*Research Article*

# Optical Interpretation of Diminishing and Non-Diminishing of Remote Objects and their Representation in Painting Lands Cape

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*Abstract*

*The geometry of painting land scape remains one of the long standing problems that cannot be explained theatrically. This work is devoted to use geometrical optics to explain this geometry. It utilizes eye geometry to explain why the horizon appears as an infinite line. It also explains why sky meat earth surface at a point, besides showing the reason that makes large mountains does not converge to a point far from us.*

*Keywords: geometrical optics, remote objects, horizon, sky, painting landscape*

## 1. Introduction

Architects since 18th up to 21th century continued to use projection for drawing their linear perspective on two dimensional surfaces. To do that they move projecting line from a point called the station point to pass cress the points that shape a gerund plane. These projecting lines, which are called orthogonal lines, continue to move across a drawn line that represents the ground level, and then change their direction up right to cross, the line that represents the picture plane. Then to draw a one vanishing point perspective, architects use a herizenat semi-central line called eye level and a vertical central line where they meet in the vanishing point. [Martin Kemp, Prof et al, 2007]

Our day life experience shows that the actual geometry of the systems is different from that seen by our necked eye. For example the width of the high way road appears to became smaller and smaller, when their distances from us becomes larger and larger.
This is in direct conflict with the fact that this width is constant over the whole high way road. This discrepancy is closely related to the optical properties of the eye [A. N. Matveer *et al*, 2008, 2014].

One needs a theoretical explanation of this conflict and to answer many questions. For instance, why the horizon appears as an infinite line, while the sky appears to be in constant with the earth surface. At a vanishing point instead of an infinite line. It is not also clear why some objects resists vanishing such as mountains, while some vanishes before on in the horizon line.

All these questions are answered in the subsequent sections. Section 2, is concerned with showing how eye focal length and macula area on which the light pyramid or cone vision is projected, affect the real geometry of the world. Section 3, 4 and 5are devoted for explaining why horizon appear as an infinite line, why the sky meets the earth surface at a point and large mountains resists vanishing respectively.

## 2. Optical Properties of the Human Eye

The optical properties of the final image you see in front of you is closely related to the properties of the image formed on the retina. The properties of the retina image are constrained by many factors. For instance two points appears distinct when they fall on two distinct visual discs, where they can be visualized as one point when they both fall on the same visual disc. These statements can be put in a rigorous concrete sense, with the aid of mathematics. If the radius of the visual disc macula lute is $d_0$ and the distance between the images of the two points formed on the retina is $L_0$. The two points appear distinct to us if [A. N. Matveer, 2014]
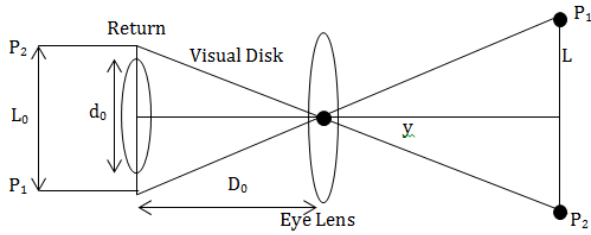
$$L_0 > d_0 \qquad (1)$$

But if

$$L_0 \leq d_0 \qquad (2)$$

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The two points are seen by us as one single point



**Figure 1:** Distance between the images of the two points $P_1$ and $P_2$ compared to the visual disc.

The visual zone which can be seen by us depends on the total area oh the visual disc receptors known as macula one can simplify the situation by assuming this area to be bounded by a circle of radius $r_o$ thus the maximum zone area radius $r_m$ can be found by using the relation [ M. M. Elhusainm,2014]

$$\frac{1}{u_m} = \frac{1}{f_m} - \frac{1}{D_O} \qquad (3)$$

With $u_m$ standing for the maximum object distance, which requires the eye focal length to be also maximum and equal to $f_m$. The distance between retina and eye lens is constant and equal to $D_O$ , equaling to the image distance.

