International Journal of Current Engineering and Technology

E-ISSN 2277 – 4106, P-ISSN 2347 – 5161
Available at http://inpressco.com/category/ijcet

Research Article

Probabilistic Design of Spur Gear

Lijesh K.P.†* and Harish Hirani‡

†Mechanical Engineering Department, Indian Institute of Technology, Delhi, New Delhi, India

Accepted 10 April 2015, Available online 15 April 2015, **Vol.5, No.2 (April 2015)**

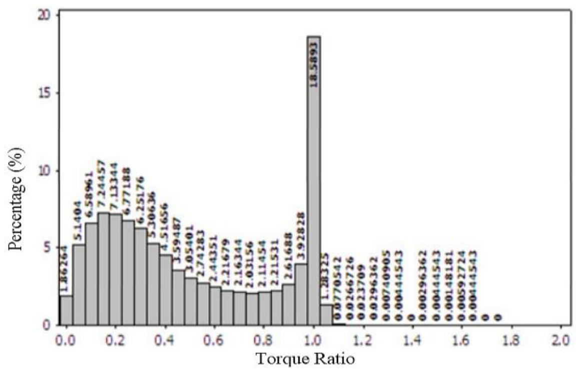
Abstract

Among the various mechanical power transmission means, gears are the most rugged and durable, but proper designing of the gears is a challenging task. It is expected to follow the pertinent standards of AGMA (American Gear Manufacturing Association) to design a reliable gear. These standards include a large number of empirical design factors accounting for various uncertainties. Due to subjective nature of those factors, often wrong input design factors lead to over-design or under-design of the gear. A portion of the present study is to analyze the various correction factors used in AGMA equations. To decide the appropriate values of correction factors, it is advisable to account statistical variations in operating and geometric conditions in the form of their mean and standard deviation values. In the present work, a methodology of probabilistic design of spur gear has been presented. A case study has been detailed to compare the procedure suggested by AGMA standard and the proposed methodology.

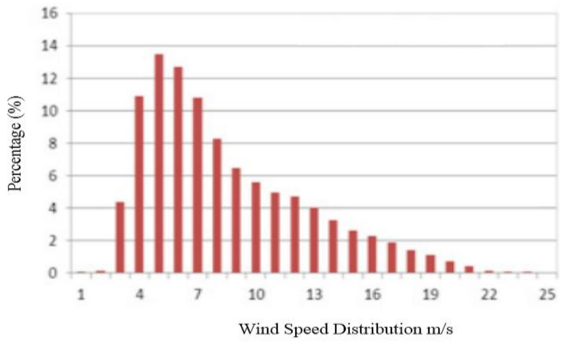
Keywords: *Reliability, gear, statistical variation, AGMA.*

1. Introduction

Among the various devices of mechanical power transmission, gears are the most rugged and durable. In the present work a study on external spur (convex-convex mechanical contact) gears has been presented. Such gears are subject to a combination of high speed and heavy load conditions, and improper design unnecessarily increases the noise level and reduces the gear-life. Proper design of gears is a challenging task and it becomes more challenging in case of fluctuating operating conditions, variable material properties and statistical geometric dimensions. Variation in operating conditions in one of typical gearbox used in wind-power-generator is presented in figure 1.



(a) Torque ratio variation



(b) Wind Speed variation

Fig. 1 Torque and wind speed variation of wind turbine (Issa *et al* 2014)

As per figure 1(a) torque ratio varies from 0 to 18.589%. As per figure 1(b) speed varies from 0 to 13.5%. There is a need to develop a statistical design approach to deal with such variations in operating conditions. It is interesting to note that statistical variation in operating conditions is very common in most of mechanical components such as bearings [Hirani, 2009, Hirani *et al*, 2000, Hirani *et al*, 1999, Hirani *et al*, 1998, Muzakkir *et al*, 2011, Hirani, 2005, Hirani *et al*, 2001, Muzakkir *et al*, 2013, Hirani 2004, Muzakkir *et al*, 2015, Hirani, Verma, 2009, Hirani, Suh, 2005, Hirani *et al*, 2001, Rao *et al*, 2000, Hirani *et al*, 2000, Hirani *et al*, 2002, Burla *et al*, 2004, Hirani, Samanta, 2007, Lijesh, Hirani, 2015, Lijesh, Hirani, 2014, Shankar *et al*, 2006, Lijesh, Hirani, 2015,

*Corresponding author: **Lijesh K.P.**

Muzakkir *et al*, 2014, Lijesh, Hirani, 2015], seals [Hirani and Goilkar, 2011, Goilkar and Hirani, 2010, Hirani and Goilkar, 2009, Goilkar and Hirani, 2009, Goilkar and Hirani, 2009], brakes [Sarkar and Hirani, 2015, Sarkar and Hirani, 2015, Sarkar and Hirani, 2013, Sarkar and Hirani, 2013, Sarkar and Hirani, 2013, Sukhwani *et al*, 2009, Sukhwan and Hirani, 2008, Sukhwani *et al*, 2008, Sukhwani *et al*, 2008, Hirani and Manjunath, 2007, Sukhwani *et al*, 2007], gears [Shah and Hirani, 2014, Hirani, 2009]. The proposed probabilistic design approach can be applied to any machine component that necessarily involves consideration of statistical material strength and the complicated stress state to which the component is subjected.

In the current industry practice, it is expected to follow the pertinent standards of AGMA (American Gear Manufacturing Association) to design a reliable gear. The AGMA based gear design process includes a large number of empirical design factors accounting for various uncertainties related to correction for geometry, precision requirements, dynamic loading, over loading, material, reliability, etc. Many of these factors are based on large numbers of tests and many years of experience and do not consider the actual uncertainties in the input data, make these factors less robust. The selection of the values of design factors is very subjective. The design factors do not incorporate all practical conditions, for example dynamic correction factor for two different scenarios (i) velocity 10±0.5 m/s and (ii) 10±2 m/s remain same.

