

Research Article

Effect of Beam Pre-Bunching on the Excitation of Electromagnetic Waves in Cerenkov Free Electron Laser

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Abstract

The Cerenkov free electron laser (CFEL) model under consideration consists of a pre-bunched relativistic electron beam and a dielectric loaded waveguide. The increase in the growth rate using a pre-bunched electron beam in a Cerenkov free electron laser (CFEL) has been studied. The growth rate as a function of modulation index has been studied and calculated at experimentally known CFEL parameters. The growth rate of the CFEL is found to increase with the modulation index and reaches maximum when the modulation index approaching unity in addition to the frequency and wave number of the pre-bunched beam are comparable to that of the radiation wave, i.e., when the pre-bunched beam velocity is comparable to the phase velocity of the radiation wave.

Keywords: Pre-bunched electron beam, Cerenkov free electron laser, Cerenkov emission, efficiency, Growth rate, modulation index, slow wave device.

1. Introduction

A slow wave structure is needed to slow down the phase velocity of electromagnetic modes. A Cerenkov free electron laser generally employs a dielectric whose dielectric constant is $|\epsilon| > 1$, as a slow wave medium to reduce the phase velocity of transverse electric (TE) or transverse magnetic (TM) modes to less than c (velocity of light). A moderately relativistic electron beam can excite the electromagnetic radiation by Cerenkov emission. In this device, an electron beam passing through wave structure resonantly interacts with wave whose phase velocity equals the drift velocity of electrons and the wave grows at the expense of energy of the beam.

The Cerenkov free electron laser (CFEL) [J Wlask *et al.* 1984] is an attractive source of high power microwave generation at short wavelengths

Recently, there has been strong motivation for generating sub-millimeter waves via Cerenkov emission in a dielectric loaded waveguide [B Johnson *et al.* 1985, Garate *et al.* 1986, Tripathi *et al.* 1989, Krall *et al.* 1988]. Tripathi and Lui [Tripathi *et al.* 1989] have proposed the operation of a free electron laser in a dielectric loaded waveguide and have found that the main effect in free electron lasers is shortening of operating wavelengths by using the same beam energy as in vacuum free electron lasers.

More recently, a lot of research work has been carried out in studying the free electron laser by pre-bunched electron beams. A high power microwave free electron laser experiment has been performed using pre-bunched electron beam of 35 MeV [A Doria *et al.* 1993]. Here when the electron beam is pre-bunched at a frequency close to an eigen frequency of the cavity, the oscillation build process is speed up and the radiation build time is shortened significantly. Free electron maser experiment with a pre-bunched electron beam is also under way at Tel Aviv University [M Cohen *et al.* 1995]. In this case, they utilize a 1.0 A current pre-bunched electron beam obtained from a microwave tube. The electron beam is bunched at 4.87 GHz frequency and is subsequently accelerated to 70 keV. The bunched beam is injected into a planar wiggler ($B_w = 300$ gauss, $\lambda_w = 4.4$ cm, where B_w is the wiggler field and λ_w is the wiggler wavelength) constructed in a Halbach configuration with 17 periods.

Simulation of enhanced pre-bunching in free electron lasers for the generation of high gain radiation at high frequencies has been demonstrated by Freund *et al.* [Freund *et al.* 2003]. It is demonstrated that free electron laser with the pre-bunched beam combines the best characteristics of amplifiers and oscillators.

A theoretical model for gain and efficiency enhancement in a FEL using pre-bunched electron beam has been developed and studied by Beniwal *et al.* [Beniwal *et al.* 2004]. It is seen that growth rate

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increases with the increase in the modulation index. Bhasin and Sharma have studied the gain and efficiency enhancement in a slow wave FEL using pre-bunched electron beam in a dielectric loaded waveguide [Anuradha Bhasin et al, 2009]. They have found that the growth rate and gain of a slow wave FEL increase with the modulation index and is maximum when the pre-bunched beam velocity is comparable to the phase velocity of the radiation wave.