The maximum distance that can be visualized clearly is equal to $u_m$ and is given by equation (3). The maximum seen zone radius $r_m$ can be given from the relation between image beside object size on one hand and image beside object distances from the lens, i.e

$$\frac{r_m}{r_o} = \frac{u_m}{D_O}$$
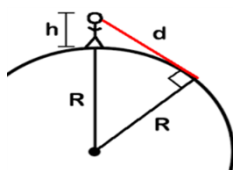$$r_m = \frac{r_o}{D_O} u_m \qquad (4)$$

The maximum visualized area is thus given by

$$A_m = \pi r_{m^2} = \pi \left(\frac{r_o}{D_o}\right) u_{m^2} \qquad (5)$$

### 3. Why the Horizon Looks as an Infinite Line

The horizon radius y is the distance between the observer and the maximum point that can be observed considering the observer height is h and the earth radius is R



**Figure 2**: Horizon Radius y

Then from the geometry of a spherical shape and rectangle triangles, one gets:
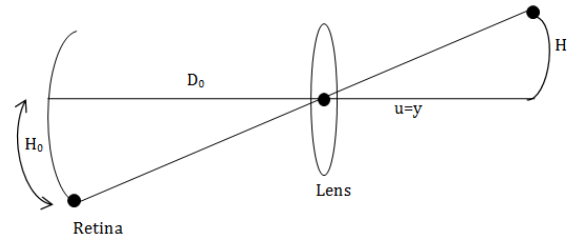
$$x^2 = (R + h)^2 - R^2 = 2Rh + h^2 \approx 2Rh \qquad (6)$$

Where we neglect h compared to R. Also

$$y^2 = x^2 - h^2 = 2Rh + h^2 - h^2 = 2Rh$$
$$y = (2Rh)^{\frac{1}{2}} \qquad (7)$$

To see why the horizon looks as an infinite line see fig (3)



**Figure 3:** The Horizon Light Pyramid falling on the Eye.

From fig 3, by considering the Horizon as a semicircle of length H , then

$$\frac{H_o}{D_o} = \frac{H}{y}$$
$$H_O = \frac{\left(\frac{\pi}{2}\right)y}{y} D_O = \frac{\pi}{2} D_o \qquad (8)$$

The image of the horizon appears as an infinite line when

$$H_o \geq r_o \qquad (9)$$

Where $r_o$ is the radius of the visual zone (macula) at the retina its radius is about 2.5mm but the distance between the eye lens and the retina is about 17mm. Thus according to equation (8)

$$H_o = \frac{\pi}{2} D_o = \frac{3.14 \times 17}{2} = 26.69 \text{ mm}$$

But $r_o = 5$ mm

Thus

$$H_o = 26.69 > r_o = 5 \text{ mm} \qquad (10)$$

Hence the horizon image on the retina is large than the macula size, which make horizon appears infinite.

### 4. Why the Sky appears to be in Contact with the Earth Surface at a point

The sky color comes from light scattering at ozone layer which has average highest of about 40 km from the earth surface, let us denote this highest by S, y is the object distance from fig (4)
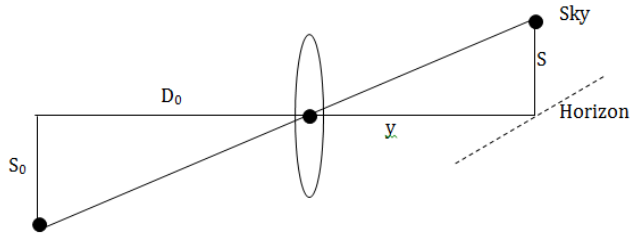
**Figure 4**: The sky view by the eye

$$\frac{S_o}{D_o} = \frac{S}{y} \quad , \quad S_o = D_o \frac{S}{y} \tag{11}$$

Where $S_o$ is the image height of the retina?

Since

$y >> S$

Thus one can easily assume

$y \to \infty \qquad S = limited$

To get

$$S_o \to \frac{D_o \, S}{\infty} \to 0 \tag{12}$$

Thus the image of the vertical line connecting the sky to the earth surface converges to a point on the retina This can explain why the sky appears to meet the earth surface at a point.