With growing scientific knowledge and measuring facilities, it is possible to estimate the statistical variation in operating conditions and quantify the uncertainties related to material-properties and geometric features. There is need to develop a design methodology to incorporate statistical variations related to load, speed, strength, dimension, etc. Implementation of such methodology will eliminate the need empirical design factors used in AGMA gear design procedure. In the present work, a probabilistic approach based on mean and standard deviation values has been presented. To illustrate the proposed design procedure, a case-study considering the statistical variations is presented. Each step has been detailed and obtained results are matched with results obtained from AGMA procedure.

2. Gear Design

The involute profile is the most commonly used system for gears. An important advantage of the involute profile is that it provides theoretically perfect conjugate action even when the distance between shaft centers is not exactly correct. The designer involved with gears is expected to follow the pertinent standards of the AGMA using involute gearing profile. The AGMA approach can be named as classical approach to design the gears. In the present work, a summary of classical (AGMA) approach has been presented. In addition, the proposed statistical (probabilistic) approach to design reliable gears has been detailed. A case study has been presented to compare the design-approaches presented in this paper.

(i) Classical approach (Budynas and Nisbett, 2014)

The fundamental stress equation used in classical (AGMA) approach for estimating bending stress is given by equation (1).

$$\sigma_b = \frac{K_v W_t}{F m Y_j} K_o K_B K_H K_s \qquad (1)$$

Here; W_t is tangential component of the applied load (N), F is the face width, m is the module of the gear and K_v is the dynamic factor. The dynamic factor (K_v) is a function of pitch line velocity (V) and manufacturing accuracy. As per the current AGMA standards K_v is represented as:

$$K_V = \frac{3.05 + V}{3.05} \quad \text{(Cast iron, cast profile)} \qquad 2(a)$$

$$K_V = \frac{6.1 + V}{6.1} \quad \text{(Cut or milled profile)} \qquad 2(b)$$

$$K_V = \frac{3.56 + \sqrt{V}}{3.56} \quad \text{(Hobbed or shaped profile)} \qquad 2(c)$$

$$K_V = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \quad \text{(Shaved or ground profile)} \qquad 2(d)$$

K_o is the overload factor which accounts the degree of non-uniformity in driving torques. The table for the overload factor is given in table 1.

Table 1 Table of overload Factor

Power Source	Uniform	Moderate shock	Heavy Shock
Uniform	1	1.25	1.75
Light Shock	1.25	1.5	2
Medium Shock	1.5	1.75	2.25

K_s is the size factor reflects non-uniformity of material property due to size. It depends upon, tooth size, diameter, ratio of tooth size to diameter of part, face width, area of stress etc. The value of K_s is determined by using equation (3). It was recommended that if the value of K_s is less than 1, use K_s=1.

$$K_S = 1.192\left(F m \sqrt{Y}\right)^{0.0535} \qquad (3)$$

K_H is the load distribution factor, which account non-uniformity in distribution of load across the line of contact. The load factor value is determined by equation (4).

$$K_H = 1 + C_{mc}\left(C_{pf} C_{pm} + C_{ma} C_e\right) \qquad (4)$$

Where;

$$C_{mc} = \begin{cases} 1 & \text{For uncrowned teeth} \\ 0.8 & \text{For crowned teeth} \end{cases}$$

Crowning is provided in the gear teeth (as shown in figure (2)) to avoid stress concentration at the corners.

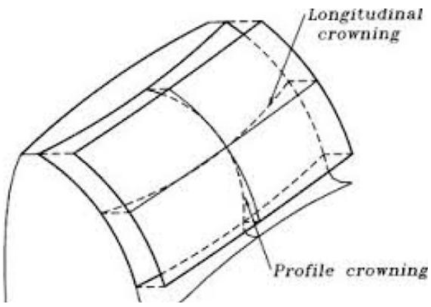


Fig. 2 Crowning of gear teeth (Litvin, 2002)

$$C_{pf} = \begin{cases} \frac{F}{10d} - 0.025 & F < 25mm \\ \frac{F}{10d} - 0.0375 + F4.92\times10^{-4} & 25mm < F < 425mm \\ \frac{F}{10d} - 0.1109 + F8.15\times10^{-4} - F^2 3.53\times10^{-4} & 425mm < F < 1000mm \end{cases}$$

If F/(10d)<0.05, F/(10d)=0.05 is used

$$C_{pm} = \begin{cases} 1 & \text{For straddle mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{For straddle mounted pinion with } S_1/S \geq 0.175 \end{cases}$$

S is the center distance between two bearings and S_1 is the distance between the center line of the gear face and mid-point of shaft-system as shown in figure 3.

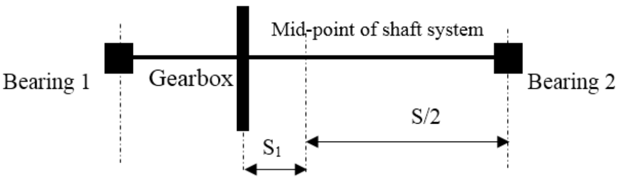


Fig.3 Definition of distance S and S_1 used in evaluation C_{pm}, $C_{ma}=A+BF+CF^2$

The value of A, B, and C is given in table 2.