It is the purpose of the present work to present a similar theoretical description of CFEL operation using a pre-bunched electron beam for far infrared wavelength generation. We present the analytical analysis for the excitation of electromagnetic waves by pre-bunched beam in a slow wave device. A dielectric loaded waveguide is considered. Boundary effects have been ignored.

The organization of the paper is as follows: We employ fluid theory and follow perturbation techniques to obtain the growth rate and efficiency of CFEL. We have studied the increase in growth rate and efficiency with the increase in modulation index calculated at experimentally known CFEL parameters. Finally, we have provided the discussion of the results.

2. Instability Analysis

A dielectric loaded waveguide of effective permittivity ϵ_1 has been considered. A pre-bunched relativistic electron beam of density n_{b0} , velocity $v_b z$, relativistic gamma factor $\gamma = 1 + \frac{eV_b}{mc^2}(1 + \Delta \sin \omega_0 \tau) \approx \gamma_0(1 + \Delta \sin \omega_0 \tau)$ [where Δ is the modulation index (its value lie from 0 to 1), mc^2 is the rest mass energy of the electrons, e is the electronic charge, $\omega_0 (\approx k_{z0} v_b)$ and k_{z0} are the modulation frequency and wave number of the pre-bunched electron beam], respectively propagates through the waveguide (cf. Fig. 1).

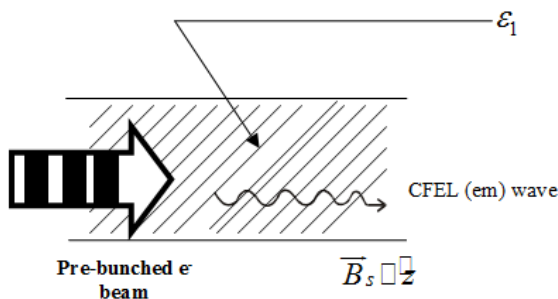


Fig.1 A schematic diagram of the Cerenkov free electron laser

An electromagnetic signal E_1 is also present in the interaction region.

$$\vec{E}_1 = \vec{E}_0 e^{-i(\omega_1 t - \vec{k}_1 \cdot \vec{x})} \tag{1}$$

$$\vec{B}_1 = \frac{c}{\omega_1} \vec{k}_1 \times \vec{E}_1, \tag{2}$$

where \vec{E}_0 and \vec{k}_1 lie in the x-z plane, i.e.,

$$\frac{\partial}{\partial y} = ik_y = 0. \tag{3}$$

The response of the beam electrons to the signal is governed by the relativistic equation of motion

$$\frac{\partial}{\partial t} (\gamma \vec{v}) + \vec{v} \cdot \nabla (\gamma \vec{v}) = -\frac{e}{m} (\vec{E} + \frac{1}{c} \vec{v} \times \vec{B}). \tag{3}$$

Expanding

$$\vec{v} = v_b z + \vec{v}_1, \quad \gamma' = \gamma + \gamma^3 \frac{\vec{v}_b \cdot \vec{v}_1}{c^2}$$

and linearizing equation (3), we get

$$\gamma v_1 + \gamma^3 \frac{v_b}{c^2} v_{z1} z = \frac{e}{im(\omega_1 - k_z v_b)} \left[E_1 \left(1 - \frac{k_z v_b}{\omega_1} \right) + k_{x1} \frac{v_b E_z}{\omega_1} \right]. \tag{4}$$

