Balancing of Stephenson’s Mechanisms

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Abstract

This paper deals with the problem of shaking force and shaking moment balancing of high speed planar mechanisms. The design equations and techniques for the complete shaking force and shaking moment balancing of two types of six-bar linkages due to both linear and rotary inertia but without considering external loads are developed. Shaking force is balanced by the method of redistribution of mass and shaking moment by the geared inertia counterweight and planetary-gear-train inertia counterweight. These geared inertia counter weights are very useful for balancing of multi-bar linkages. The proposed techniques produce better results than that of the previous techniques.

Keywords: Shaking force, shaking moment, dynamic balancing, vibration, and Stephenson’s mechanisms.

1. Introduction

A mechanical system with unbalanced shaking force and shaking moment transmits considerable vibration to the frame and foundation of the machine. Different approaches and solutions (G.G.Lowen et al 1983 and V.Arakelian et al 2005) devoted to this problem have been developed and documented for various planar mechanisms. The dynamic balancing of a mechanism is generally achieved by two steps. First step is the cancellation or reduction of the shaking force and the second the cancellation or reduction of the shaking moment. Generally the cancellation of the shaking force transmitted to the frame can be achieved by adding counterweights in order to make the total centre of mass of moving links stationary by additional structures (V.Vander Wijk et al 2012) or by elastic components (G.Alici et al 2003).


The present paper is the extension work of authors (V.Arakelian et al 1999, GaoFeng 1989, GaoFeng 1990). The results obtained are better than the previous method results. The paper is organized as follows: section 1 deals with introduction, section 2 presents articulation dyad, Section 3 deals with Asymmetric link with three rotational pairs. Dynamic balancing of Stephenson’s mechanisms is given in section 4.

Numerical examples and results are discussed in section 5. Conclusions are given in section 6.

2. Articulation dyad

A. Complete shaking force and shaking moment balancing of an articulation dyad

An open kinematic chain of two binary links and one joint is called a dyad. When two links are articulated by a joint so that movement is possible that arrangement of links is known as articulation dyad.

Fig.1 Complete shaking force and shaking moment balancing of an articulation dyad

The familiar scheme of complete shaking force and shaking moment balancing of an articulation dyad [10]-[13] is shown in Fig.1. A counterweight is attached to link 2 which permits the displacement of the center of mass of link 2 to joint A. Then, by means of a counter
weight with mass $m_{cw}$ [Fig.1] a complete balancing of shaking force is achieved. A complete shaking moment balance is obtained by four gear inertia counter weights 3-6, one of them is of the planetary type and mounted on link 2.

B. Complete shaking force and shaking moment balancing of an articulation dyad by gear inertia counterweights mounted on the base

The scheme used in the present work [Fig.2] is distinguished from the earlier scheme by the fact that gear 3 is mounted on the base and is linked kinematically with link 2 through link 1.

![Fig.2](image)

**Fig.2** Complete shaking force and shaking moment balancing of an articulation dyad by gear inertia counterweights mounted on the base

To prove the advantages of such a balancing, the application of the new system with the mass of link 1 not taken into account is considered. In this case (compared to the usual method Fig.1), the mass of the counter weight of link 1 will be reduced by an amount

$$\Delta m_{cw1} = \frac{m_3 l_{OA}}{r_{cw1}}$$

where,

$m_3$ is the mass of gear 3, $l_{OA}$ is the distance between the centers of hinges O and A, $r_{cw1}$ is the rotation radius of the center of mass of the counter weight.

It is obvious that the moment of inertia of the links is correspondingly reduced. If the gear inertias are made in the form of heavy rims in order to obtain a large moment of inertia, the moments of inertia of the gear inertia counter weights may be presented as

$$I = \frac{m_i D_i^2}{4} \quad (i=3...6).$$

Consequently, the mass of gear 6 will be reduced by an amount

$$\Delta m_6 = 4(m_3 l_{OA}^2 + \Delta m_{cw1} r_{cw1}^2) \frac{T_6}{D_6^2 T_5}$$

where, $T_5$ and $T_6$ are the numbers of teeth of the corresponding gears. Thus, the total mass of the system will be reduced by an amount

$$\Delta m = \Delta m_{cw1} + \Delta m_6$$

Here the complete shaking force and shaking moment balancing of the articulation dyad with the mass and inertia of link 1 taken into account is considered. For this purpose initially, statically replace mass $m_1'$ of link 1' by two point masses $m_B$ and $m_c$ at the centers of the hinges B and C.