Table 2 Empirical Constants A, B, and C to find C_{ma}

Condition	A	B	C
Open gearing	0.247	6.57×10^{-4}	-3.01×10^{-6}
Commercial, enclosed units	0.127	6.22×10^{-4}	-3.66×10^{-6}
Precision, enclosed units	0.0675	5.03×10^{-4}	-3.65×10^{-6}
Extra precision enclosed gear units	0.00360	4.015×10^{-4}	-3.24×10^{-6}

$$C_e = \begin{cases} 0.8 & \text{For gearing adjusted at assembly or compatability is improved by lapping or both} \\ 1 & \text{For all other conditions.} \end{cases}$$

The value of K_H is can also be decided based on the face width given in table 3 (Gopinath and Mayuram) and table 4 (Norton, 2001). The wider the face width, the more difficult it is to manufacture and mount the gear to bring uniformity in distribution of load across the full face width is the load distribution factor, which account non-uniformity in distribution of load across the face width of gears.

Table 3 Table of load distribution factor (Gopinath and Mayuram)

Characteristics of Support	Face width (F) (mm)			
	0-50	150	225	400
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across the full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists	Over 2.2	Over 2.2	Over 2.2	Over 2.2

Table 4 Table of load distribution factor (Norton, 2001)

Face width (F) (mm)	K_H
<50	1.6
150	1.7
250	1.8
>500	2

In previous paragraph three sources to estimate K_H have been mentioned. All three sources provide different values of K_H, for example gear having face width 50mm and F/D<0.5 the values of K_H using equation (5) is 1.13, using Table 3 it is 1.3 and using Table 4 it comes out to be 1.6. There is huge variation between the values predicted by three methods. Designer often find difficulty is adopting in one of the method.

K_B is the rim thickness factor. When the rim thickness (t_r) is not sufficient to provide support for the tooth root, the location of the bending fatigue failure may be through the gear rim rather than at the tooth fillet. The value of rim thickness factor (K_B) is selected from table 5 based on the ratio $m_B = t_r/h_t$, where h_t is the height of the tooth.

Table 5 Table of Rim Thickness Factor

m_B	K_B
0.5	2.4
2.2	0.6
0.8	1.7
1	1.3
>1.2	1

Y_j is AGMA bending geometry factor and it is the modified form of Lewis form factor 'Y'. The value of Y_j depends on number of teeth in mating gear and its value is estimated from the graph given by (Richard, Budynas, 2014).

The safety factor (S_F) guarding against bending fatigue failure is given by equation (6)

$$S_F = \frac{S_T Y_N}{K_T K_R \sigma_B} \quad (6)$$

Where; S_T is the allowable bending stress, Y_N is the stress cycle factor for bending stress, K_T is the temperature factor, K_R is the reliability factor.

The value of allowable bending stress is given by equation 7(a).

$$S_T = \begin{cases} 0.533 H_B + 88.3 MPa & \text{For Grade 1} \\ 0.703 H_B + 113 MPa & \text{For Grade 2} \end{cases} \quad 7(a)$$

The stress cycle factor (Y_N) is given by the graph provided in figure 14-14 of book of Richard, Budynas (2014). The reliability factor (K_R) is estimated based on the required reliability (R) and the value is estimated from equation 7(b).

$$K_R = \begin{cases} 0.658 - 0.0759\, ln(1-R) & 0.5 < R < 0.99 \\ 0.50 - 0.109\, ln(1-R) & 0.99 < R < 0.9999 \end{cases} \quad 7(b)$$

For oil or gear blank temperature up to 120^{0}C, K_T is considered to be unity. For higher temperature the factor should be greater than unity.

(ii) Proposed Approach

The AGMA approach uses a single value for predefined conditions which may result in wrong design if the condition is slightly changed. To overcome such problem, probabilistic approach incorporating the statistical variation in various design factors can be used. In this approach, it is assumed that the standard deviation of each variable follows the probability density function of Gaussian distribution, expressed in terms of its mean μ_Q and its standard deviation $\hat{\sigma}_Q$ as given in equation (6):

$$f(Q) = \frac{1}{\hat{\sigma}_Q \sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{Q-\mu_Q}{\hat{\sigma}_Q}\right)^2\right] \quad (8)$$

To normalize the Eq. (68, variables can be expressed in terms of 'normal variable' having a mean of zero and a standard deviation of unity, such as.

$$Z = \frac{Q-\mu_Q}{\hat{\sigma}_Q} \quad (9)$$

Reliability is the probability that any machine system will perform its intended function satisfactorily without failure. In mechanical design, the reliability means the chances of strength greater than the stress. In other words, stress and strength are statistical in nature and very much tied to the reliability of the stressed component. To understand it let us consider the probability density functions for stress and strength, σ and S_y. The mean values of stress and strength are μ_σ and μ_S respectively. Here, the average factor of safety is μ_S/μ_σ. The margin of safety for any value of stress σ and strength S_y is defined as $Q=S_y-\sigma$. In classical approach, the part safety is decided based on the positive value of $\overline{Q} = \mu_S - \mu_\sigma$. However, there are chances of overlap in stress and strength distributions, as shown by the shaded area in figure 4. In this area the stress exceeds the strength, the margin of safety is negative, and part is expected to fail. The hatched area is called the interference of σ and S_y. The reliability (R) that a part will perform without failure is the area of the margin of safety distribution for Q > 0. The interference is the area (1−R) where part fails.