Velocity components in the x and z directions are given by

$$v_{x1} = \frac{e}{im\gamma(\omega_1 - k_z v_b)} \left[E_{x1} - \frac{k_z v_b E_{x1}}{\omega_1} + \frac{k_{x1} v_b E_{z1}}{\omega_1} \right]. \tag{5}$$

$$v_{z1} = \frac{e E_{z1}}{im(\omega_1 - k_z v_b) \gamma^3}. \tag{6}$$

On linearizing and solving equation of continuity, we obtain density perturbation

$$n_1 = n_{b0} \frac{\vec{k}_1 \cdot \vec{v}_1}{(\omega_1 - k_z v_b)}. \tag{7}$$

Using the value of v_{x1} and v_{z1} from equations (5) and (6) in equation (7), we get

$$n_1 = \frac{en_{b0}}{im(\omega_1 - k_z v_b)^2} \left[\frac{E_{x1} k_{x1} (1 - \frac{k_z v_b}{\omega_1}) + \frac{k_{x1}^2 v_b E_{z1}}{\omega_1} + \frac{k_{z1} E_{z1}}{\gamma^3} \right]. \tag{8}$$

The perturbed current density is given by

$$\vec{J}_1 = -n_{b0} e \vec{v}_1 - n_1 e v_b \hat{z}. \tag{9}$$

Substituting the values of \vec{v}_1 and n_1 from equations (5), (6) and (8) in equation (9), and keeping the value in the wave equation, we obtain

$$k_1^2 E_1 - k_1 \left(\vec{k}_1 \cdot \vec{E}_1 \right) - \frac{\omega_1^2}{c^2} \epsilon E_1 = \frac{4\pi i \omega_1 \vec{J}_1}{c^2} \tag{10}$$

and writing x and z components of the latter, we obtain

$$\left(k_{z1}^2 - \frac{\omega_1^2}{c^2} \epsilon + \frac{\omega_{pb}^2}{\gamma c^2} \right) E_{x1} = \left(k_{x1} k_z - \frac{\omega_{pb}^2}{\gamma c^2} \frac{k_{x1} v_b}{(\omega_1 - k_z v_b)} \right) E_{z1}, \tag{11}$$

where $\omega_{pb}^2 = \frac{4\pi n_{b0}}{m} e^2$.

Equation (11) gives the dispersion relation and can be further rearranged by taking ω_{pb}^2 terms to the right hand side and retaining only those terms which have a resonance denominator $(\omega_1 - k_z v_b)^2$, we get

$$(\omega_1^2 - \frac{k_1^2 c^2}{\epsilon})(\omega_1 - k_z v_b)^2 = \frac{\omega_{pb}^2}{\gamma^3 \epsilon} (\omega_1^2 + k_{x1}^2 v_b^2 \gamma^2). \tag{12}$$

The two factors on the left-hand side of equation (12) when equated to zero $\omega_1 - \frac{k_1 c}{\sqrt{\epsilon}} = 0, \omega_1 - k_z v_b = 0$, give radiation and beam modes, respectively. To determine the growth rate of the CFEL instability, we use the first order perturbation techniques. In the presence of the right hand side terms (i.e., $n_{b0} \neq 0$), we assume that the eigen functions are not modified but their eigen value are. We expand ω_1 as

$$\omega_1 = \omega_{1r} + \delta = k_z v_b + \delta = k_{z0} v_b + \delta,$$

where δ is the small frequency mismatch and

$$\omega_{1r} = \frac{k_1 c}{\sqrt{\epsilon}}.$$

On further solving equation (12) we obtain

$$\delta = \left[\frac{\omega_{pb}^2 (\omega_{1r}^2 + k_{x1}^2 v_b^2 \gamma^2)}{2\omega_{1r} \gamma^3 \epsilon} \right]^{1/3} e^{i \frac{2n\pi}{3}}, n = 0, 1, 2, 3, \dots \tag{13}$$

Hence the growth rate, i.e., the imaginary part of δ is given as

$$\Gamma = \left[\frac{\omega_{pb}^2 (\omega_{1r}^2 + k_{x1}^2 v_b^2 \gamma^2)}{2\omega_{1r} \gamma^3 \epsilon} \right]^{1/3} \frac{\sqrt{3}}{2} \tag{14}$$

where $\gamma = \gamma_0(1 + \Delta \sin \omega_0 \tau)$.

