

Research Article

Hydro-Magnetic Flow and Heat Transfer in an Inclined Composite Porous Medium Bounded by Parallel Plates

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Accepted 30 Nov 2014, Available online 01 Dec 2014, Vol.4, No.6 (Dec 2014)

Abstract

This paper deals with non-Darcian convective hydromagnetic flow in an inclined composite porous medium. It is assumed that the flow is steady, laminar fully developed and that fluid properties are constant. The flow in both phases is assumed to be driven by a common constant pressure gradient $-\frac{\partial p}{\partial x}$ and temperature gradient $\Delta T = T_{w1} - T_{w2}$ where T_{w1} and T_{w2} are the temperatures of upper and lower plates. It is also assumed that at any given instant, the temperatures of fluid and the solid are the same. The viscous and Darcy dissipation terms are included in the energy equation. The governing equation is coupled and non-linear because of inclusion of dissipation terms and buoyancy force. The equations are solved using perturbation method. The effects of various parameters such as Hartmann number, porous parameter, inclination angle and Grashoff number on the flow quantities are discussed.

Keywords: Hydromagnetic flow, Darcy dissipation, buoyancy force, Hartmann number.

1. Introduction

Convection in a porous medium has attracted a great deal of research attention because of the fundamental nature of the problem and a broad range of applications, including geothermal systems, thermal insulation, storage of nuclear waste materials, solidification of castings, direct contact heat exchangers, enhanced recovery of petroleum resources, and geothermal energy extraction. These applications together with the fact that the porous media occur naturally in many practical problems have motivated an increasing number of different types of studies in the last two decades. Extensive reviews of prior work on convective flows in porous media have been presented by Combrarnous and Bories (1975), Cheng (1978), Rudraiah (1984, 1988), Niedl (1984), Kariang (1991) and Neild and Bejan (1992). Most of the theoretical work has been based on Darcy's law and as noted by Prasad *et al.*, (1985). But the experimental results did not agree with the theoretical results obtained with the Darcy model. This has led to inclusion of inertia and viscous effects in studies of convection in porous medium (Vafai and Tien 1981, 1982, Becker man *et al.*, 1986, Geurgiadias and Cattan 1986, Lauriant and Prasad 1986, Rudraiah 1988 and Prasad 1990).

Based on our review of the current literature, it is evident that very few studies are available on conventional flow and heat transfer in inclined porous layer (Bories and Cambarnous 1973; Chaeng 1977; Rudraiah 1986;

Oosthuizen and Paul, 1987) and further studies on composite porous layers with inclined geometry are rare. Malashetty *et al.*, (2001) discussed convective flow and heat transfer in an inclined composite porous medium. Thus it is the objective of the present studies to investigate the effect of magnetic field on convection in composite porous layer with inclined geometry. As a first step towards introducing this complexity into the problem. We consider only the Brinkman extended Darcy-Lapwood model to obtain realistic predictions.

Mathematical Model

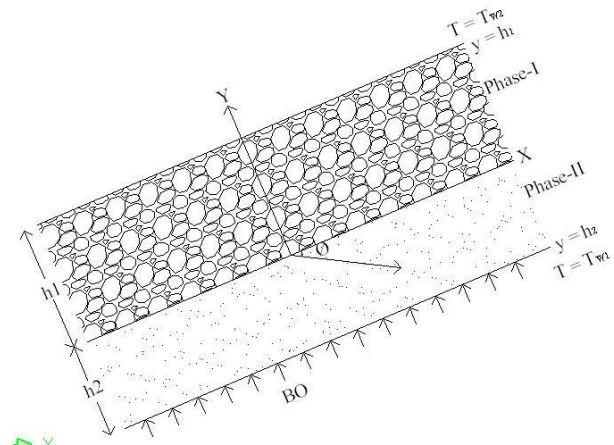


Fig. 1 Physical Model

Consider the steady laminar conducting flow of a viscous incompressible fluid through differentially permeable

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porous layers bounded two infinite inclined parallel plates maintained at different constant temperatures, extending in the Z and X directions making an angle α with the horizontal. (Fig. 3.1) The region $0 \leq y \leq h_1$ (Phase I) is filled with a homogeneous isotropic porous material having permeability K_1 . This phase is saturated with a viscous fluid having density, ρ_1 viscosity μ_1 and thermal conductivity k_1 . The region $-h_2 \leq y \leq 0$ (Phase II) is filled with another homogeneous isotropic porous material having permeability K_2 . This Phase is saturated with a different viscous fluid having density, ρ_2 Viscosity μ_2 and thermal conductivity k_2 . A uniform transfer magnetic field of strength B_0 is applied perpendicular to the plates.