$$m_B = m_1' l_{CS} / l_{BC}$$

$$m_C = m_1' l_{BS} / l_{BC}$$

Where, $l_{BC}$ is the length of link 1, $l_{CS}$ and $l_{BS}$ are the distances between the centers of joints C and B and the center of mass $S_1'$ of link 1', respectively.

After such an arrangement of masses the moment of inertia of link 1' will be equal to

$$I^*_1 = I_{S1} - m_B l_{CS} l_{BS}$$

where, $I_{S1}$ is the moment of inertia of link 1' about the center of mass $S_1'$ of the link. Thus a new dynamic model of the system is obtained, where the link 1' is represented by two point masses $m_B$, $m_C$ and has a moment of inertia $I^*_1$.

This fact allows for an easy determination of the parameters of the balancing elements as follows:

$$m_{cw2} = (m_B l_{AS2} + m_C l_{AB}) / r_{cw2}$$

where, $m_2$ is the mass of link 2, $l_{AB}$ is the distance between the centers of the hinges A and B, $l_{AS2}$ is the distance of the center of hinge A from the center mass of $S_2$ of link 2, $r_{cw2}$ is the rotation radius of the center of mass of the counterweight with respect to A, and

$$m_{cw1} = (m_3 + m_{cw2} + m_B l_{OA} + m_C l_{OC}) / r_{cw1}$$

where, $m_1$ is the mass of link 1, $l_{OS1}$ is the distance of the joint center O from the center of mass $S_1$ of link 1.

$$m_{cw3} = m_C l_{OC} / r_{cw3}$$

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where, \( l_{oc} = l_{ap}, \) is the rotation radius of the center of mass of the counterweight. Taking into account the mass of link 1 brings about the correction in Eq. (3) in this case,

\[
\Delta m = \Delta m_{CW1} + \Delta m_b - \Delta m_1'
\]

(9)

where,

\( \Delta m_1' \) is the value deciding the change in the distribution of the masses of the system links resulting from the addition of link 1'.

3. Asymmetric link with three rotational pairs

A link with three nodes is called ternary link, where nodes are points for attachment to other links. In the earlier research by Gao Feng relating to balancing of linkages with a dynamic substitution of the masses of the link by three rotational pairs shown in Fig. 3 two replacement points A and B are considered. This results in the need to increase the mass of the counter weight. However, such a solution may be avoided by considering the problem of dynamic substitution of link masses by three point masses. Usually the center of mass of such an asymmetric link is located inside a triangle formed by these points.

Fig. 3 Dynamic substitution of the masses of the link by three rotational pairs

The conditions for dynamic substitution of masses are the following:

\[
\begin{bmatrix}
1 & 1 & 1 \\
l_A e^{\theta_A} & l_B e^{\theta_B} & l_C e^{\theta_C} \\
l_A^2 & l_B^2 & l_C^2
\end{bmatrix}
\begin{bmatrix}
m_A \\
m_B \\
m_C
\end{bmatrix}
=
\begin{bmatrix}
m_i \\
0 \\
I_s
\end{bmatrix}
\]

where, \( m_1, m_2, \) and \( m_c \) are point masses, \( l_A, l_B, \) and \( l_C \) are the moduli of radius vectors of corresponding points, \( \theta_A, \theta_B, \) and \( \theta_C \) are angular positions of radius vectors; \( m \) is the mass of link, \( I_s \) is the moment of inertia of the link about an axis through \( S \) (axial moment of inertia of link).

From this system of equations the masses are obtained

\[
m_A = D_s / D_i; m_B = D_B / D_i; m_C = D_C / D_i
\]

(10)

where, \( D_s, D_B, D_C \) and \( D_i \) are determinants of the third order obtained from the above system of equations.

