

Research Article

## Adaptive Beamformers for Cellular Radio Systems using Smart Antenna

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Accepted 10 Dec 2014, Available online 15 Dec 2014, Vol.4, No.6 (Dec 2014)

### Abstract

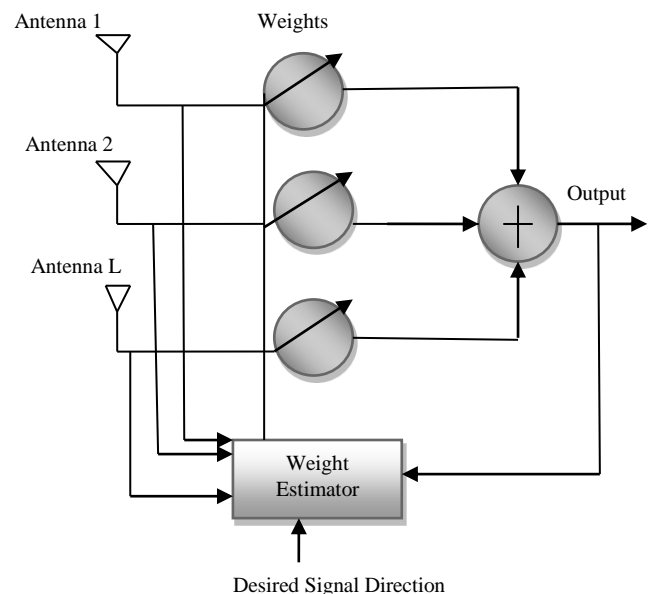
In this paper, three different adaptive beamforming algorithms namely Least Mean Square (LMS), Recursive Least Square (RLS) and Conjugate Gradient Method (CGM) for the array antenna systems are studied and their performance is analysed. Literature on adaptive beamforming for array antenna system was focused more on first two methods for the performance analysis. In this work, we focused more on CGM and the compressive comparison is done. All three adaptive beamforming algorithms are developed using MATLAB software. The performance analysis of beam pattern for different array elements are studied, then the convergence speed are compared, Mean Square Error (MSE) and Bit Error Rate (BER) are computed and the results are compared.

**Keywords:** BER, CGM, LMS, MSE, QAM, RLS, SNR.

### 1. Introduction

Adaptive beamforming is widely used in the wireless communication for receiving the required signal in the desired direction without knowing the prior information (Prerna Saxena *et al*, 2014). Many digital beamforming algorithms have been proposed for array antenna systems, but the adaptive algorithms suitable for practical smart antenna base station installation are of more interest (Shiann-Jeng Yu *et al*, 1996). Single Input Single Output (SISO) antennas suffer from inadequate performance, accuracy, data rates etc, hence Multiple Input Multiple Output (MIMO) systems are popular antenna systems for mobile and wireless communication application which counterparts the SISO systems (S.F. Shaukat *et al*, 2009; G. KranthiKumar, 2012). Defense applications like RADAR, Satellite etc require most sophisticated antennas for wireless communication; this is accomplished by the use of smart antenna as major component (N.S. Grewal *et al*, 2014). Application of smart antenna systems in recent years has brought a huge interest in mobile communication to overcome the problem of multiple fading, co-channel interference, BER, outage probability and system complexity (Lal C. Godara, July 1997). The key aspect of the smart antenna system is the generating the main beam towards the desired signal and null towards the undesired direction (Hema Singh *et al*, 2012). Proposal of fast converging algorithms are indeed required to meet the demands of 3G and 4G networks for the high speed data transfer in mobile communication. The fast convergence can be obtained by reducing the MSE and

BER which effectively improves the system performance that can be used for practical mobile base stations (Md.Salman Razaak *et al*, 2010). Figure 1 shows the basic block diagram of adaptive beamformer for smart antenna systems.



**Fig 1:** Adaptive Beamformer for Smart Antenna System.

It mainly consists of receive antennas, weight estimator and processor. The receive antennas receive the signal, which is then multiplied by adjustable weights. The individual results are combined and the output signal is given to the digital signal processor. The processor calculates the weights for the each channel (Prerna Saxena *et al*, 2014). The simulated results are compared in terms

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 Global Impact Factor (2014): 4.550

of beam width, depth of nulls, sidelobe levels (SLL), convergence rate and MSE as a function of signal to noise ratio (SNR) and BER. After the comprehensive comparison of all three methods, the most practically suitable technique for smart antenna based station is proposed.