It is assumed that the flow is fully developed and that fluid properties are constant. The flow in the both phases is assumed to be driven by a common constant pressure gradient $\left(-\frac{\partial p}{\partial x}\right)$ and temperature gradient $\Delta T = T_{w1} - T_{w2}$ where T_{w1} is the temperature of the boundary at $y = h_1$, and T_{w2} that at $y = -h_2$. It is also assumed that at any given instant, the temperatures of the fluid and the temperature of the solid are the same.

2. Nomenclature

Cp	-	Specific heat at constant pressure
Ec	-	Eckert number
G	-	Acceleration due to gravity
K	-	Permeability of the porous medium
K	-	Thermal conductivity of the fluid saturated porous medium
P	-	Non dimensional pressure gradient
Pr	-	Prandtl number
Re	-	Reynolds number
U	-	Velocity
\bar{u}_1	-	Average velocity
T	-	Temperature
T _w	-	Wall temperature
β	-	Coefficient of thermal expansion
σ	-	Porous parameter
α	-	Angle of inclination
ρ	-	Density of the fluid
V	-	Kinematic viscosity
μ	-	Viscosity
ε	-	Product of prandtl number and Eckert number (Pr.Ec)
ΔT	-	Difference of temperature ($T_{w1} - T_{w2}$)
θ	-	Non dimensional temperatures $(T - T_{w2}) / \Delta T$

3. Mathematical formulation of the problem

The momentum and energy equations governing the flow in the two phases are given below

Phase I

$$\mu_1 \frac{d^2 u_1}{dy^2} + \rho_1 g \beta_1 (T_1 - T_{w2}) \sin \alpha - \left(\sigma_{e1} B_0^2 + \frac{\mu_1}{k_1} \right) u_1 = \frac{\partial p}{\partial x} \quad (1.1)$$

$$\frac{d^2 T_1}{dy^2} + \frac{\mu_1}{k_1} \left(\frac{du_1}{dy} \right)^2 + \frac{\mu_1}{K_1 k_1} u_1^2 = 0 \quad (1.2)$$

Phase II

$$\mu_2 \frac{d^2 u_2}{dy^2} + \rho_2 g \beta_2 (T_2 - T_{w2}) \sin \alpha - \left(\sigma_{e2} B_0^2 + \frac{\mu_2}{k_2} \right) u_2 = \frac{\partial p}{\partial x} \quad (1.3)$$

$$\frac{d^2 T_2}{dy^2} + \frac{\mu_2}{k_2} \left(\frac{du_2}{dy} \right)^2 + \frac{\mu_2}{K_2 k_2} u_2^2 = 0 \quad (1.4)$$

where u is the x component of fluid velocity. T is the fluid temperature, g is acceleration due to gravity and β is the coefficient of thermal expansion. The subscripts 1 and 2 denote the values of Phase I and Phase II respectively.

In reality by the experimental observations of Givler and Attobellie (1994), they are not equal. But it is assumed that the fluid viscosity and the Brinkman viscosity (i.e., effective viscosity) are same. This assumption has been made to restrict the number of parameters governing the system.

The boundary conditions on velocity are no-slip conditions requiring that the velocity must vanish at the wall. The boundary conditions on temperature are isothermal conditions. The two boundaries are held at constant different temperatures. In addition the continuity of velocity, shear stress, and temperature and heat flux at the interface between the two porous layers is assumed. The boundary conditions on velocity are

$$\begin{aligned} u_1(h_1) &= 0 \\ u_1(0) &= u_2(0) \\ u_2(-h_2) &= 0 \\ \mu_1 \frac{du_1}{dy} &= \mu_2 \frac{du_2}{dy} \quad \text{at } y = 0 \end{aligned} \quad (1.5)$$

The boundary and interface conditions for temperature are

$$\begin{aligned} T_1(h_1) &= T_{w1} \\ T_1(0) &= T_2(0) \\ T_2(h_2) &= T_{w2} \\ K_1 \frac{dT_1}{dy} &= K_2 \frac{dT_2}{dy} \quad \text{at } y = 0 \end{aligned} \quad (1.6)$$