4. Dynamic Balancing of Stephenson’s mechanisms

4.1 Stephenson’s mechanism with two fixed points (Mechanism with high degree of complexity)

The Stephenson’s mechanism with two fixed points shown in Fig. 4 is obtained when one of the binary links in the basic Stephenson’s chain is fixed. This is a mechanism with high degree of complexity as more than one radii of path curvature of motion transfer points C and D are not known. This mechanism is used in the reversing gear of steam engines. The balanced Stephenson’s mechanism with two fixed points is shown in Fig. 4

Fig. 4 Stephenson’s mechanism with two fixed points

Link 5 is replaced by dynamic substitution of link masses by three point masses \( m_{c3}, m_{d5} \) and \( m_{e5} \); The conditions for dynamic substitution of masses are the following:

\[
\begin{bmatrix}
1 & 1 & 1 \\
l_A e^{\theta_A} & l_B e^{\theta_B} & l_C e^{\theta_C} \\
l_A^2 & l_B^2 & l_C^2
\end{bmatrix}
\begin{bmatrix}
m_c \\
m_d \\
m_e
\end{bmatrix}
=
\begin{bmatrix}
m_i \\
0 \\
I_s
\end{bmatrix}
\]

(11)

Where,
m_{c5}, m_{65} and m_{6} are point masses; 
l_{c}, l_{d}, l_{e} are the moduli of radius vectors of 
corresponding points. 
$\theta_{c}, \theta_{d}, \theta_{e}$ are the angular positions of radius vectors. 
m_{s} is the mass of link 5. 

\[ l_{5s} \text{is the moment of inertia of link 5 about an axis} \]
\[ \text{through} \ s_{5} \text{ (axial moment of inertia of link 5)} \]

\[ m_{cs} = D_{c}/D_{s}; m_{ps} = D_{p}/D_{s}; m_{es} = D_{e}/D_{s} \]

(11)

Where  \( D_{c}, D_{p}, D_{e} \) are determinants of the third order obtained from the above system of equations.

Link 6 is dynamically replaced by two point masses \( m_{cs} \) and \( m_{ps} \) and attached a counterweight \( m_{cw6} \). For link 6 to be dynamically replaced by two point masses the condition to be satisfied is \( k_{4}^{2} = l_{DS6}^{2} - l_{t6s6}^{2} \). Where, \( k_{4} \) is the radius of gyration of link 6 about its center of mass \( l_{DS6} \) is obtained from the above condition.

\[ m_{cw6} = \frac{(m_{c}l_{Ds6} + m_{ps}l_{ps6})}{r_{cw6}} \]

\[ m_{Es} = \frac{m_{Es6}l_{Es6} + l_{ps6}}{r_{Es}^{2}} \]

\[ m_{Ps} = \frac{m_{Ps6}l_{Ps6} + l_{ps6}}{r_{Ps}^{2}} \]

Where, \( r_{cw6} = l_{Ds6} - l_{t6s6} \) is the radius of rotation of counterweight \( m_{cw6} \).

Now, link 2 is dynamically replaced by three point masses \( m_{o2}, m_{a2}, m_{b2} \).

\[ \begin{bmatrix} 1 & l_{o2}e^{i\theta_{o2}} & l_{a2}e^{i\theta_{a2}} & l_{b2}e^{i\theta_{b2}} \\
1 & l_{o2}^{*} & l_{a2}^{*} & l_{b2}^{*} \\
m_{o2} & m_{a2} & m_{b2} & l_{s2} \end{bmatrix} = \begin{bmatrix} m_{2} \\
0 \\
1 \end{bmatrix} \]

(12)

where, \( m_{o2}, m_{a2}, m_{b2} \) are the point masses, \( l_{o2}, l_{a2}, l_{b2} \) are the moduli of radius vectors of the corresponding points, \( \theta_{o2}, \theta_{a2}, \theta_{b2} \) are angular positions of radius vectors, \( m_{s} \) is the mass of link 2. \( l_{s2} \) is the mass moment of inertia of link 2 about its center of mass.

\[ m_{o2} = D_{o2}/D_{2}; m_{a2} = D_{a2}/D_{2}; m_{b2} = D_{b2}/D_{2}; \]