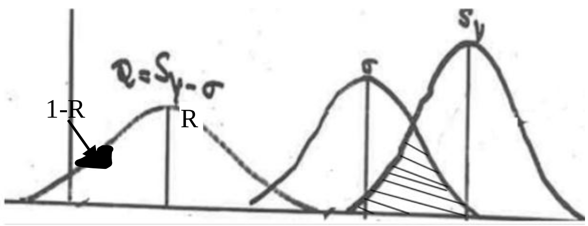


Fig.4 Stress and strength distributions

To find the chance that Q>0, the value of 'Z' variable (equation 9) is obtained for Q=0.

$$Z = \frac{Q-\mu_Q}{\sigma_Q} = -\frac{\mu_Q}{\sigma_Q} \quad (10)$$

Where $\mu_Q = \mu_S - \mu_\sigma$ and $\sigma_Q = \sqrt{\sigma_s^2 + \sigma_\sigma^2}$, in equation (10)

From the value of Z, the reliability of the component is estimated using the table given in A-10 of book authored by Budynas, Nisbett (2014).

In the proposed approach, the design factors given in equation (1) are considered with statistical variations to obtain the values of mean and standard deviation of the bending stress. The mean value (μ_σ) of stress is obtained by substituting the mean value of each variable given in equation (1). To determine the standard deviation of stress (σ_s), the stress equation (1) is differentiated w.r.t. individual independent variable. If the design variable is dependent then it is substituted as function of independent variables. To understand the procedure, consider the equation (1). The stress equation given in equation (1) is can be rewritten as a function of K_v, W_t, b, m, Y_j, K_o, K_s, K_H and K_B as given in equation (11).

$$\sigma_b = f\left(K_v, W_t, F, m, Y_j, K_o, K_s, K_H, K_B\right) \quad (11)$$

The standard deviation of equation (9) is estimated by equation (10)

$$\sigma_b = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial \sigma_b}{\partial x_i}\right)^2 \partial x_i^2} \quad (12)$$

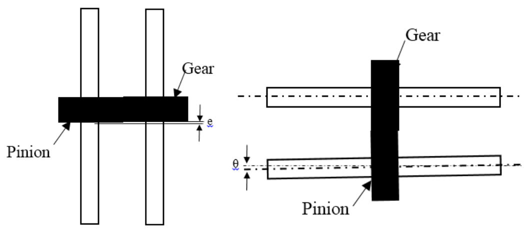
$$\sigma_b = \sqrt{\begin{array}{l}\left(\frac{\partial \sigma_b}{\partial K_v}\right)^2 \sigma_{K_v}^2 + \left(\frac{\partial \sigma_b}{\partial W_t}\right)^2 \sigma_{W_t}^2 + \left(\frac{\partial \sigma_b}{\partial F}\right)^2 \sigma_F^2 + \left(\frac{\partial \sigma_b}{\partial m}\right)^2 \sigma_m^2 + \\ \left(\frac{\partial \sigma_b}{\partial Y_J}\right)^2 \sigma_{Y_j}{}^2 + \left(\frac{\partial \sigma_b}{\partial K_o}\right)^2 \sigma_{K_o}^2 + \left(\frac{\partial \sigma_b}{\partial K_s}\right)^2 \sigma_{K_s}^2 + \left(\frac{\partial \sigma_b}{\partial K_H}\right)^2 \sigma_{K_H}^2 + \left(\frac{\partial \sigma_b}{\partial K_B}\right)^2 \sigma_{K_B}^2\end{array}}$$

As explained earlier the variables must be independent, the variable load (W_t) is dependent on the applied torque (T) and pitch diameter (D) (W_t=2T/D). Therefore *f(Wt)* is replaced by *f(T,D).* Similarly the K_v is function pitch line velocity (V) (given equation (2)) and V is function of angular speed (N) and pitch diameter (D) (V=πDN/60). Equation (10) is rewritten as equation (13)

$$\sigma_b = \sqrt{\begin{array}{l}\left(\frac{\partial \sigma_b}{\partial T}\right)^2 \sigma_T^2 + \left(\frac{\partial \sigma_b}{\partial F}\right)^2 \sigma_F^2 + \left(\frac{\partial \sigma_b}{\partial m}\right)^2 \sigma_m^2 + \left(\frac{\partial \sigma_b}{\partial Y_J}\right)^2 \sigma_{Y_j}{}^2 + \left(\frac{\partial \sigma_b}{\partial K_o}\right)^2 \sigma_{K_o}^2 + \\ \left(\frac{\partial \sigma_b}{\partial K_s}\right)^2 \sigma_{K_s}^2 + \left(\frac{\partial \sigma_b}{\partial K_H}\right)^2 \sigma_{K_H}^2 + \left(\frac{\partial \sigma_b}{\partial K_B}\right)^2 \sigma_{K_B}^2 + \left(\frac{\partial \sigma_b}{\partial D}\right)^2 \sigma_D^2 + \left(\frac{\partial \sigma_b}{\partial N}\right)^2 \sigma_N^2\end{array}} \quad (13)$$

In the present approach variable in load is accounted by considering variations in torque and speed; therefore there is no need to consider the over load factor (K_o).

AGMA bending geometry factor (Y_j) is function of number of teeth and nominal pressure angle, which will has zero standard deviation. Therefore, there is no need to consider standard deviation of Y_j. Similarly, deviation in value of module (m) is almost negligible, so no need to consider its standard deviation.



(a) Axial offset (b) Angular Offset

Fig. 5 axial and angular offset during gear and pinion engagement

The load distribution factor (K_H) is the due to the improper engagement of gear and pinion as shown in figure 5. Due to the assembly error in engaging the gear and pinion, as well in the mounting of bearings, the faces of gear and pinion shift and as a result effective face width in contact decreases.