4. Non-dimensionalization of the flow quantities

We introduce the following non dimensional quantities to make basic equations and boundary conditions dimensionless

$$u_1^* = \frac{u_1}{u_1}, \quad u_2^* = \frac{u_2}{u_1}, \quad y_1^* = \frac{y_1}{h_1}, \quad y_2^* = \frac{y_2}{h_2}, \quad \theta = (T - T_{w2}) / \Delta T$$

$$m = \frac{\mu_1}{\mu_2}, \quad K = \frac{K_1}{k_2},$$

$$h = \frac{h_2}{h_1}, \quad n = \frac{\rho_2}{\rho_1}, \quad b = \frac{\beta_2}{\beta_1}, \quad k = \frac{k_1}{k_2}$$

$$Da_1 = \frac{k_1}{h_1^2}, \quad Da_2 = \frac{k_2}{h_2^2},$$

$$\frac{\sigma_{e2}}{\sigma_{e1}} = \alpha,$$

In the view of the above non-dimensional quantities the equations (1.1) to (1.4) take the following form after dropping asterisks;

Phase I

$$\frac{d^2 u_1}{dy^2} + \left(\frac{Gr}{Re}\right) \text{Sin} \alpha \theta_1 - \left(\frac{1}{Da_1} + M^2 u_1\right) u_1 = P \tag{1.8}$$

$$\frac{d^2 \theta_1}{dy^2} + \text{Pr} Ec \left(\frac{du_1}{dy}\right)^2 + \frac{\text{Pr} Ec}{Da} u_1^2 = 0 \tag{1.9}$$

Phase II

$$\frac{d^2 u_2}{dy^2} + \frac{Gr}{Re} \text{Sin} \alpha u_2 - b^2 u_2 = mp \tag{1.10}$$

$$\frac{d^2 \theta_2}{dy^2} + \text{Pr} Ec \frac{k}{m} \left(\frac{du_2}{dy}\right)^2 + \frac{\text{Pr} Ec K}{m Da_2} u_2^2 = 0 \tag{1.11}$$

where

$$b^2 = \frac{k}{Da} + \alpha m M^2$$

$$Gr = \frac{g \beta_1 h_1^3 \Delta T}{v_1^2}$$

$$\text{Pr} = \mu_1 c_p / k_1$$

$$\text{Re} = \bar{u}_1 h_1 / v_1$$

$$\sigma = \frac{h_1}{\sqrt{k_1}}$$

$$Ec = (\bar{u}_1)^2 / c_p \Delta T$$

$$P = h_1^2 \left(\frac{\partial p}{\partial x}\right) / \mu_1 \bar{u}_1$$

The non dimensional term of the velocity, temperature boundary and interface conditions (3.5) and (3.6) becomes (asterisks are neglected hereafter).

$$u(1) = 0, u_1(0) = u_2(0), u_2(-1) = 0$$

$$\frac{du_1}{dy} = 1/mh \frac{du_2}{dy} \text{ at } y = 0 \tag{1.12}$$

and

$$\theta_1(1) = 1, \theta_1(0) = \theta_2(0), \theta_2(-1) = 0$$

$$\frac{d\theta_1}{dy} = \frac{1}{kh} \frac{d\theta_2}{dy} \text{ at } y = 0 \tag{1.13}$$

5. Solution of the problem

The governing equations of momentum (1.8) , (1.10) along with the energy equations (1.9) , (1.11) are solved subject to the boundary and interface conditions (1.12) and (1.13) from the velocity and temperature distributions. In this case the equations are coupled and nonlinear because of buoyancy force and the inclusion of the dissipation terms in the energy equation. In many practical problems the Eckert number is very small and is of order 10^{-5} so the product $\text{Pr} Ec = (\varepsilon)$ is very small and can be exploited to apply the regular perturbation method in view of this the solutions are assumed in the form

$$(u_i, \theta_i) = (u_{i0}, \theta_{i0}) + \varepsilon (u_{i1}, \theta_{i1}) + \dots \tag{1.14}$$

Where $i = 1, 2 \dots u_{i0}, \theta_{i0}$ are solutions for the ε equal to zero. u_{i1}, θ_{i1} are perturbed quantities to u_{i0}, θ_{i0} respectively. Substituting the above identities in equations (1.8) – (1.11) are equating the coefficients of like powers of ε to zero we obtain the zeroth and first order equations as follows:

Phase I

Zeroth – order equations

$$\frac{d^2 u_{10}}{dy^2} + \left(\frac{Gr}{Re} \text{Sin} \alpha\right) \theta_{10} - a^2 u_{10} = P \tag{1.15}$$

$$\frac{d^2 \theta_{10}}{dy^2} = 0 \tag{1.16}$$

First – order equations

$$\frac{d^2 u_{11}}{dy^2} + \left(\frac{Gr}{Re} \text{Sin} \alpha\right) \theta_{11} - a^2 u_{11} = P \tag{1.17}$$

$$\frac{d^2 \theta_{11}}{dy^2} + \left(\frac{du_{10}}{dy}\right)^2 + \frac{1}{Da} u_{10}^2 = 0 \tag{1.18}$$