Now link 4 is dynamically replaced by two point masses \( m_{o4}, m_{b4} \) and linked kinematically with gear inertia counterweight 9 by link 4’ and attached a counter weight \( m_{cw4} \) against link 4. For link 4 to be dynamically replaced by two point masses the condition to be satisfied is

\[ k_{4}^{2} = l_{DS4}^{2} - l_{t4s4}^{2} \]

where, \( k_{4} \) is the radius of gyration of link 4 about its center of mass, \( l_{DS4} \) is arbitrarily fixed and \( l_{t4s4} \) is obtained from the above condition.

\[ m_{cw4} = \frac{m_{c4}l_{Ds4} + m_{b4}l_{bs4} + m_{f4}l_{bf4}}{r_{cw4}} \]

\[ m_{cs4} = \frac{m_{cs4} + m_{cw4} + m_{f4} + m_{bs4}}{l_{cs4}} \]

(13)

where, \( m_{cw4} \) is the counterweight attached against point \( B \) \( r_{cw4} = (l_{ps4} - l_{t4s4}) \) is the radius of rotation of counterweight \( m_{cw4} \).

Now link 3 is dynamically replaced by two point masses \( m_{c3}, m_{b3} \).

For link 3 to be dynamically replaced by two point masses the condition to be satisfied is \( k_{3}^{2} = l_{CS3}l_{ps3} \) where, \( k_{3} \) is the radius of gyration of link 3 about its center of mass, \( l_{CS3} \) is arbitrarily fixed and \( l_{ps3} \) is obtained from the above condition.
Now gear 11 is mounted on the base and linked kinematically with link 3' and link 3' is statically replaced by two point masses \( m_1, m_2 \),

\[
m_1 = \frac{m_1' l_{1h}}{l_{1h}}
\]

\[
m_2 = \frac{m_2' l_{2h}}{l_{2h}}
\]

where \( l_{1h} = l_{102} \)

After static substitution of masses the moment of inertia of link 3'

\[
I_{3'} = I_{3s} - m_1 l_{1h} r_{1h},
\]

where, \( I_{3'} \) is the moment of inertia of link 3' after static substitution of masses.

The counterweight attached against mass \( m_j \) is equal to

\[
m_{CW,j} = \frac{m_j l_{1A,j}}{r_{CW,j}}
\]

\[
m_{CW,3} = \frac{(m_3 l_{A3} + m_{CW,1A} + m_1 l_{1A})}{r_{CW3}}
\]

\[
m_{ACW} = \frac{(m_3 + m_{CW3} + m_{CS} + m_1 + m_{A2}) r_{A2}^2}{I_{ACW}}
\]

Where

- \( m_{CW,j} \) is the counterweight against point mass \( m_j \)
- \( r_{CW3} = l_{A3} - l_{A5} \) is the radius of rotation of counterweight
- \( m_{ACW} \) is the counterweight mass attached against point A

4.1.2 Shaking moment balancing of the mechanism

The shaking moment generated by links 2 and 6 are given in eq. (16). The shaking moment generated by the mechanism is determined by the sum

\[
M^m = M^m_2 + M^m_3 (16)
\]

where,

- \( M^m_2, M^m_3 \) are the shaking moments of the rotating links 6 and 2 respectively with the inertia of the replaced point masses taken into account.

\[
M^m_2 = \left( l_{16} + m_1 l_{16}^2 + m_{CS} l_{16}^2 + m_{CW3} l_{16}^2 \right) \alpha_2
\]

\[
M^m_3 = \left( I_{3s}^* + I_{3s}^* + (m_3 + m_{CW} + m_{CS} + m_1 + m_{A2} + m_{ACW} r_{ACW}^2 \right) r_{O2}^2 + (m_3 + m_{CW3} + m_{CS} + m_1 + m_{A2}) r_{O2}^2 + m_{ACW} r_{ACW}^2 + 2 m_1 l_{16}^2 + 2 m_1 r_{O2}^2) \alpha_2
\]

Where, \( I_{16} \) is the mass moment of inertia of link 6 about its center of mass

- \( I_{3s}^*, I_{3s}^* \) are the changed moments of inertia of links 3' and 4' respectively
- \( \alpha_2, \alpha_3 \) are the angular accelerations of links 2 and 6 respectively

The shaking moment generated by the mechanism is balanced by using the gear inertia counterweights 7-14.

