} l'_{JS_3}$$

where, $I'^{*}_{S_3}$ is the moment of inertia of link $3'$ after static substitution of masses.

The counterweight attached against mass mJ is equal to

$$m_{CW_J} = {m_J l_{O_2J}} \big/ {r_{CW_J}} \qquad (14)$$

$$m_{CW_3} = {(m_3 l_{AS_3} + m_{C5} l_{AC} + m_H l_{AH})} \big/ {r_{CW_3}}$$

$$m_{A_{CW}} = {(m_3 + m_{CW_3} + m_{C5} + m_H + m_{A2}) l_{O_2A}} \big/ {r_{A_{CW}}} \qquad (15)$$

Where

m_{CW_J} is the counterweight against point mass m_J

$r_{CW_3} = l_{P_3S_3} - l_{AS_3}$ is the radius of rotation of counterweight

m_{CW_3} $m_{A_{CW}}$ is the counterweight mass attached against point A

4.1.2 Shaking moment balancing of the mechanism

The shaking moment generated by links 2 and 6 are given in eq. (16).The shaking moment generated by the mechanism is determined by the sum

$$M^{int} = M_6^{int} + M_2^{int} \quad (16)$$

where,

M_6^{int}, M_2^{int} are the shaking moments of the rotating links 6 and 2 respectively with the inertia of the replaced point masses taken into account.

$$M_6^{int} = (I_{S6} + m_6 l^2_{O_6S_6} + m_{E5} l^2_{O_6E} + m_{CW_6} r^2_{CW_6})\alpha_6$$

$$M_2^{int} = (I'^{*}_{S_3} + I'^{*}_{S_4} + (m_4 + m_{D5} + m_{CW_4} + m_F + m_{B2} + m_{B_{CW}} r^2_{B_{CW}}) l^2_{O_2B} + (m_3 + m_{CW_3} + m_{C5} + m_H + m_{A2}) l^2_{O_2A} + m_{A_{CW}} r^2_{A_{CW}} + 2 m_J l^2_{O_2J} + 2 m_G l^2_{O_2G})\alpha_2$$

Where, I_{S_6} is the mass moment of inertia of link 6 about its center of mass

$I'^{*}_{S_3}, I'^{*}_{S_4}$ are the changed moments of inertia of links $3'$ and $4'$ respectively.

α_2, α_6 are the angular accelerations of links 2 and 6 respectively

The shaking moment generated by the mechanism is balanced by using the gear inertia counterweights 7-14.

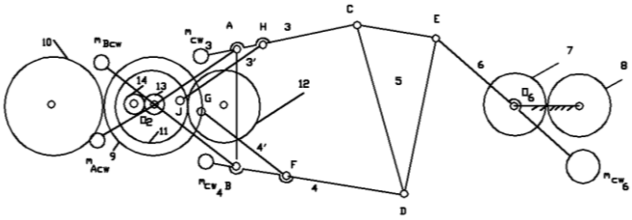

Fig.5 Balanced Stephenson's mechanism with two fixed points

4.2 Stephenson's mechanism with three fixed points

Stephenson's mechanism with three fixed points has six links out of them four are binary and two are ternary links. In the Stephenson's mechanism two ternary links are connected by binary links. This Stephenson's mechanism with three fixed points shown in Fig.6 is obtained when one of the ternary links is fixed. Stephenson's six-bar linkage can be thought of as two four-bar linkages connected in parallel and sharing two links in common. This mechanism is mostly used in Steam engines. The balanced Stephenson's mechanism with three fixed points is shown in Fig.7.

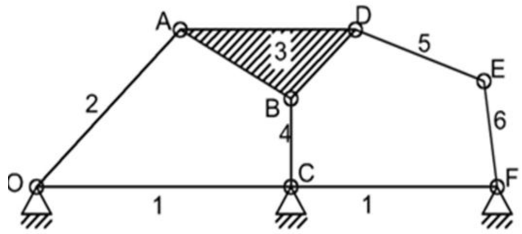

Fig.6 Stephenson's mechanism with three fixed points

In the Stephenson's mechanism with three fixed points shown in Fig.6.