Phase II

Zeroth – order equations

$$\frac{d^2 u_{20}}{dy^2} + \frac{Gr}{Re} \text{Sin} \alpha \theta_{20} - b^2 u_{20} = mp \tag{1.19}$$

$$\frac{d^2 \theta_{20}}{dy^2} = 0 \tag{1.20}$$

First order equations

$$\frac{d^2 u_{21}}{dy^2} + \frac{Gr}{Re} \theta_{21} \text{Sin} \alpha - b^2 u_{21} = 0 \tag{1.21}$$

$$\frac{d^2 \theta_{21}}{dy^2} + \frac{k}{m} \left(\frac{du_{20}}{dy}\right)^2 + \frac{k}{m Da_2} (u_{20})^2 = 0 \tag{1.22}$$

The corresponding boundary and interface conditions (1.12) and (1.13) reduce to

$$\begin{aligned}
 u_{10}(1) &= 0 \\
 u_{10}(0) &= u_{20}(0) \\
 u_{20}(-1) &= 0 \\
 \frac{du_{10}}{dy} &= \frac{1}{mh} \frac{du_{20}}{dy} \text{ at } y=0
 \end{aligned}
 \tag{1.23}$$

$$\begin{aligned}
 \theta_{10}(1) &= 1 \\
 \theta_{10}(0) &= \theta_{20}(0) \\
 \theta_{20}(-1) &= 0 \\
 \frac{d\theta_{10}}{dy} &= \frac{1}{kh} \frac{d\theta_{20}}{dy} \text{ at } y=0
 \end{aligned}
 \tag{1.24}$$

$$\begin{aligned}
 u_{11}(1) &= 0 \\
 u_{11}(0) &= u_{21}(0) \\
 u_{21}(-1) &= 0 \\
 \frac{du_{11}}{dy} &= \frac{1}{mh} \frac{du_{21}}{dy} \text{ at } y=0
 \end{aligned}
 \tag{1.25}$$

$$\begin{aligned}
 \theta_{11}(1) &= 0 \\
 \theta_{11}(0) &= \theta_{21}(0) \\
 \theta_{21}(-1) &= 0 \\
 \frac{d\theta_{11}}{dy} &= \frac{1}{kh} \frac{d\theta_{21}}{dy} \text{ at } y=0
 \end{aligned}
 \tag{1.26}$$

Solutions of the zero-order equations (1.15), (1.16), (1.19) and (1.20) subject to the boundary conditions (1.23) and (1.24) are

$$\theta_{10} = \frac{y+kh}{kh+1} \tag{1.27}$$

$$\theta_{20} = \frac{kh(y+1)}{(kh+1)} \tag{1.28}$$

$$u_{10} = B_1 \cosh ay + B_2 \text{Sinh} ay + \frac{1}{a^2} (a_1 y + a_2) \tag{1.29}$$

$$u_{20} = B_3 \cosh ay + B_4 \sinh ay - \frac{1}{b^2} (a_3 y + a_4) \tag{1.30}$$

Solution of the first order perturbation equations (1.17), (1.18), (1.21) and (1.22) subject to the boundary conditions (1.25) and (1.26) are

$$\begin{aligned}
 \theta_{11} &= a_{19} \cosh ay + a_{20} \sinh ay + a_{21} y \sinh ay + a_{22} y \cosh ay \\
 &+ a_{23} \cosh ay + a_{24} \sinh ay + a_{25} y^4 + a_{26} y^3 + a_{27} y^2 + E_1 y + E_2
 \end{aligned}
 \tag{1.31}$$

$$\begin{aligned}
 \theta_{21} &= f_1 \cosh 2by + f_2 \sinh 2by + f_3 y \cosh by + f_4 y \sinh by + f_5 \cosh by + \\
 &f_6 \sinh by + f_7 y^4 + f_8 y^3 + f_9 y^2 + d_1 y + d_2
 \end{aligned}
 \tag{1.32}$$

$$\begin{aligned}
 U_{11} &= A_1 \cosh ay + A_2 \sinh ay + g_1 \cosh 2ay + g_2 \sinh 2ay + g_3 y^2 \cosh ay \\
 &+ g_4 y^2 \sinh ay + g_5 y \cosh ay + g_6 y \sinh ay + g_7 y^4 + g_8 y^3 + g_9 y^2 + g_{10} y + g_{11}
 \end{aligned}
 \tag{1.33}$$