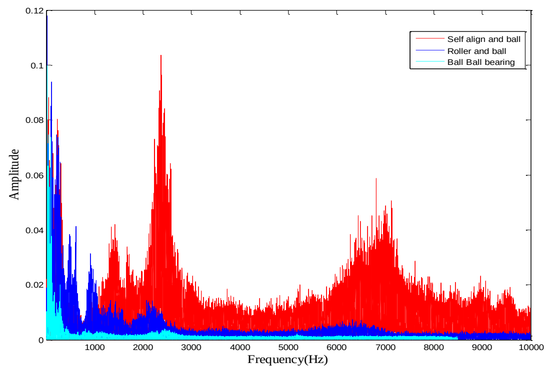


Fig. 6 FFT for different type of bearings

It is interesting to note that gear effective width depends on the alignment and rigidity of the support bearings. Figure 5(a) depicts an axial offset between the gears, which results in reduction in contacting face width. Similarly an angular offset is shown in figure 5(b). These offsets arise due to the clearance in the bearings and coupling (i.e. Jaw coupling) between driving and driven shafts. Therefore to incorporate the design factor for load distribution, the full assembly has to be considered. To illustrate it, let us consider three scenarios of an assembly: (i) Assembly 1: shaft supported on two deep groove ball bearing of bore diameter 50mm; (ii) Assembly 2: shaft supported on one deep groove ball bearing and one radial roller bearing; and (iii) shaft supported on one deep groove ball bearing and one self-aligning ball bearing. The FFT plot obtained from acceleration signals of three assemblies have been plotted in figure 6. This problem can be overwhelmed by including the statistical variation considering the full system. However to get the accurate results, intensive study has to be performed which is out of scope of the present work.

In statistical approach, this can be accounted by consider the minimum value of face width. To understand this let us consider mean value of width as μ_b and standard deviation as σ_b. In statistical approach, the minimum value of face width will be μ_b-3σ_b. By considering this, there is no need to account K_H and standard deviation in face width.

In statistical approach variation in material properties is accounted. In addition, the minimum possible value of face width is considered. Therefore, there is no need to account size factor K_s. There is very rare chance of designing thin rim of gear. Generally t_r is far greater than the tooth thickness (m_B>1.2). The basic reason for stating m_B>1.2 is that the diameter of shaft is generally much smaller than the pitch diameter of the gear. Therefore, there is no need to account K_B.

As per discussions provides in previous paragraphs, Eq (11) and Eq (13) can be expressed as:

$$\sigma_b = f\left(K_v, T, D, F, m, Y_j\right) \tag{14}$$

$$\sigma_b = \sqrt{\left(\frac{\partial \sigma_b}{\partial T}\right)^2 \sigma_T^2 + +\left(\frac{\partial \sigma_b}{\partial D}\right)^2 \sigma_D^2 + \left(\frac{\partial \sigma_b}{\partial N}\right)^2 \sigma_N^2 + \left(\frac{\partial \sigma_b}{\partial F}\right)^2 \sigma_F^2} \tag{15}$$

To examine the variation of solutions attained using with and without statistical variation of AGMA equation (1), a case study is considered and comparison of results is presented.

Case Study: Determine safety of a high quality (shaved and grounded) pinion of spur gear system to transmit a torque of 1130Nm @ 4500 rpm with speed reduction of 3.5. Assume the pressure angle = 20°, number of teeth (Z_p)=29 and module(m)=6mm. The corrected endurance strength of gear material varies in the range of 300 MPa to 400 MPa. Standard deviations in torque is σ_T= 300 Nm, in speed 100 rpm and in pitch diameter 0.5mm. Due to crowning and manufacturing errors, the face width varies from 50mm to 52mm. Due to possibilities of angular and parallel offset, effective face width may varies in the range of 40mm to 52mm.

Solution: To determine the safety of spur gear, one of the two approaches (i) Proposed probabilistic approach and (ii) AGMA classical approach can be used. In the present case, both the approaches have been used and comparison among the obtained results has been presented.

(i) Probabilistic Approach

(i) As per provided information, the dynamic factor for shaved and grounded pinion is given as:

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \tag{16}$$

The pitch line velocity (V) is a function of pitch circle diameter of pinion (D) and speed (N). On incorporating these values, equation (16) is rewritten as:

$$K_v = \sqrt{\frac{5.56 + \sqrt{\frac{\pi DN}{60}}}{5.56}} \Rightarrow \sqrt{1 + 0.0412\sqrt{DN}} \tag{17}$$

(ii) From the graph provided in (Richard, Budynas, 2014), the value of form factor (Y_j) for 29 number of teeth in pinion and 87 teeth in gear is is 0.38.

(iii) F=μ_b-3σ_b=(52+50)/2-3(52-50)/6 = 50mm

Substituting the above estimated values in equation (1) we get:

$$\sigma_s = \frac{K_v W_t}{F m Y_j} \Rightarrow \frac{877 \times T}{FD}\sqrt{1 + 0.0412\sqrt{DN}} \tag{18}$$

Partial differentiation equation (18) w.r.t. D, N, T and F, we get

$$\frac{\partial \sigma_s}{\partial N} = \frac{877 \times T}{FD}\frac{1}{2}\left(1 + 0.0412\sqrt{DN}\right)^{-0.5}\frac{1}{2}\sqrt{D}N^{-0.5} \Rightarrow 4.9 \times 10^3$$

$$\frac{\partial \sigma_s}{\partial T} = \frac{877}{FD}\sqrt{1 + 0.0412\sqrt{DN}} \Rightarrow 1.47 \times 10^5$$

$$\frac{\partial \sigma_s}{\partial D} = -\frac{877 \times T}{FD^{-2}}\sqrt{1 + 0.0412\sqrt{DN}} + \frac{877 \times T}{FD}\frac{1}{2}\left(1 + 0.0412\sqrt{DN}\right)^{-0.5}\frac{1}{2}\sqrt{N}D^{-0.5} \Rightarrow -8.3 \times 10^8$$

$$\frac{\partial \sigma_s}{\partial F} = -\frac{877 \times T}{F^{-2}D}\sqrt{1 + 0.0412\sqrt{DN}} \Rightarrow -3.34 \times 10^9$$