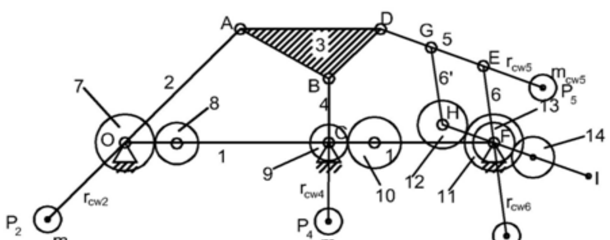

Fig. 7 Balanced Stephenson's mechanism with three fixed points

Link 1 is the fixed ternary link. The other ternary link 3 is connected to fixed ternary link by binary links 2, 4 and 6. The links 3 and 5 are not directly connected to the frame. The geared inertia counterweights required to balance the shaking moment generated by links 3 and 5 are mounted on the base of the mechanism by kinematically linking the geared inertia counterweights and the links by links of known mass and center of mass.

4.2.1 Shaking force balancing of the mechanism

The shaking force of the mechanism is balanced by dynamically replacing the links by point masses and mass counterweights. Here link 3 is dynamically replaced by three point masses m_{A3}, m_{B3}, m_{D3} by using the following conditions

$$\begin{bmatrix} 1 & 1 & 1 \\ l_A e^{i\theta_A} & l_B e^{i\theta_B} & l_D e^{i\theta_D} \\ {l_A}^2 & {l_B}^2 & {l_D}^2 \end{bmatrix} \begin{bmatrix} m_{A3} \\ m_{B3} \\ m_{D3} \end{bmatrix} = \begin{bmatrix} m_3 \\ 0 \\ I_{S_3} \end{bmatrix}$$

$$m_{A3} = \frac{D_{A_3}}{D_3}, m_{B3} = \frac{D_{B_3}}{D_3}, m_{D3} = \frac{D_{D_3}}{D_3} \quad (17)$$

Where,

l_A, l_B, l_D are the moduli of radius vectors of corresponding points
$\theta_A, \theta_B, \theta_D,$ are the angular positions of radius vectors
m_3 is the mass of link 3
I_{S_3} is the mass moment of inertia of link 3 about its centre of mass
$D_{A_3}, D_{B_3}, D_{D_3}$ and D_3 are the third order determinants obtained from the system of equations.

For link 2 to be dynamically replaced by two point masses m_{A2} *and* m_{P2} the condition to be satisfied is

$${k_2}^2 = l_{AS_2} l_{P_2S_2}$$

Where l_{AS_2} is arbitrarily taken and $l_{P_2S_2}$ is obtained from the above condition

$$m_{A2} = \frac{m_2 l_{P_2S_2}}{(l_{AS_2} + l_{P_2S_2})}$$

$$m_{P2} = \frac{m_1 l_{AS_2}}{(l_{AS_2} + l_{P_2S_2})}$$

Counterweight m_{CW_2} against link 2 can be obtained as

$$m_{CW_2} = \frac{(m_2 l_{OS_2} + m_{A3} l_{OA})}{r_{CW_2}} \quad (18)$$

Where $r_{CW_2} = \left(l_{P_2S_2} - l_{OS_2}\right)$, is the radius of rotation of counterweight m_{CW_2}

For link 4 to be dynamically replaced by the point masses m_{B4} and m_{P4} the condition to be satisfied is

$${k_4}^2 = l_{BS_4} l_{P_4S_4}$$

Where l_{BS_4} is arbitrarily taken and $l_{P_4S_4}$ is obtained from above condition

$$m_{B4} = \frac{m_4 \, l_{P_4S_4}}{(l_{BS_4} + l_{P_4S_4})}$$

$$m_{P4} = \frac{m_4 \, l_{BS_4}}{(l_{BS_4} + l_{P_4S_4})}$$

Counterweight m_{CW_4} against link 4 can be obtained as

$$m_{CW_4} = \frac{(m_{B3} l_{BC} + m_4 l_{CS_4})}{r_{CW_4}} \quad (19)$$

Where $r_{CW_4} = \left(l_{P_4S_4} - l_{CS_4}\right)$, is the radius of rotation of counterweight m_{CW_4}