$$\begin{aligned}
 U_{21} &= A_3 \cosh by + A_4 \sinh by + e_1 \cosh 2by + e_2 \sinh 2by + e_3 y^2 \sinh by \\
 &+ e_4 y^2 \cosh by + e_5 y \cosh by + e_6 y \sinh by + e_7 y^4 + e_8 y^3 + e_9 y^2 + e_{10} y + e_{11}
 \end{aligned}
 \tag{1.34}$$

where

$$\begin{aligned}
 c_1 &= \frac{c_3}{kh}, c_3 = \frac{kh}{kh+1}, c_1 = c_3, c_2 = c_4 \\
 a_1 &= -\frac{G_r}{\text{Re}} \text{Sin} \phi c_1, a_2 = p - \frac{G_r}{\text{Re}} \text{Sin} \phi c_2 \\
 a_3 &= -\frac{G_r}{\text{Re}} \text{Sin} \phi mn \beta h^2 c_3, a_4 = m p h^2 - \frac{G_r}{\text{Re}} \text{Sin} \phi mn \beta h^2 c_4
 \end{aligned}$$

$$\begin{aligned}
 a_5 &= B_2 \text{Sin} ha + B_3 \text{Cos} ha, \\
 a_6 &= B_2 \text{Sin} ha \text{Cosh} b + B_4 \text{Sin} hb \text{Cos} ha \\
 a_7 &= -\left(-\frac{a_3}{b^2} + \frac{m a_1 h}{a^2} \right) \text{Sin} ha \text{Cosh} b + m a h a_6
 \end{aligned}$$

$$\begin{aligned}
 a_8 &= \frac{1}{b \sinh a \cosh b + m a h \sinh b \cosh a} \\
 B_1 &= -\frac{\text{Sin} ha (B_2) + (a_1 + a_2)}{a^2 \text{Cos} ha}
 \end{aligned}$$

$$\begin{aligned}
 B_2 &= \frac{1}{m a h} \left[b B_4 - \frac{a_3}{b^2} + \frac{m h a_1}{a^2} \right] \\
 B_3 &= B_1 + \frac{a_4}{b^2} - \frac{a_2}{a^2}, B_4 = a_7 a_8
 \end{aligned}$$

$$\begin{aligned}
 a_9 &= -\left(a^2 B_1^2 + \frac{B_2^2}{D a} \right), a_{10} = -\left(a^2 B_2^2 + \frac{B_1^2}{D a} \right)
 \end{aligned}$$

$$\begin{aligned}
 a_{11} &= -\left(a^2 B_1 B_2 + \frac{B_1 B_2}{D a} \right), a_{12} = -\left(2 \frac{a_1}{a} B_1 - \frac{2 B_2 a_2}{a^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 a_{13} &= -\left(2 \frac{a_1}{a} B_2 - \frac{2 B_1 a_2}{a^2} \right), a_{14} = -\frac{a_1^2}{a^4 (D a)}
 \end{aligned}$$

$$\begin{aligned}
 a_{15} &= -\frac{2 a_1 a_2}{a^4 (D a)}, a_{16} = \frac{2 a_1 B_2}{a^2 (D a)}
 \end{aligned}$$

$$\begin{aligned}
 a_{17} &= \frac{2 a_1 B_1}{a^2 (D a)}, a_{18} = -\left[\frac{a_1^2}{a^4} + \frac{1}{D a} \frac{a_2^2}{a^4} \right]
 \end{aligned}$$

$$\begin{aligned}
 a_{19} &= \frac{a_9}{8 a^2} + \frac{a_{10}}{8 a^2}, a_{20} = \frac{a_{11}}{4 a^2}, a_{21} = \frac{a_{12}}{a^2},
 \end{aligned}$$

$$\begin{aligned}
 a_{22} &= \frac{a_{13}}{a^2}, a_{23} = \frac{a_{14}}{a^2} - \frac{2 a_9}{a^3}, a_{24} = \frac{a_9}{a^2} - \frac{2 a_{11}}{a^3}
 \end{aligned}$$

$$\begin{aligned}
 a_{25} &= \frac{a_{14}}{12}, a_{26} = \frac{a_{15}}{6}, a_{27} = -\frac{a_9}{4} + \frac{a_{10}}{4} + \frac{a_{11}}{2}
 \end{aligned}$$

$$\begin{aligned}
 r_1 &= -\frac{k}{m} \left(b^2 B_2^2 + \frac{B_4^2}{D a} \right), r_2 = -\frac{k}{m} \left(b^2 B_4^2 + \frac{B_3^2}{D a} \right)
 \end{aligned}$$