Given σ_T=300 Nm, σ_N= 100 rpm and σ_D= 0.5mm. Substituting the values obtained and given values in equation (15), the estimated value of σ_s is 44.78 MPa, Now to calculate the value of normalized factor 'Z', μ_{sy}, σ_{sy}, μ_Q, σ_Q is calculated as follows

σ_{sy}=(400-300)/6=16.66MPa,
μ_{sy}=(400+300)/2=350MPa.

$$\mu_\sigma = 2\frac{\sqrt{1 + 0.0412\sqrt{DN}}\,T}{F_{avg} m Y D} \Rightarrow$$

$$2260 \times \frac{\sqrt{1 + 0.0412\sqrt{0.174 \times 4500}}}{0.046 \times 0.006 \times 0.38 \times 0.174} => 181 MPa$$

μ_Q = $\mu_{Sy}-\mu_\sigma$ = 350-181.22=169MPa and $\sigma_Q = \sqrt{\sigma_s^2 + \sigma_\sigma^2}$ =47.56MPa. The value of 'Z' is estimated by equation (8)

$$Z = \frac{Q - \mu_Q}{\sigma_Q} = -\frac{\mu_Q}{\sigma_Q} = -\frac{169}{47.56} = -3.55$$

For the value of Z =-3.55, the estimated reliability from from table A-10 (Budynas and Nisbett, 2014) is 0.9998.

(ii) Classical Approach

The stress calculated using the AGMA equation given in equation (1) is estimated as follows:

$$\sigma_b = \frac{K_v W_t}{b m Y_j} K_o K_B K_H K_s$$

$$= \frac{T\sqrt{1 + 0.0412\sqrt{DN}}}{D b m Y_j} 1.25 \times 1 \times 1.13 \times 1 = 256 MPa$$

$$FOS = \frac{(400 + 300)}{2\sigma_Q} = 1.36$$

FOS of safety is 1.36, which is indicating overdesign of gear. Generally after accounting all correction factors, FOS should not be more than 1. This observation is similar to the observation made in previous (probabilistic) approach.

In the present only bending failure of spur gears has been considered. However, in practice other failure modes such as abrasion, scoring and galling also occur. For complete design of gear, all failure modes must be considered.

Conclusion

Gears designed using AGMA (American Gear Manufacturing Association) method requires a number of subjective design correction factors. The proposed probabilistic approach reduces the subjectivity by eliminating correction design factors and provides more realistic results. A case study to illustrate the procedure to use the proposed statistical approach has been considered. Based on the obtained results it can be said that the proposed approach is a robust approach and shall be used to design various machine elements.

References

Hirani, H., Athre, K., Biswas, S. (1998), Rapid and Globally Convergent Method for Dynamically Loaded Journal Bearing Design, *Proc. IMechE (UK), Journal of Engineering Tribology*, vol. 212, pp. 207-214.

Hirani, H., Athre, K., Biswas, S. (1999), Dynamic Analysis of Engine Bearings, *International Journal of Rotating Machinery*, vol. 5, no.4, pp. 283-293.

Hirani, H., Athre, K., Biswas, S. (2000), A Hybrid Solution Scheme for Performance Evaluation of Crankshaft Bearings, *Trans. ASME, Journal of Tribology*, vol. 122, no. 4, pp. 733-740.

Hirani, H., Athre, K., Biswas, S. (2000), Transient Trajectory of Journal in Hydrodynamic Bearing, *Applied Mechanics and Engineering*. vol. 5, no 2.

Hirani, H., Athre, K., Biswas, S. (2001), A Simplified Mass Conserving Algorithm for Journal Bearing under Dynamic Loads, *International Journal of Rotating Machinery*, vol. 1, pp. 41-51.

Hirani, H., Athre, K., Biswas, S. (2001), Lubricant Shear Thinning Analysis of Engine Journal Bearings, *STLE, Journal of Tribology Transaction*, vol 44, no. 1, pp 125-131.

Muzakkir, S.M., Hirani, H., Thakre, G.D., Tyagi, M.R. (2011), Tribological Failure Analysis of Journal Bearings used in Sugar Mill, *Engineering Failure Analysis*, vol. 18, no. 8, pp. 2093-2103.

Hirani, H., Athre, K., Biswas, S. (2002), Comprehensive Design Methodology for Engine Journal Bearing, *IMechE (UK), Part J, Journal of Engineering Tribology*, vol 214, pp. 401-412.

Hirani, H. (2004), Multi-objective Optimization of a journal bearing using the Pareto optimal concept, *Proc. Institute Mech. Engineers., Part J, Journal of Engineering Tribology*, vol. 218, no. 4, pp. 323-336.

Hirani, H. (2005), Multiobjective optimization of journal bearing using mass conserving and genetic algorithms, *Proc. Institute Mech. Engineers., Part J, Journal of Engineering Tribology*, vol. 219, no. 3, pp. 235-248.

Hirani, H., and Suh, N. P. (2005), Journal Bearing Design using Multi-objective Genetic Algorithm and Axiomatic Design Approaches, *Tribology International*, vol. 38, no. 5, pp. 481-491.

Hirani, H., and Verma, M. (2009), Tribological study of elastomeric bearings of marine shaft system, *Tribology International*, vol. 42, no. 2, pp. 378-390.

Muzakkir, S.M., Hirani, H., Thakre, G.D., (2013), Lubricant for Heavily-Loaded Slow Speed Journal Bearing, Tribology Transactions, vol.56, no. 6, pp. 1060-1068.