For link 5 to be dynamically replaced by two point masses m_{E5} *and* m_{P5} the condition to be satisfied is

$${k_5}^2 = l_{DS_5} l_{P_5S_5}$$

Where l_{DS_5} is arbitrarily taken and $l_{P_5S_5}$ is obtained from the above condition

$$m_{D5} = \frac{m_5 l_{P_5S_5}}{(l_{DS_5} + l_{P_5S_5})}$$

$$m_{P5} = \frac{m_5 l_{DS_5}}{(l_{DS_5} + l_{P_5S_5})}$$

After link 5 is dynamically replaced by two point masses it is kinematically connected to its corresponding gear inertia counter weight 12 by link $6'$, more over link $6'$ is statically replaced by two point masses m_G *and* m_H

$$m_G = \frac{m'_6 l_{FS_6}}{l_{EF}}$$

$$m_H = \frac{m'_6 l_{ES_6}}{l_{EF}}$$

Counterweight m_{CW_5} against link 5 is calculated by using the formula

$$m_{CW_5} = \frac{(m_{D3} l_{DE} + m_G l_{GE} + m_5 \, l_{ES_5})}{r_{CW_5}} \quad (20)$$

Where $r_{CW_5} = l_{P_5S_5} - l_{ES_5}$ is radius of rotation of counterweight m_{CW_5}

For link 6 to be dynamically replaced by two point masses m_{E6} and m_{P6} the condition to be satisfied is

$${k_6}^2 = l_{ES_6} l_{P_6S_6}$$

Where l_{ES_6} is arbitrarily taken and $l_{P_6S_6}$ is obtained from the above condition

$$m_{E6} = \frac{m_6 l_{P_6S_6}}{(l_{P_6S_6} + l_{ES_6})}$$

$$m_{P6} = \frac{m_6 l_{ES_6}}{(l_{P_6S_6} + l_{ES_6})}$$

Counterweight against link 6 can be calculated as

$$m_{CW_6} = \frac{\left(m_{D3} + m_G + m_{CW_5} + m_5\right) l_{EF}}{r_{CW_6}} \tag{21}$$

Where $r_{CW_6} = l_{P_6S_6} - l_{FS_6}$, is the radius of rotation of counterweight m_{CW_6}

4.2.2 Shaking moment balancing of the mechanism

The shaking moments generated by links 2, 4, 5 and 6 are shown in eq. (22) .The shaking moment generated by the linkage is determined by the sum

$$M^{int} = M_2^{int} + M_4^{int} + M_6^{int} + M_5^{int} \tag{22}$$
$$M_2^{int} = (I_{S_2} + m_2 l_{OS_2}^2 + m_{A3} l_{OA}^2 + m_{CW_2} r_{CW_2}^2)\alpha_2$$
$$M_4^{int} = (I_{S_4} + m_{B3} l_{BC}^2 + m_{CW_4} r_{CW_4}^2 + m_4 l_{CS_4}^2)\alpha_4$$
$$M_6^{int} = (I_{S_6}'^* + I_{S_6} + m_6 l_{FS_6}^2 + (m_{CW_5} + m_G + m_{D3} + m_5) l_{EF}^2 + m_6' l_{FS_6}^2 + m_{CW_6} r_{CW_6}^2)\alpha_6$$
$$M_5^{int} = (2 m_H l_{FH}^2)\alpha_5$$

Where

$M_2^{int}, M_4^{int}, M_5^{int}, M_6^{int}$ are the shaking moments generated by links 2, 4, 5 and 6 respectively

I_{S_2}, I_{S_4} *and* I_{S_6} are the mass moments of inertias of links 2, 4 and 6 respectively

$I_{S_6}'^*$ is the changed moment of inertia of links $6'$ respectively.

$\alpha_2, \alpha_4, \alpha_5$ *and* α_6 are the angular accelerations of links 2, 4, 5 and 6 respectively.

For shaking moment balancing 8 gear inertia counterweights are used, two at $'O'$, two at 'C' and four at $'F'$.