$$\begin{aligned}
 r_3 &= -\frac{k}{m} \left(b^2 + \frac{B_3 B_4}{D a} \right), r_4 = \frac{k}{m} \left(\frac{2 a_3 B_3}{b^2 D a} \right)
 \end{aligned}$$

$$\begin{aligned}
 r_5 &= -\frac{k}{m} \left(\frac{2 a_3 B_4}{b^2 D a} \right), r_6 = -\frac{k}{m} \left(-\frac{2 a_3}{b} + \frac{2 a_4 B_3}{b^2 D a} \right)
 \end{aligned}$$

$$\begin{aligned}
 r_7 &= -\frac{k}{m} \left(-\frac{2 a_3}{b} + \frac{2 a_4 B_4}{b^2 D a} \right), r_8 = -\frac{k}{m} \left(\frac{a_3^2}{b^4 D a_2} \right)
 \end{aligned}$$

$$\begin{aligned}
 r_9 &= -\frac{k}{m} \left(\frac{a_3^2}{b^4} + \frac{a_4^2}{b^4 D a_2} \right), \\
 f_1 &= \frac{r_1 + r_2}{4 b^2}, f_2 = \frac{r_3}{4 b^2}, f_3 = \frac{r_4}{b^2}
 \end{aligned}$$

$$\begin{aligned}
 f_4 &= \frac{r_5}{b^2}, f_5 = -\frac{2 r_5}{b^3} + \frac{r_6}{b^2}, \\
 f_6 &= -\frac{2 r_4}{b^3} + \frac{r_7}{b^2}, f_7 = \frac{r_8}{12}, f_8 = \frac{r_9}{6}, f_9 = -\frac{r_1}{2} + \frac{r_2}{2} + \frac{r_{10}}{2}
 \end{aligned}$$

$$e_1 = \frac{sf_1}{3b^2}, e_2 = \frac{sf_2}{3b^2}, e_3 = \frac{sf_3}{4b}, e_4 = \frac{sf_4}{4b}$$

$$e_5 = \frac{sf_6}{2b} - \frac{sf_3}{4b^2}, e_6 = \frac{sf_5}{2b} - \frac{sf_4}{4b^2}$$

$$e_7 = -\frac{sf_7}{b^2}, e_8 = -\frac{sf_8}{b^2}, e_9 = -\frac{12f_7s}{b^4} - \frac{sf_9}{b^2}$$

$$j_1 = -\frac{6sf_8}{b^4} - \frac{sE_5}{b^2}, j_2 = -\frac{24sf_7}{b^6} - \frac{2sf_9}{b^4} + \frac{se_4}{b^2}$$

$$g_1 = \frac{st_1}{3a^2}, g_2 = \frac{st_2}{3a^2}, g_3 = \frac{st_3}{4a}, g_4 = \frac{st_4}{4a}$$

$$g_5 = \frac{st_5}{2a} - \frac{st_3}{4a^2}, g_6 = \frac{st_6}{2a} - \frac{st_4}{4a^2}, g_7 = -\frac{st_7}{a^2}$$

$$g_8 = -\frac{st_8}{a^2}, g_9 = -\frac{12st_7}{a^4} - \frac{st_9}{a^2}$$

$$F_1 = mah \cosh a, F_2 = b \sinh a$$

$$F_3 = \cosh b, F_4 = -\sinh b$$

$$J_1 = mah I_5 - I_4 \text{ Sinha}$$

$$I_5 = I_1 - I_3 \text{ Cosha}$$

$$A_3 = \frac{F_2 I_2 - F_4 J_1}{F_1 F_4 - F_2 F_3}, A_4 = \frac{F_3 J_1 - F_1 I_2}{-F_2 F_3 + F_1 F_4}$$

$$A_1 = A_3 - I_3, A_2 = -\frac{(I_1 + A_1 \text{ Cosha})}{\text{Sinha}}$$

6. Deductions and discussions

An analytic solution for convection flow and heat transfer for a composite fluid saturated inclined porous layer is obtained using the regular perturbation method. The flow is modeled with the Darcy – Lapwood - Brinkman equation. The viscous and Darcy dissipation terms are included in the energy equation. The product $Pr.Ec(=\epsilon)$ is used as a perturbation parameter.

The effect of the ratio of the heights of the two porous layers on velocity field is shown in fig. (1.2). for fixed $K = 1, L = 5, \theta = \pi/4, Da = 3, \alpha = 2, P = 0.5, m = 2, M = 2, K = 2, \epsilon = 0.01, n = 0.1$. We observe that the larger the height of the upper phase compared to the lower phase the smaller the velocity of the fluid flow between the parallel plates.