![Fig.5 Balanced Stephenson’s mechanism with two fixed points](image-url)

4.2 Stephenson’s mechanism with three fixed points

Stephenson’s mechanism with three fixed points has six links out of them four are binary and two are ternary links. In the Stephenson’s mechanism two ternary links are connected by binary links. This Stephenson’s mechanism with three fixed points shown in Fig.6 is obtained when one of the ternary links is fixed. Stephenson’s six-bar linkage can be thought of as two four-bar linkages connected in parallel and sharing two links in common. This mechanism is mostly used in Steam engines. The balanced Stephenson’s mechanism with three fixed points is shown in Fig.7.

![Fig.6 Stephenson’s mechanism with three fixed points](image-url)

![Fig.7 Balanced Stephenson’s mechanism with three fixed points](image-url)

Link 1 is the fixed ternary link. The other ternary link 3 is connected to fixed ternary link by binary links 2, 4 and 6. The links 3 and 5 are not directly connected to the frame. The geared inertia counterweights required to balance the shaking moment generated by links 3 and 5 are mounted on the base of the mechanism by
kinematically linking the geared inertia counterweights and the links by links of known mass and center of mass.

4.2.1 Shaking force balancing of the mechanism

The shaking force of the mechanism is balanced by dynamically replacing the links by point masses and mass counterweights. Here link 3 is dynamically replaced by three point masses \(m_{A3}, m_{B3}, m_{D3}\) by using the following conditions

\[
\begin{bmatrix}
1 & 1 & 1 \\
\theta_A & \theta_B & \theta_D \\
l_A^2 & l_B^2 & l_D^2
\end{bmatrix}
\begin{bmatrix}
m_{A3} \\
m_{B3} \\
m_{D3}
\end{bmatrix}
= \begin{bmatrix}
m_3 \\
0 \\
l_3
\end{bmatrix}
\] (17)

Where,

\(l_A, l_B, l_D\) are the moduli of radius vectors of corresponding points

\(\theta_A, \theta_B, \theta_D\) are the angular positions of radius vectors

\(m_3\) is the mass of link 3

\(l_3\) is the mass moment of inertia of link 3 about its centre of mass

\(D_{A3}, D_{B3}, D_{D3}\) and \(D_3\) are the third order determinants obtained from the system of equations.

For link 2 to be dynamically replaced by two point masses \(m_{A2}\) and \(m_{P2}\) the condition to be satisfied is

\[k_2^2 = l_{AS2}^2 p_{S2}\]

Where \(l_{AS2}\) is arbitrarily taken and \(p_{S2}\) is obtained from the above condition

\[m_{A2} = \frac{m_3 l_{P2} S_2}{(l_{AS2} + l_{P2} S_2)}\]

\[m_{P2} = \frac{m_3 l_{AS2}}{(l_{AS2} + l_{P2} S_2)}\]

Counterweight \(m_{CW2}\) against link 2 can be obtained as

\[m_{CW2} = \frac{(m_3 l_{AS2} + m_3 l_{P2})}{r_{CW2}}\] (18)

Where \(r_{CW2} = (l_{P2} S_2 - l_{AS2})\), is the radius of rotation of counterweight \(m_{CW2}\)

For link 4 to be dynamically replaced by the point masses \(m_{B4}\) and \(m_{P4}\) the condition to be satisfied is

\[k_4^2 = l_{BS4}^2 p_{S4}\]

Where \(l_{BS4}\) is arbitrarily taken and \(p_{S4}\) is obtained from above condition

\[m_{B4} = \frac{m_4 l_{P4} S_4}{(l_{BS4} + l_{P4} S_4)}\]

\[m_{P4} = \frac{m_4 l_{BS4}}{(l_{BS4} + l_{P4} S_4)}\]

Counterweight \(m_{CW4}\) against link 4 can be obtained as

\[m_{CW4} = \frac{(m_4 l_{BS4} + m_4 l_{P4})}{r_{CW4}}\] (19)

Where \(r_{CW4} = (l_{P4} S_4 - l_{BS4})\), is the radius of rotation of counterweight \(m_{CW4}\).