For maximum gain it is assumed that all electrons are bunched in the decelerating zone, i.e., $\omega_0 \tau = -\pi/2$. This gives $\gamma = \gamma_0(1 - \Delta)$, where Δ is the modulation index, its value lies between 0 to 1 and $\Delta \neq 1$.

Using Equation (13) the real part of δ is given as

$$|\delta_r| = \frac{\Gamma}{\sqrt{3}}, \quad \text{i.e., } \omega_1 = k_z v_b - \frac{\Gamma}{\sqrt{3}}$$

$$\text{or } v_b = \frac{\omega_1}{k_z} + \frac{\Gamma}{\sqrt{3} k_z} \text{ i.e., } v_b > \frac{\omega_1}{k_z}.$$

This is the necessary condition for electron bunching and net energy transfer from beam electrons to the radiation wave.

3. Results and Discussions

In the numerical calculations we have used typical parameters of Cerenkov free electron laser (CFEL). (e.g., beam energy = 1.35 MeV and beam-current $I_b = 900A$, modulation frequency $\omega_0 = 4.87GHz$ and effective permittivity $\epsilon = 1.7$)

In Fig. 2, we have plotted the variation of the growth Γ (in rad /sec) as a function of modulation index Δ when the phase of the pre-bunched beam is $-\pi/2$, i.e., when the electron beam is in the

decelerating zone. From Fig. 2, it can be seen that the growth rate increases with the modulation index, when $\Delta \approx 0.80$ and beyond this value of modulation index, i.e., when Δ increases from 0.80 to 0.98, the growth rate increases by a factor of 9 for CFEL parameters. For modulation index $\Delta = 0$, i.e., without modulated beam, the value of the growth is found to be 2.696×10^{10} rad/sec for CFEL parameters. The growth rate of the CFEL instability scales as one third power of the beam current and minus one third power of the effective permittivity [Vivek Beniwal et al, 2004].

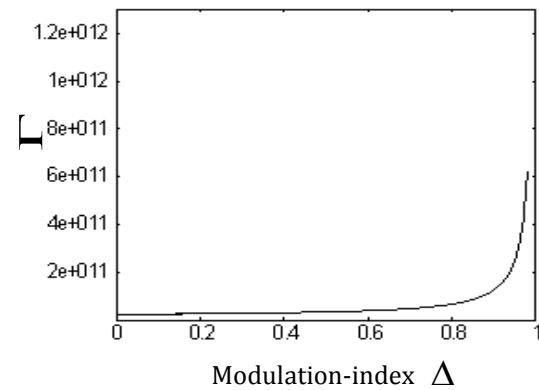


Fig.2. Growth rate Γ (in rad /sec) as a function of modulation index Δ for (a) CFEL parameters with $E_b = 1.35$ MeV, $I_b = 900A$ and for $\sin \omega_0 \tau = -1$

If we introduce plasma in the interaction region of CFEL then further reducing the requirements on beam energy for generating shorter wavelength radiation.

We have neglected the radial spread in the beam such as dc and ac space charge effects. We have also neglected the dc self generated magnetic field as we have chosen the beam parameters in such a way that dc self-generated magnetic field may not play an important role.

The growth rate of the pre-bunched CFEL increases with the modulation index and consequently the gain and efficiency of the device also increases with the modulation index.

Conclusion

In conclusion, we can say that the beam pre-bunching on Cerenkov free electron laser offers considerable enhancement in growth rate, hence efficiency when the beam electrons are in the retarding zone and when the phase velocity of the radiation is comparable to the pre-bunched beam velocity. By increasing the modulation index the growth rate, gain, and efficiency of the pre-bunched CFEL increases, but at the same time the beam energy spread will also be increased, and consequently the efficiency and gain will be affected.

In addition to this it is seen that by using pre-bunched CFEL, requirement for beam energy can be reduced drastically for generating high frequency radiations.

The scheme seems to work well at millimeter and sub-millimeter wavelengths.

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