The effect of the Grashoff number on velocity field is shown in fig. (1.3) for fixed $K=2, Da=5, \theta = \pi/4, \alpha = 2, p=0.5, m=2, M=2, k=1, \epsilon=0.01, n=0.1$. The effect of increasing Grashoff number is to increase the convective motion. Physically an increase is the value of the Grashoff number means on increase of buoyancy for which supports the motion.

The effect of Darcy number on the velocity field is shown in figure (1.4) for fixed $K=2, L=3, \theta = \pi/4, \alpha = 2, p=0.5, m=2, M=2, k=1, \epsilon=0.01, n=0.1$. increasing the Darcy number increases the velocity fields.

The velocity distribution for different inclination angles is shown in fig. (1.5) for fixed $K=2, L=5, Da=5, \alpha = 2, p=0.5, m=2, M=2, k=1, \epsilon=0.01, n=0.1$. We observed that the increase in the inclination angle increase the flow.

In figure (1.6) for fixed $K = 1, L = 5, \theta = \pi/4, Da = 3, \alpha = 2, P = 0.5, m = 2, M = 2, K = 2, \epsilon = 0.01, n = 0.1$, we find that an increase in the ratio of the permeabilities decreases the velocity fields.

The effect of Grashoff number on the velocity field is shown in fig.3. We observe that the effect of increasing Grashoff number is to increase the convective motion. An increase in the value of Grashoff number. Physically means an increase of buoyancy force which supports the motion.

The effect of Darcy Number on the temperature field as shown in fig. (1.7). for fixed $K=1, L=5, \theta = \pi/4, \alpha = 2, p=0.5, m=2, M=2, k=1, \epsilon=0.01, n=0.1$. It is observe that an increase in the value of Darcy number increases the temperature field.

The effects of the ratio of the heights of the two porous layers on the temperature field is shown in fig (1.8) for fixed $K=1, L=5, \theta = \pi/4, \alpha = 2, p=0.5, m=2, M=2, k=1, \epsilon=0.01, n=0.1, Da=3$. It is observed that an increase in the ratio of heights of the porous layers.

The effects of temperature profiles for different values of the ratio of the permeabilities is discussed in fig. (1.9) for fixed $L=5, \theta = \pi/4, \alpha = 2, p=0.5, m=2, M=2, k=1, \epsilon=0.01, n=0.1, Da=3$. Increasing the ratio of permeabilities increases an increasing in the temperature. In fig (1.10) we observe that the temperature increases from the lower plate $y=-1$, to upper plate $y=1$. For fixed y , the temperature increases L i.e. increasing with the ratio of G_r and R_e .

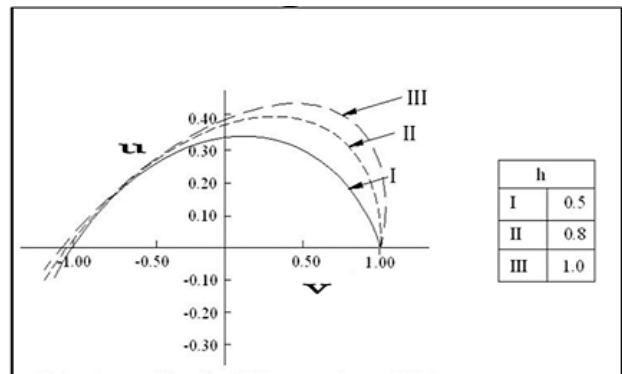


Fig.2

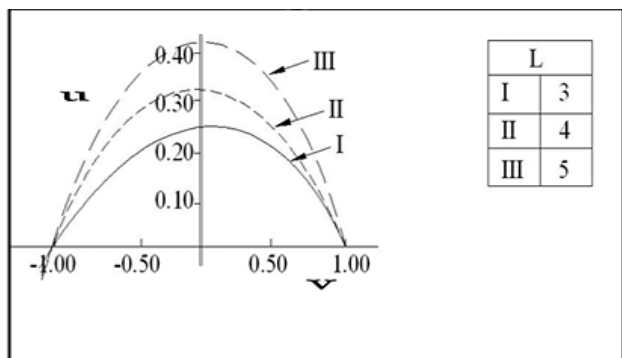


Fig.3

From in Fig. (1.11) we observe that the temperature increases with increasing θ for a given value of angle of inclination ϕ , for fixed y , temperature increases with increasing ϕ .