## 2. LMS Algorithm

The LMS algorithm can be considered to be the most popular adaptive algorithm for continues adaptation, but less convergence rate is its major drawback (Veerendra et al, 2014). Let us consider the array correlation matrix ( $\bar{R}_{xx}$ ) and the signal correlation vector ( $\bar{r}$ ) over a range of snapshots or for each instant in time. The instantaneous estimates are given as

$$\hat{R}_{xx}(k) \approx \bar{x}(k)\bar{x}^H(k) \quad (1)$$

and

$$\hat{r}(k) \approx d^*(k)\bar{x}(k) \quad (2)$$

The steepest descent iterative approximation is given by (Frank B Gross, 2005) as

$$\bar{w}(k+1) = \bar{w}(k) - \frac{1}{2} \mu \nabla_{\bar{w}}(J(\bar{w})) \quad (3)$$

Where,  $J(\bar{w})$  is the cost function,  $\mu$  is the step-size parameter and  $\nabla_{\bar{w}}$  is the gradient of the performance surface. The cost function is given by (Veerendra et al, 2014).

$$J(\bar{w}) = D - 2\bar{w}^H \bar{r} + \bar{w}^H R_{xx} \bar{w} \quad (4)$$

Where:  $D = E[|d|^2]$

Substituting the instantaneous correlation approximations, we have the *Least Mean Square* (LMS) solution.

$$\bar{w}(k+1) = \bar{w}(k) + \mu e^*(k)\bar{x}(k) \quad (5)$$

Where,  $e(k) = d(k) - \bar{w}^H(k)\bar{x}(k)$  = error signal

The convergence of the LMS algorithm is directly related to the *step-size parameter*  $\mu$ . If the step-size is inversely proportional to the largest eigenvalue of  $\hat{R}_{xx}$  given by

$$0 \leq \mu \leq \frac{1}{\lambda_{\max}} \quad (6)$$

Where  $\lambda_{\max}$  is the largest eigenvalue of  $\hat{R}_{xx}$ . This can be approximated as (Frank B Gross, 2005),

$$0 \leq \mu \leq \frac{1}{2 \text{trace}[R_{xx}]} \quad (7)$$

## 3. RLS Algorithm

RLS (Recursive Least Squares) algorithms, is well known to pursue fast convergence even when the eigen value spread of the input signal correlation matrix is large. We

can recursively calculate the required correlation matrix and the required correlation vector.

The correlation matrix and the correlation vector  $K$  is

$$\hat{R}_{xx}(k) = \sum_{i=1}^k \bar{x}_k(i)\bar{x}_k^H(i) \quad (8)$$

$$\hat{r}(k) = \sum_{i=1}^k d^*(i)\bar{x}(i) \quad (9)$$

Where,  $k$  is the block length and last time sample  $k$  and  $\hat{R}_{xx}(k)$ ,  $\hat{r}(k)$  is the correlation

Thus weighted estimate of above equations can be calculated as

$$\hat{R}_{xx}(k) = \sum_{i=1}^k \alpha^{k-1} \bar{x}_k(i)\bar{x}_k^H(i) \quad (10)$$

$$\hat{r}(k) = \sum_{i=1}^k \alpha^{k-1} d^*(i)\bar{x}_k(i) \quad (11)$$

Where,  $\alpha$  is the forgetting factor.

The forgetting factor is also sometimes referred to as the *exponential weighting* factor [Frank B Gross, 2005].  $\alpha$  is a positive constant such that  $0 \leq \alpha \leq 1$ .

$$\hat{R}_{xx}(k) = \alpha \hat{R}_{xx}(k-1) + \bar{x}(k)\bar{x}^H(k) \quad (12)$$

$$\hat{r}(k) = \alpha \hat{r}(k-1) + d^*(k)\bar{x}(k) \quad (13)$$

Thus, future values for the array correlation estimate and the vector correlation estimate can be found using previous values.