Shaking force of the mechanism by the proposed method

$$F_{Proposed} = -(m_2 A_{G2} + m_3 A_{G3} + m_4 A_{G4} + m_5 A_{G5} + m_6 A_{G6} + m_6' A_{G6}')$$

Shaking moment of the mechanism by the proposed method

$$M_{proposed}^{int} = M_2^{int} + M_4^{int} + M_5^{int} + M_6^{int}$$

Shaking force of the mechanism by GaoFeng's method

$$F_{Gaofeng} = -(m_2 A_{G2} + m_3 A_{G3} + m_4 A_{G4} + m_5 A_{G5} + m_6 A_{G6} + m_{G12} A_{G12})$$

Shaking moment of the mechanism by GaoFeng's method

$$M_{Gaofeng}^{int} = M_2^{int} + M_4^{int} + M_5^{int} + M_6^{int} + (I_{S12} + 2 m_{G12} l_{EF}^2)\alpha_6$$

5. Numerical example

The Stephenson's mechanism with three fixed points shown in Fig.6 has the following parameters

$m_2 = 2kg, k_2 = 0.2345m, m_3 = 5.3kg, k_3 = 0.1197m, m_4 = 3kg, k_4 = 0.7345m, m_5 = 3.5kg,$
$k_5 = 0.8912m, m_6 = 4kg, k_6 = 0.7654, l_A = 4.5m, l_B = 1.1m, l_D = 5.7m, \theta_A = 0^0, \theta_B = 248^0,$
$\theta_D = 159^0, l_{AS_2} = 2m, l_{OA} = 4m, l_{BC} = 6m, l_{BS_4} = 3m, l_{DE} = 11m, l_{DS_5} = 5.5m, l_{EF} = 9m, l_{FS_6} = 4.5m,$
$l_{GE} = 2.6m, l_{FH} = 2.6m, \omega_2 = 10rad/s, \alpha_2 = 10rad/s^2$

5.1 Comparison between the results of Proposed and GaoFeng's methods

The results of shaking force in the mechanism by Proposed and GaoFeng's method are shown in table 1.The results from table 1 show that except at crank angle 0 and 50 degrees the shaking forces in the mechanism by proposed method are less than that of by GaoFeng's method and there is a little improvement in shaking force balancing. The results of shaking moment in the mechanism by proposed and GaoFeng's methods are shown in table 2. On comparison of shaking moment values it can be observed that except at 0 and 60 degrees of the crank angle, shaking moment values by the proposed method are less than that of by GaoFeng's method and considerable improvement in shaking moment can be noticed. When shaking force and shaking moment values are compared for Stephenson's mechanism with three fixed points it can be observed that improved results are produced by the proposed method than that of by GaoFeng's method.

Table 1 Shaking force comparison of Stephenson's mechanism with three fixed points

Crank angle (deg)	Shaking force generated in proposed method N	Shaking force generated in GaoFeng method N
0	-1206.43	-1201.70
10	-11885.50	-11886.39
20	2118.27	2133.67
30	1617.82	1629.48
40	-24927.78	-25317.72
50	11.07	2.70
60	3680.60	3727.03
70	2298.29	2324.49

Table 2 Shaking moments comparison of Stephenson's mechanism with three fixed points

Crank angle(deg)	Shaking moment generated in proposed method N-m	Shaking moment generated in GaoFeng's method N-m
0	-3043.32	-2356.63
10	15581.41	20262.32
20	4032.73	4223.91
30	8537.94	8578.43
40	276880.32	280870.24
50	28007.22	28236.34
60	-6796.12	-6425.32
70	28531.34	28862.45

The shaking forces and shaking moments of Stephenson's mechanism with three fixed points are determined at an interval of 10^0. At number of positions of crank angle the results are almost equal. Shaking force is maximum, 24927.78 N, at 40^0 and minimum, 11.074 N, at 50^0. Shaking moment of the mechanism is maximum, 276880.22 N-m, at 40^0 and minimum, 3043.3 N-m, at 0^0. But at higher value of shaking moment i.e. at 10^0, the proposed method offered a great improvement, 23% over GaoFeng method.

Conclusions

Shaking force is balanced by the method of redistribution of mass and shaking moment by geared inertia counterweights. All the planetary gears used for balancing the shaking moment generated by links not directly to the frame are mounted on the base of the mechanism, which is constructively more efficient and makes the balanced mechanism compact.

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