Muzakkir, S.M., Lijesh, K. P., Hirani, H., Thakre, G.D. (2015) Effect of Cylindricity on the Tribological Performance of Heavily-Loaded Slow Speed Journal Bearing, Proc. Institute Mech. Engineers., *Part J, Journal of Engineering Tribology*, 2015, vol 229, no.2, pp.178-195.

Muzakkir, S.M. and Hirani, H. (2015), A Magnetorheological Fluid Based Design Of Variable Valve Timing System For Internal Combustion Engine Using Axiomatic Design, International Journal of Current Engineering Research, Vol.5, No.2 (April 2015), pp 603-612.

Rao, T. V. V. L. N., Hirani, H., Athre, K., Biswas, S. (2000), An Analytical Approach to Evaluate Dynamic Coefficients and Non-linear Transient Analysis of a Hydrodynamic Journal Bearing, *STLE Tribology Transactions*, vol. 23, no.1, pp. 109-115.

Muzakkir, S. M., and Harish Hirani. (2015)Maintenance Free Bearings. *International Journal of Engineering Research,* vol 4, no.3, pp.133-136.

Muzakkir, S. M., Harish Hirani, and G. D. Thakre. (2015), Experimental Investigations on Effectiveness of Axial and Circumferential Grooves in Minimizing Wear of Journal Bearing Operating in Mixed Lubrication Regime. *International Journal of Current Engineering and Technology*, vol.5, no.1, pp.486-489.

Muzakkir, S. M., K. P. Lijesh, and Harish Hirani. (2015).Effect of Base Oil on the Anti-Wear Performance of Multi-Walled Carbon Nano-tubes (MWCNT), *International Journal of Current Engineering and Technology*, vol.5, no.2, pp.681-684.

H Hirani, T Rao, K Athre and S Biswas, (1997), Rapid performance evaluation of journal bearings, *Tribology international*, vol. 30, no.11, pp. 825-834.

H Hirani, K Athre, S Biswas, (1999), Dynamically loaded finite length journal bearings: analytical method of solution. *Journal of tribology*, vol. 121, no. 4, pp. 844-852.

Hirani, H. (2009),Root cause failure analysis of outer ring fracture of four row cylindrical roller bearing, *Tribology Transactions*, vol.52, no.2, pp.180-190.

Lijesh, K. P., S. M. Muzakkir, and Harish Hirani. (2015), Failure Analysis of Rolling Contact Bearing for Flywheel Energy Storage Systems., vol.5, no.1, pp. 439-443.

Lijesh K.P. and Harish Hirani, (2015), Shaft Alignment Systems: A Comparative Study, vol.5, no.2, pp. 1009-1014.

Lijesh, K. P., Hirani, H., (2014), Stiffness and Damping Coefficients for Rubber mounted Hybrid Bearing, *Lubrication Science*, vol. 26, no.5, pp. 301-314.

Lijesh, K. P., Hirani, H., (2015), Development of Analytical Equations for Design and Optimization of Axially Polarized Radial Passive Magnetic Bearing, *ASME, Journal of Tribology*, vol. 137, no. 1, (9 pages).

Lijesh, K. P., Hirani, H., (2015), Optimization of Eight Pole Radial Active Magnetic Bearing, *ASME, Journal of Tribology*, vol.137, no.2.

Lijesh, K. P. and Hirani, H. (2015), Design and Development of Halbach Electromagnet for Active Magnetic Bearing, *Progress In Electromagnetics Research C*, vol. 56, 173–181.

Muzakkir, S.M., Lijesh, K. P. and Hirani, H. (2014), Tribological Failure Analysis of a Heavily-Loaded Slow Speed Hybrid Journal Bearing, *Engineering Failure Analysis*, vol. 40, pp.97–113.

Samanta, P., Hirani, H., (2007), A Simplified Optimization Approach for Permanent Magnetic Journal Bearing, *Indian Journal of Tribology*.

Samanta, Pranab, and Harish Hirani. (2008), Magnetic bearing configurations: Theoretical and experimental studies. Magnetics, IEEE Transactions on vol. 44, no. 2, pp. 292-300.

Shankar, S., Sandeep, Hirani, H. (2006), Active Magnetic Bearing, *Indian Journal of Tribology*, pp 15-25.

Hirani, H., Samanta, P. (2007), Hybrid (Hydrodynamic + Permanent Magnetic) Journal Bearings, Proc. Institute Mech. Engineers., *Part J, Journal of Engineering Tribology*, vol. 221, no. J8, pp. 881-891.

Lijesh, K. P., and Harish Hirani. (2015), The Performance of Hybrid Journal Bearing under Extreme Operating Conditions. International Journal of Current Engineering and Technology, vol. 5, no.1, pp. 277-282.

Lijesh, K. P., and Hirani, H. (2014). Design of eight pole radial active magnetic bearing using monotonicity. In Industrial and Information Systems (ICIIS), 2014 9th International Conference on IEEE, pp. 1-6.

Lijesh K.P., Harish Hirani, Samanta P., (2015), Theoretical and Experimental Study for Hybrid Journal Bearing, International Journal of Scientific and Engineering Research, Volume 6, Issue 2, pp. 133-139.

Lijesh K. P, Harish Hirani, (2015), Magnetic Bearing Using RMD Configuration, Journal of Tribology.

Sarkar, C. and Hirani, H., (2015), Development of magnetorheological brake with slotted disc, *Proc. IMechE, Part D: Journal of Automobile Engineering*, DOI: 10.1177/0954407015574204.