## 4. Constant Gradient Method

The convergence rate can be accelerated by use of the conjugate gradient method (CGM). The CGM is an iterative method whose goal is to minimize the quadratic cost function.

$$J(\bar{w}) = \frac{1}{2} \bar{w}^H \bar{A} \bar{w} - \bar{d}^H \bar{w} \quad (14)$$

Where,

$$\bar{A} = \begin{bmatrix} x_1(1) & x_2(1) & \dots & x_M(1) \\ x_1(2) & x_2(2) & \dots & x_M(2) \\ \vdots & \vdots & \ddots & \vdots \\ x_1(K) & x_2(K) & \dots & x_M(K) \end{bmatrix} \quad \text{K} \times \text{M matrix}$$

Where,

$K$  = number of snapshots

$N$  = number of array elements

$\bar{w}$  = unknown weight vector

$\bar{d} = [d(1) \ d(2) \ \dots \ d(K)]^T$  = desired signal vector of  $K$  snapshots.

We may take the gradient of the cost function and set it to zero in order to find the minimum. It can be shown that

$$\nabla_{\bar{w}} J(\bar{w}) = \bar{A} \bar{w} - \bar{d} \quad (15)$$

The general weight update equation is given by

$$\bar{w}(n+1) = \bar{w}(n) - \mu(n)\bar{D}(n) \quad (16)$$

Where the step size is determined by

$$\mu(n) = \frac{\bar{r}^H(n)\bar{A}\bar{A}^H r(n)}{\bar{D}^H(n)\bar{A}\bar{A}^H \bar{D}(n)} \quad (17)$$

We may now update the residual and the direction vector as follows

$$\bar{r}(n+1) = \bar{r}(n) + \mu(n)\bar{A}\bar{D}(n) \quad (18)$$

The direction vector update is given by

$$\bar{D}(n+1) = \bar{A}^H \bar{r}(n+1) - \alpha(n)\bar{D}(n) \quad (19)$$

We can use a linear search to determine  $\alpha(n)$  which minimizes  $\bar{J}(\bar{w}(n))$ . Thus

$$\alpha(n) = \frac{\bar{r}^H(n+1)\bar{A}\bar{A}^H \bar{r}(n+1)}{\bar{r}^H(n)\bar{A}\bar{A}^H \bar{r}(n)} \quad (20)$$

Array factor of uniform linear array (ULA) can be calculated using the following formula given by (Frank B Gross, 2005) as

$$AF(\phi) = \sum_{n=1}^N a_n e^{an} e^{i2\pi \cos(\phi)x_n} \quad (21)$$

Where,  $a_n$  is the excitation amplitude, and  $x_n$  is the x coordinate.

### 5. Mean Square Error (MSE)

The squared

$$J_{MSE} = \sigma_n^2 + \lim_{t \rightarrow \infty} E \left[ |w^H(i)x(i)|^2 \right] \quad (22)$$

Where,  $\sigma_n$  is the variance of additive Noise.

Let us consider  $y(n)$  as the output response for ULA, then

$$y'(n) = w^H r'(n) \quad (23)$$

Where, 'w' is the vector of complex weight and 'r' (n)' is vector of received signal respectively.

The error signal which is actually the difference between the required signal and the reference signal which is given by (S.F. Shaukat et al, 2009) as

$$\varepsilon(n) = \frac{y'(n)}{|y'(n)|} - y'(n) \quad (24)$$

### 6. Bit Error Rate (BER)

BER is the measure of quality of digital signal. For multiple antennas it is given by (Lal C. Godara, July 1997) as

$$BER = P_e = Q \left( \sqrt{\frac{3GD}{K(1+8\beta)-1}} \right) \quad (25)$$

Where, G is the processing gain, K is the number of users  $\beta = 0.005513$  for CDMA systems and D is the diversity of beam.

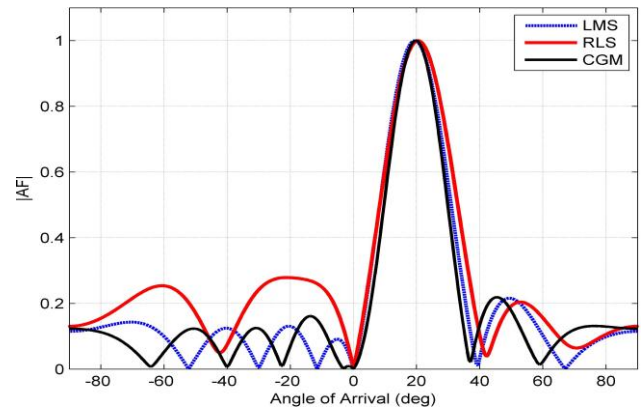
### 7. Results and Discussion

The proposed algorithms are simulated using MATLAB software. The Minimum Shifting Keying is used as digital modulation technique for all three algorithms. Table 1 show the parameters used in the simulation.