For link 5 to be dynamically replaced by two point masses \(m_{ES}\) and \(m_{PS}\) the condition to be satisfied is

\[k_5^2 = l_{DS5}^2 p_{S5}\]

Where \(l_{DS5}\) is arbitrarily taken and \(p_{S5}\) is obtained from the above condition

\[m_{DS5} = \frac{m_5 l_{P5} S_5}{(l_{DS5} + l_{P5} S_5)}\]

\[m_{PS} = \frac{m_5 l_{DS5}}{(l_{DS5} + l_{P5} S_5)}\]

Counterweight \(m_{CW5}\) against link 5 can be calculated by using the formula

\[m_{CW5} = \frac{(m_5 l_{DS5} + m_5 l_{P5})}{r_{CW5}}\] (20)

Where \(r_{CW5} = (l_{P5} S_5 - l_{ES5})\) is radius of rotation of counterweight \(m_{CW5}\).

For link 6 to be dynamically replaced by two point masses \(m_{ES}\) and \(m_{PS}\) the condition to be satisfied is

\[k_6^2 = l_{ES6}^2 p_{S6}\]

Where \(l_{ES6}\) is arbitrarily taken and \(p_{S6}\) is obtained from the above condition

\[m_{ES} = \frac{m_6 l_{P6} S_6}{(l_{P6} S_6 + l_{ES6})}\]
\[ m_{P6} = \frac{m_{I\text{gs}}}{l_{P6\text{S}}+l_{S6}} \]

Counterweight against link 6 can be calculated as

\[ m_{\text{cw6}} = \left( \frac{m_{\text{d3}}+m_{\text{g}}+m_{\text{cw}}r_{\text{cw6}}^2}{r_{\text{cw6}}} \right) \alpha \]

Where \( r_{\text{cw6}} = l_{P6\text{S}} - l_{S6} \) is the radius of rotation of counterweight \( m_{\text{cw6}} \).

4.2.2 Shaking moment balancing of the mechanism

The shaking moments generated by links 2, 4, 5 and 6 are shown in eq. (22). The shaking moment generated by the linkage is determined by the sum

\[ M^{\text{int}} = M^{\text{int}}_2 + M^{\text{int}}_4 + M^{\text{int}}_5 + M^{\text{int}}_6 \]  

\[ M^{\text{int}}_2 = (l_{s2} + m_2l_{s2}^2 + m_{s3}l_{s3}^2 + m_{\text{cw}}r_{\text{cw2}}^2)\alpha_2 \]

\[ M^{\text{int}}_4 = (l_{s4} + m_4l_{s4}^2 + m_{\text{cw}}r_{\text{cw4}}^2 + m_4l_{s4}^2)\alpha_4 \]

\[ M^{\text{int}}_5 = (l_{s5} + m_5l_{s5}^2 + m_{\text{cw}}r_{\text{cw5}}^2 + m_5l_{s5}^2)\alpha_5 \]

\[ M^{\text{int}}_6 = (l_{s6} + m_6l_{s6}^2 + m_{\text{cw}}r_{\text{cw6}}^2 + m_6l_{s6}^2)\alpha_6 \]

Where

\( M^{\text{int}}_2, M^{\text{int}}_4, M^{\text{int}}_5, M^{\text{int}}_6 \) are the shaking moments generated by links 2, 4, 5 and 6 respectively. \( l_{s2}, l_{s4}, l_{s5}, l_{s6} \) are the mass moments of inerts of links 2, 4 and 6 respectively. \( l_{s6}' \) is the changed moment of inertia of links 6’. \( \alpha_2, \alpha_4, \alpha_5 \) and \( \alpha_6 \) are the angular accelerations of links 2, 4, 5 and 6 respectively.

For shaking moment balancing 8 gear inertia counterweights are used, two at ‘0’, two at ‘C’ and four at ‘F’.