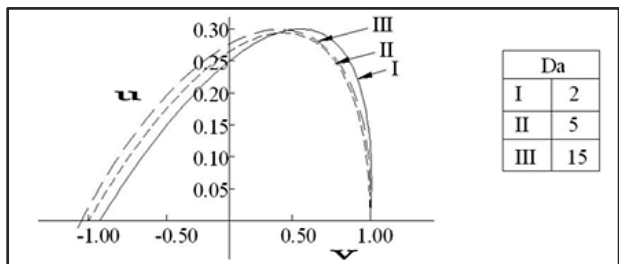


Fig.4

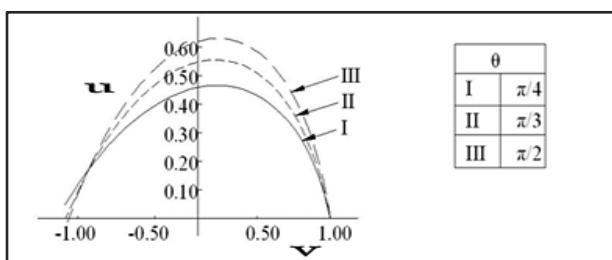


Fig.5

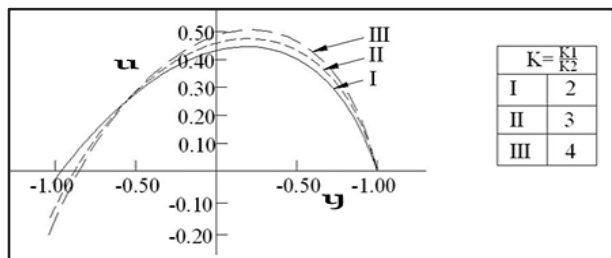


Fig.6

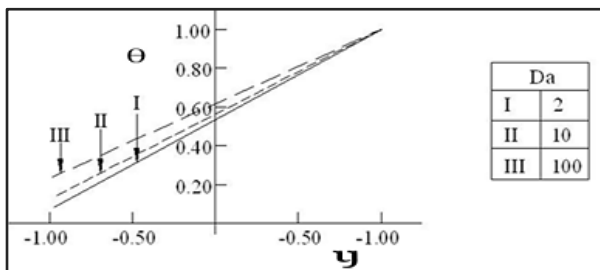


Fig. 7

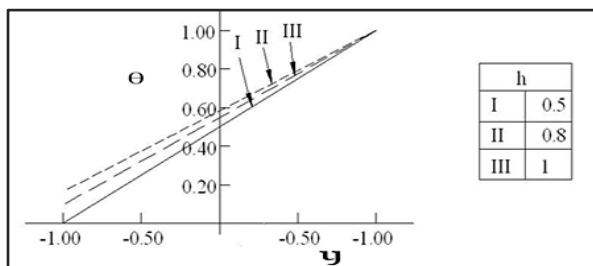


Fig.8

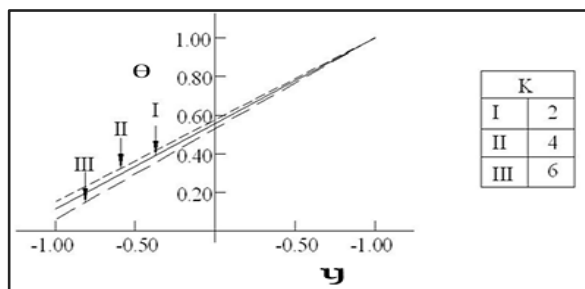


Fig.9

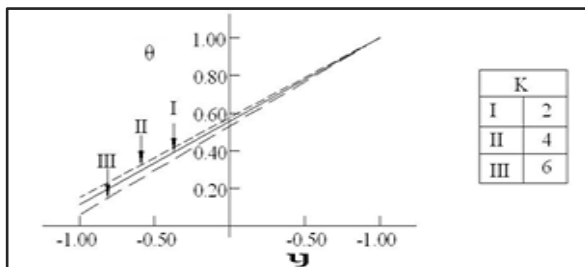


Fig. 10

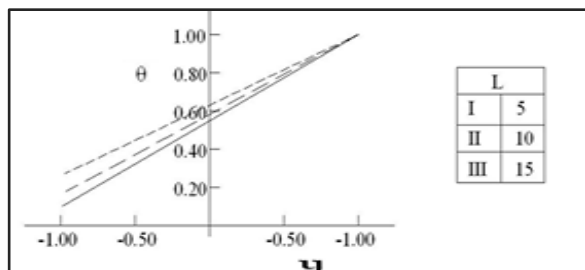


Fig. 11

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