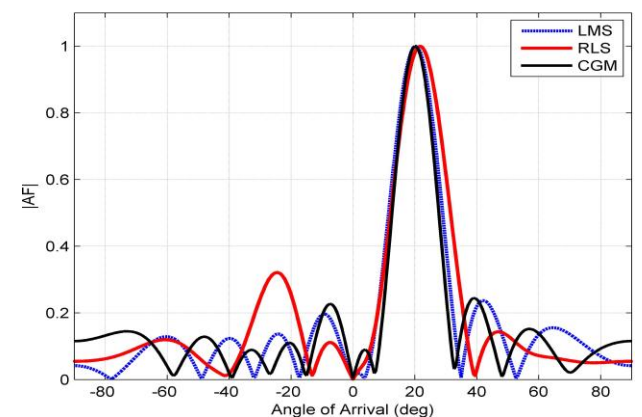
**Table 1** Parameters used in the simulation

S. No	Parameters	Values
1	Number of elements (N)	8 & 10
2	Spacing between the array elements (d)	$\lambda / 2$
3	Angle of Arrival (AOA) of signal	$20^\circ$
4	Angle of Interference (AOI) of signal	$10^\circ$
5	Forgetting factor ( $\alpha$ )	0.91
6	Number of data samples (K)	20
7	Step Size ( $\mu$ )	0.01

Figure 2 and 3 shows the normalized array factor versus AOA for N=8 and 10 respectively.



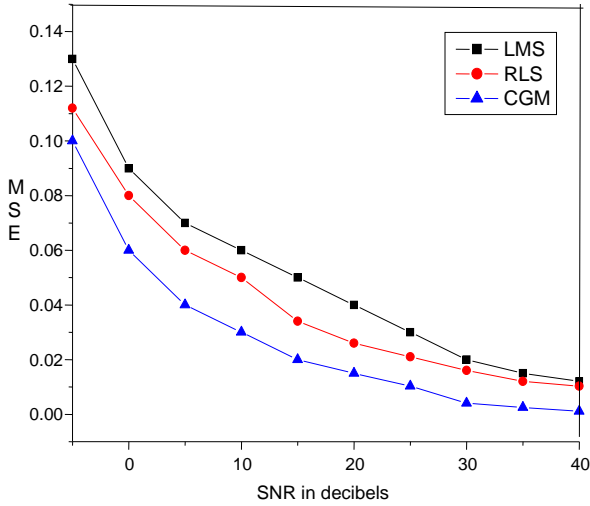
**Fig. 2:** Plot of normalized array factor versus Angle of Arrival for array elements N=8, array spacing  $d = \lambda / 2$ , AOA= $20^\circ$  and AOI= $10^\circ$ .



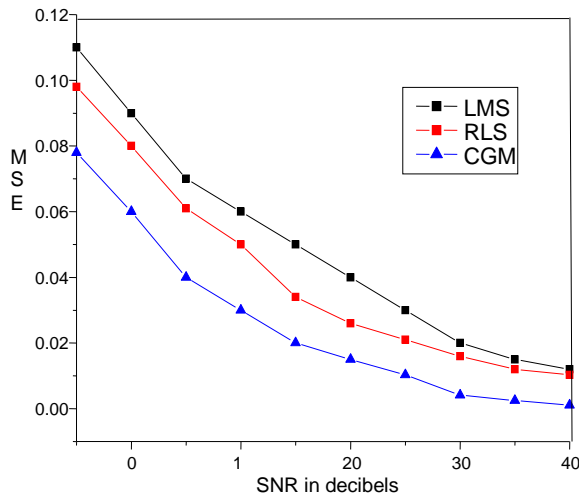
**Fig. 3:** Plot of normalized array factor versus Angle of Arrival for array elements N=10, array spacing  $d = \lambda / 2$ , AOA= $20^\circ$  and AOI= $10^\circ$ .