**Shaking force of the mechanism by the proposed method**

\[ F_{\text{proposed}} = -(m_2\dot{A}_{22} + m_4\dot{A}_{41} + m_5\dot{A}_{53} + m_6\dot{A}_{65} + m_6\dot{A}_{66}) \]

**Shaking moment of the mechanism by the proposed method**

\[ M^{\text{int}}_{\text{proposed}} = M^{\text{int}}_2 + M^{\text{int}}_4 + M^{\text{int}}_5 + M^{\text{int}}_6 \]

5. Numerical example

The Stephenson’s mechanism with three fixed points shown in Fig. 6 has the following parameters

\[ m_2 = 2.4\text{kg}, k_2 = 0.3845\text{m} \]

\[ l_{s2} = 2\text{m}, l_{s4} = 2\text{m}, l_{s5} = 2\text{m}, l_{s6} = 2\text{m} \]

**Table 1** Shaking force comparison of Stephenson’s mechanism with three fixed points

<table>
<thead>
<tr>
<th>Crank angle (deg)</th>
<th>Shaking force generated in proposed method N</th>
<th>Shaking force generated in GaoFeng method N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-120.43</td>
<td>-120.70</td>
</tr>
<tr>
<td>10</td>
<td>-11885.50</td>
<td>-11886.39</td>
</tr>
<tr>
<td>20</td>
<td>2118.27</td>
<td>2133.67</td>
</tr>
<tr>
<td>30</td>
<td>1617.82</td>
<td>1629.48</td>
</tr>
<tr>
<td>40</td>
<td>-24927.78</td>
<td>-25317.72</td>
</tr>
<tr>
<td>50</td>
<td>11.07</td>
<td>2.70</td>
</tr>
<tr>
<td>60</td>
<td>3680.60</td>
<td>3727.03</td>
</tr>
<tr>
<td>70</td>
<td>2298.29</td>
<td>2324.49</td>
</tr>
</tbody>
</table>

**Table 2** Shaking moments comparison of Stephenson’s mechanism with three fixed points

<table>
<thead>
<tr>
<th>Crank angle (deg)</th>
<th>Shaking moment generated in proposed method N-m</th>
<th>Shaking moment generated in GaoFeng’s method N-m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3043.32</td>
<td>-2356.63</td>
</tr>
<tr>
<td>10</td>
<td>15581.41</td>
<td>20262.32</td>
</tr>
<tr>
<td>20</td>
<td>4022.73</td>
<td>4223.91</td>
</tr>
<tr>
<td>30</td>
<td>8537.94</td>
<td>8578.43</td>
</tr>
<tr>
<td>40</td>
<td>27680.32</td>
<td>280870.24</td>
</tr>
<tr>
<td>50</td>
<td>28007.22</td>
<td>28236.34</td>
</tr>
<tr>
<td>60</td>
<td>-6796.12</td>
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</tr>
<tr>
<td>70</td>
<td>28531.34</td>
<td>28862.45</td>
</tr>
</tbody>
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The shaking forces and shaking moments of Stephenson's mechanism with three fixed points are determined at an interval of 10°. At number of positions of crank angle the results are almost equal. Shaking force is maximum, 24927.78 N, at 40° and minimum, 11.074 N, at 50°. Shaking moment of the mechanism is maximum, 276880.22 N·m, at 40° and minimum, 3043.3 N·m, at 0°. But at higher value of shaking moment i.e. at 10°, the proposed method offered a great improvement, 23% over GaoFeng method.

Conclusions

Shaking force is balanced by the method of redistribution of mass and shaking moment by geared inertia counterweights. All the planetary gears used for balancing the shaking moment generated by links not directly to the frame are mounted on the base of the mechanism, which is constructively more efficient and makes the balanced mechanism compact.

References

V.Van der wijk, J.L.Herder, Aug 15-18 2010, active dynamic balancing unit for controlled shaking force and shaking moment balancing, proceedings of the ASME 2010 International in Engineering conference (IDETC/CIE 2010), Montreal, Quebec, Canada.
V.Van der wijk, J.L.Herder, Aug 15-18 2010, active dynamic balancing unit for controlled shaking force and shaking moment balancing, Proceedings of ASME 2010 International in Engineering conference (IDETC/CIE 2010), Montreal, Quebec, Canada.