Sarkar, C. and Hirani, H., (2013),Theoretical and experimental studies on a magnetorheological brake operating under compression plus shear mode, *Smart Materials and Structures*, vol.22. no.11, art. no. 115032

Sarkar, C. and Hirani, H., (2013), Synthesis and Characterization of Antifriction Magnetorheological Fluids for Brake, *Defense Science Journal*, vol. 63, no.4, pp.408-412.

Sarkar, C. and Hirani, H., (2013), Design of a Squeeze Film Magnetorheological Brake Considering Compression Enhanced Shear Yield Stress of Magnetorheological Fluid, *Journal of Physics: Conference Series*, vol.412, no.1, 012045.

Sarkar, C. and Hirani, H., (2014), Effect of particle size on shear stress of magnetorheological fluids, Smart Science, http://dx.doi.org/10.6493/SmartSci.2015.317.

Sukhwani, V. K., Hirani, H., (2008),A Comparative Study of Magnetorheological-Fluid-Brake and Magnetorheological-Grease-Brake, *Tribology Online*, vol 3, no.1, pp. 31-35.

Sukhwani, V. K., Hirani, H., and Singh, T. (2008), Synthesis and Performance Evaluation of MR Grease, *NLGI Spokesman,* vol. 71, no.10.

Sukhwani, V. K. and Hirani, H., (2008), Design, Development and Performance Evaluation of High Speed MR Brake, *Proc. Institute Mech. Engineers., Part L, Journal of Materials: Design and Applications*, vol. 222, no.1, pp. 73-82.

Sukhwani, V. K., Hirani, H., and Singh, T. (2009), Performance Evaluation of a Magnetorheological Grease Brake, *Greasetech India*, vol. 9, no.4, pp. 5-11.

Sukhwani, V. K., Hirani, H., and Singh, T. (2007),Synthesis of Magnetorheological (MR) Grease, *Greasetech* India.

Muzakkir, S.M. and Hirani, H. (2015), A Magnetorheological Fluid Based Design Of Variable Valve Timing System For Internal Combustion Engine Using Axiomatic Design, International Journal of Current Engineering Research, vol.5, no.2, pp. 603-612.

Hirani, H. and Goilkar, S.S., (2011), Rotordynamic Analysis of Carbon Graphite Seals of a Steam Rotary Joint , *Book on IUTAM Symposium on Emerging Trends in Rotor Dynamics*, 253-262, Springer Netherlands.

Goilkar S S, and Hirani H., (2009) ,Design and development of test setup for online wear monitoring of mechanical face seals using torque sensor, *Tribology Transactions*, vol. 52, no.1, pp. 47-58.

Goilkar, S.S. and Hirani, H. (2009), Tribological Characterization of Carbon Graphite Secondary Seal *Indian Journal of Tribology*, vol. 4, no. 2, pp. 1-6.

Goilkar, S.S. and Hirani, H. (2009) Tribological Characterization of Carbon Graphite Secondary Seal *Indian Journal of Tribology*, vol. 4, no. 2, pp. 1-6.

Goilkar, S.S. and Hirani, H. (2010),Parametric Study on Balance Ratio of Mechanical Face Seal in Steam Environment, *Tribology International*, vol. 43, no. 5-6, pp. 1180-1185.

Hirani, H. and Goilkar, S.S., (2009),Formation of Transfer Layer and its Effect on Friction and Wear of Carbon-Graphite Face Seal under Dry, Water and Steam Environments, *Wear*, vol. 226, no. 11-12, pp.1141-1154.

Shah, H. and Hirani, H. (2014), Online Condition Monitoring of Spur Gear, *International Journal of Condition Monitoring*, vol 4, no. 1, pp.15-22.

Hirani, H. (2009), Online Wear Monitoring of Spur Gear, *Indian Journal of Tribology*, vol 4, no.2, pp.38-43.

Hirani, H. and Manjunatha, C. S., (2007),Performance Evaluation of Magnetorheological Fluid Variable Valve, *Proc. of the Institution of Mechanical Engineers, Part D, Journal of Automobile Engineering*, vol. 221, no.1, pp. 83-93.

Muzakkir, S.M. and Hirani, H., (2015), Design of Innovative Engine Valve: Background and Need, International Journal of Scientific Engineering and Technology, Volume No.4 Issue No.3, pp : 178-181.

Muzakkir, S.M. and Hirani, H., (2015), Design of Innovative Engine Valve, International Journal of Scientific Engineering and Technology, Volume No.4 Issue No.3, pp: 212-217.

Al-Tubi, I., Long, H., Tavner, P., Shaw, B., & Zhang, J. (2015). Probabilistic analysis of gear flank micro-pitting risk in wind turbine gearbox using supervisory control and data acquisition data. IET Renewable Power Generation. pp. 1-5

Gautham, B. P., Gupta, P., Kulkarni, N. H., Panchal, J. H., Allen, J. K., & Mistree, F. (2013). Robust Design of Gears With Material and Load Uncertainties. In ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (pp. V03BT03A046-V03BT03A04.

R. G. Budynas and J.K. Nisbett, (2014), Mechanical Engineering Design, Indian dition,The McGraw-Hill Companies, Inc., New York

K.Gopinath and M.M.Mayuram, NPTEL, Lecture 8, Spur Gears, http://nptel.ac.in/courses/IITMadras/Machine_Design_II/pdf/2_8.pdf.

R. L. Norton, (2001), Machine Design, An integrated approach Second edition, Addison Wesley Longman, Singapore.

F. L. Litvin, D. Vecchiato, A. Demenego, E. Karedes, B. Hansen and R. Handschuh, (2002), Design of One Stage Planetary Gear Train With Improved Conditions of Load Distribution and Reduced Transmission Errors, J. Mech. Des., vol. 124, no.4, pp. 745-752.