From the figures 2 and 3, we can analyse that the LMS algorithm has low sidelobes compared to RLS and CGM, at the same time array pattern of RLS technique has prominent sidelobes compared to other two techniques. The array pattern of CGM is excellent which has slightly narrow main beam and less sidelobes compared to other two, giving the accurate AOA of required signal. Beamforming analysis are tabulated in Table 2, 3 and 4 for LMS, RLS and CGM respectively.

Figures 4 and 5 show the plot of MSE versus SNR of output signal for array elements N=8 and 10 respectively.



**Fig. 4:** Plot of MSE versus SNR of output signal for array elements N=8, array spacing  $d= \lambda /2$ , AOA= $20^\circ$  and AOI= $10^\circ$



**Fig. 5:** Plot of MSE versus SNR of output signal for array elements N=10, array spacing  $d= \lambda /2$ , AOA= $20^\circ$  and AOI= $10^\circ$ .

**Table 2** Beamforming Analysis for LMS algorithm.

S. No	Array Element N	Beamwidth (degree)	SLL (dB)
1	8	18.12	-13.24
2	10	17.51	-14.12

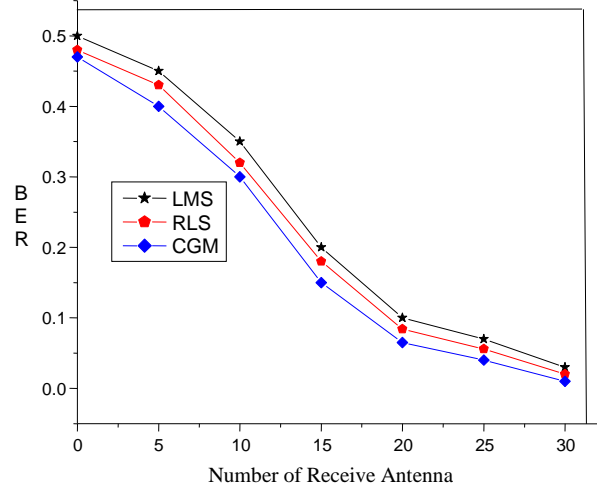
**Table 3** Beamforming Analysis for RLS algorithm.

S. No	Array Element N	Beamwidth h (degree)	SLL (dB)
1	8	19.12	-8.16
2	10	18.51	-8.35

**Table 4** Beamforming Analysis for CGM algorithm.

S. No	Array Element N	Beamwidth h (degree)	SLL (dB)
1	8	17.78	-11.16
2	10	17.21	-11.35

Figure 6 shows the plot BER versus number of receive antennas.



**Fig. 6:** Plot of BER versus number of receive antennas

From figure 4 and 5 it is clear that, as the SNR of output signal increases, the MSE decreases. It can be noticed that the fall of MSE is sharp as the number of array elements increases. The error is more in LMS as its convergence rate is less and it is less in the CGM. Hence CGM adaptive beamforming algorithm can perform accurately even in worst case situations for the practical installation of base station smart antenna system.

In figure 6, we observe that the BER decreases as the number of antenna elements at the receiver are increased. Since CGM has the highest convergence rate among the three, fall of BER is sharp for increasing array elements. This mitigates the co-channel interference, multipath fading and makes the system robust.

Minimum BER of all three algorithms are tabulated in Table 5 and 6 for N=8 and 10 respectively.

**Table 5** Results of minimum BER with N=8 array elements, for different AOA by varying SINR from -20 to 2dBs.

AOA (deg)	Minimum BER of LMS	Minimum BER of RLS	Minimum BER of CGM
20	0.0126	0.0120	0.0104

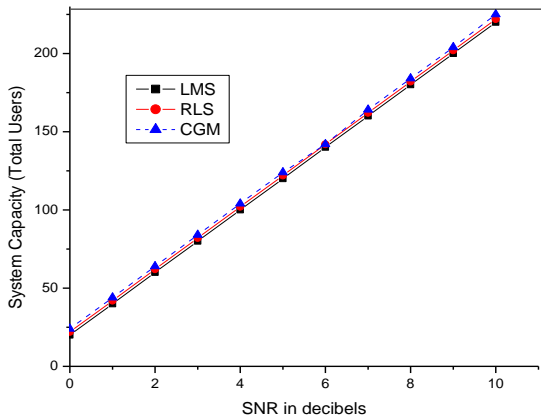
30	0.0116	0.0112	0.0108
40	0.0128	0.0121	0.0109
50	0.0119	0.0116	0.0110
60	0.0118	0.0113	0.0109
80	0.0126	0.0118	0.0103