### 5. Why Large Mountains does not Vanish

In our real life, when we go to deserts large mountains resists vanishing and becomes fainter and fainter till it disappears. How can one explain this and why it does not converge to a vanishing point like sky. The answer of this question is firstly related to the fact that the sky which scatters sun light directly is much brighter than large mountains which reflects very less intense light that looses most of its intensity upon reflected, scatted and absorbed by the atmosphere. Thus large mountains disappear before reaching the vanishing distance due to its low light intensity emerging from them.

To understand this, it is important to use mathematics and physical laws. Assuming light intensity obeying the exponential decay law let the minimum light intensity reaching retina, and which beyond it an object disappear, is equal to $I_e$ . If the intensity of a large body like a mountain is very law and equal to $I_b$ and that of the sky is $I_s$ at their surfaces. Then the distances $y_b$ and $y_s$ which just bayed them they disappear satisfies

$$I_e = I_b e^{-\mu y_b} = I_s e^{-\mu y_s} \tag{13}$$

There fore

$$e^{\mu(y_b - y_s)} = I_b / I_s$$

$$y_b - y_s = (\ln I_b / I_s) / \mu$$

$$y_s - y_b = \frac{1}{\mu} (\ln I_s / I_b) \tag{14}$$

If

$$I_s >> I_b \tag{15}$$

Then

$$y_s >> y_b \tag{16}$$

Thus according to equation (10)

$$S_{os} = \frac{D_o S_s}{y_s} << \frac{D_o S_b}{y_b} = S_{ob}$$

I.e

$$S_{os} << S_{ob} \tag{17}$$

This means that the more brighter remote object lira the sky appears much smaller and can vanish but less brighter fact objects disappear before vanishing.

### 6. Why Long Roads Vanishes on the Horizon to a point

It is familiar to anybody to observe that very long high way roads and the railway roads vanish on the horizon. This can also be explained here by using equation (11), S = road width (small, y = road location at horizon (very large)

$$y \to \infty \tag{18}$$

Thus the image size of road width on the retina is (see equation 11)

$$S_o = \frac{D_o S}{y} \to \frac{D_o S}{\infty} \to 0 \tag{19}$$

Thus the road appears to vanish at horizon.

### 7. Discussion

The vanishing of remote objects depends on the relation between the size of macula cell and the size of the image formed by the eye. According to equation (2) the object appears vanishing to the eye if its size is smaller than the visual cell size.

Section (3) discusses the horizon appearance to the eye. It shows that the horizon appears to us as an infinite line since the horizon image size on the macula is larger than the macula size as shown mathematically be equation (9) and numerically by equation (10).

The non-vanishing of the horizon is due to the fact that its size which is a semicircle increases as its distance from the eye increases as shown by figure (3), where

$$H = \frac{\pi}{2} y.$$

The geometry of the horizon is related to the spherical shape of the earth. The vanishing of any object requires the constancy of the object size upon increasing the object distance y.

The sky height, contrary to horizon size, keep itself constant, while y increases. This makes its image size diminishes gradually according to equation (11), where it appears vanishing when the image size $S_o$ in equation (11) satisfies $S_o < d_o$.

On the other hand mountains which are very large does not vanish to a point since the light comes from it disappear before it reach the vanishing point according to equation (17). The railway and ordinary roads vanishes at horizon since $S_o$ becomes vanishingly small, i.e $S_o \to 0$, and its size is less than the visual cell size $d_o$, $S_o < d_o$.

## Conclusion

The above discussion shows that the geometry of painting land scape can be easily described by using geometrical optics. Geometrical optics shows that the vanishing and non-vanishing of objects is related to the eye geometry in which the distance between eye lens and retina is constant.

The vanishing of the objects happens when their image size is less than the visual cell size. The horizon or any object appears infinite if its image size on the retina is larger than macula size.

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