**Table 6** Results of minimum BER with N=10 array elements, for different AOA by varying SINR from -20 to 2dBs.

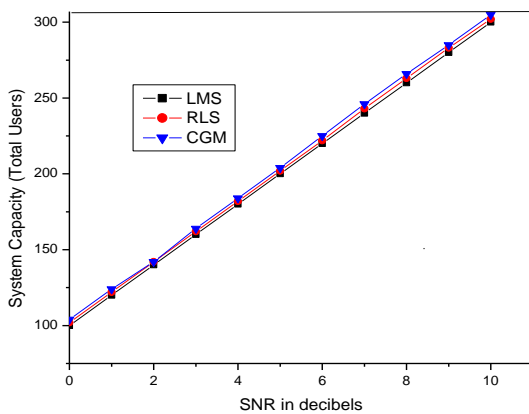
AOA (deg)	Minimum BER of LMS	Minimum BER of RLS	Minimum BER of CGM
20	0.0120	0.0118	0.0101
30	0.0114	0.0110	0.0103
40	0.0121	0.0118	0.0105
50	0.0116	0.0112	0.0108
60	0.0117	0.0111	0.0107
80	0.0120	0.0114	0.010

The results of Table 5 and 6 clears the fact, CGM technique has minimum BER for various AOA. It is also noticed that as the array elements increased the BER decreases.

Figure 7 and 8 are the plots of system capacity (Number of users) as a function of SNR from 2dBs to 10dBs for N=8 and 10 with AOA=20° and AOI=10° respectively.



**Fig. 7:** Plot of system capacity versus SNR for array elements N=8, array spacing  $d= \lambda / 2$ , AOA=20° and AOI=10°.



**Fig. 8:** Plot of system capacity versus SNR for array elements, N=10, array spacing  $d= \lambda / 2$ , AOA=20° and AOI=10°.

The system capacity (Number of Users) increases with increasing SNR. For more array elements, more will be the system capacity (Number of Users). Theoretical simulation shows that there is slight variation in system capacity, but in real life, all the three algorithms have identical system capacity. Table 7 gives the comparison between LMS, RLS and CGM.

**Table 7** Comparison of Algorithms

	LMS	RLS	CGM
AOA(deg)	20	20	20
AOI(deg)	10	10	10
Convergence Rate	60	15	5
Null Depth	More	Moderate	More
Complexity	Less	More	More
Null steering	Very Good	Poor	Good
Accuracy	Low	Medium	High

**Conclusion**

In this work, we analyzed three adaptive beamforming algorithms and their performance is studied. The LMS is the most popular method, but less convergence rate is its major drawback, it limits its use in sophisticated communication applications. The RLS algorithm shows high rate of convergence, but the side lobes are not reduced. Since it requires large number of multiplications, the system based on this technique will become more complex. The CGM algorithm calculates the array weights by orthogonal search at every iteration. It shows good beam forming pattern, high convergence rate and low BER. The beamforming analysis of LMS, RLS and CGM are tabulated in Table 2, 3 and 4 respectively. It is summarized as below.

1. As array elements (N) increases, SLL decreases for all three algorithms.
2. Beam width of CGM is narrower compared to LMS and RLS.
3. Beam width of RLS is slightly broader compared to LMS and CGM.
4. From figure 2 and from Table 3, it is clear that the SLL of RLS is more compared to LMS and CGM.
5. SLL of LMS is small compared to RLS and CGM.
6. CGM technique gives the maximum null depth.
7. RLS algorithm gives the minimum null depth.

The choice of adaptive algorithm decides the efficiency of the smart antenna system to a great extent in wireless communication. Smart antennas using CGM can provide high resolution, high convergence rate, high directivity, excellent beam pattern, low BER, low MSE, less SLL and less power level. Therefore CGM the is most suitable adaptive beamforming technique as compared to LMS and RLS for practical base station installation.

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