

## Research Article

## De-noising of 1-D Signals using Fourier Transform and Haar Wavelet

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### Abstract

Signal recovery and noise reduction are closely related signal processing problems of both theoretical and practical interests. In this project, these classical problems are studied with two tools- Fourier Transform and Wavelets. The filtering of time series data for the purpose of removing unwanted signal components is of interest to a wide variety of science disciplines. The process of mapping a time series into the frequency domain via the Fourier transform is used to separate the signal characteristics from that of the noise. In Wavelet thresholding methods in which the wavelet coefficients are threshold in order to remove their noisy part. This project report presents and discusses two de-noising methods. Their main features and limitations are discussed. A signal contaminated with Additive White Gaussian Noise (AWGN) is used for performance evaluation and simulation results are provided.

**Keywords:** Filtering, thresholding, de-noising.

### 1. Introduction

In modern communication applications such as cellular communication, image processing, medical signal processing, RADAR etc. the signals that we deal with are not purely the information that is meant to be received. In almost all applications it is unavoidable that a noise or distortion may be added to the signal. Processing of this noise is important as the noise contains unwanted information which is not desired by the end user.

The idea behind the project is to estimate the uncorrupted speech from the distorted or noisy speech signal and sine signal, and is also referred to as speech “de-noising”. There are various methods to help restore speech from noisy distortions. Selecting the appropriate method plays a very important role in getting the desired speech. We have selected one method as the Fourier transforms method which is more classical in its principle. The second method is the use of wavelets which is a relatively new concept used for de-noising.

A sine signal and audio signal is taken and white Gaussian noise is added to it. This would be given as an input to the de-noising algorithm, which produces a speech signal close to the original speech signal (high quality). In the Fourier transforms approach we calculate the Fourier transform of the noisy signal and try to separate the noise components in the frequency domain. This separation is achieved by truncating the Fourier transform coefficients to their integer values. On truncating, we bring the modified signal back to the time domain and observe the output. In the wavelets method we carry out wavelet decomposition by choosing a suitable wavelet and also

specifying up to what level we want the decomposition to take place. By applying a suitable thresholding to the noisy signal we can separate out the noise components to a good extent. The wavelet is reconstructed to give a de-noised output.

### 2. Theory

#### 2.1 Noise

Noise is defined as an unwanted signal that interferes with the communication or measurement of another signal. A noise itself is an information-bearing signal that conveys information regarding the sources of the noise and the environment in which it propagates. Signal distortion is the term often used to describe a systematic undesirable change in a signal. Here we have used a specific type of noise –**White noise**: purely random noise has an impulse auto-correlation function and a flat power spectrum. White noise theoretically contains all frequencies in equal power.

#### 2.2 Fourier Transforms Theory

The time domain representation of a signal does not give us complete information regarding the properties of the signal. Hence we prefer to take the signal into the frequency domain where we can analyze the signal and its frequency components better using the Fourier analysis. The audio signal is a continuous-time a-periodic signal having a finite energy. The frequency response obtained on computing the Fourier transform is a continuous spectrum. The addition of a white Gaussian noise not only affects the amplitude in time domain but also affects the frequency response of the signal. Additional frequency components may be added which are originally not present in the signal.

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For a simple sine signal if we add a white Gaussian noise, compute its Fourier transform and plot its spectrum we get an idea of in which region are the frequency components of the noise present. By using some approximations we can try to eliminate these frequency components by making their amplitude zero in the Fourier domain itself. If amplitude becomes zero for that frequency we can say that the effect of noise at that frequency is minimized if not eliminated. We use this approach to minimize the effect of noise by approximating the noise coefficients to zero in the frequency domain. Hence the only components which are present belong to the original signal in this case the sine wave.

When the signal is brought back into the time domain using the Inverse Fourier transform and observed then we find that modified signal will now bear a resemblance to the original signal. Due to the approximations there is a possibility that some of the signal components may also be set to zero in the process and hence we do not get the original signal back but we can observe that the effect of the noise is suppressed to a good extent.

As we are running our simulations in MATLAB, where the signal is stored in a discrete manner, therefore we compute the Discrete Fourier transform (DFT) of the signal.

### 2.2.1 Algorithm using Fourier transforms

- (1) Read the audio signal from a file using WAVREAD command
- (2) Add a White Gaussian noise to 'y' to get the noisy signal 'x' which will be analyzed further. Keep the SNR moderately high.
- (3) Compute the Fourier transform (DFT) of 'x'
- (4) Fix the noise components to zero using the FIX command. Set a threshold to fix the values and once the values are 'fixed' restore their magnitude to the original value.
- (5) Compute the Inverse Fourier Transform (IDFT) to get the de-noised signal 'z'
- (6) Consider only the real values of 'z'
- (7) Write this de-noised signal to a file using the WAVWRITE command
- (8) Plot 'x', 'y' and 'z' and compare the output
- (9) Draw inferences

### 2.3 Wavelet Theory

Fourier analysis of a signal involves the use of the complex exponential  $e^{j\omega t}$  as the Basis function. The components of the complex exponential are sine ( $\sin(\omega t)$ ) and cosine ( $\cos(\omega t)$ ). These functions are the smoothest, analytic functions with the derivative existing everywhere and anywhere. These functions have a time span ranging from  $t = -\infty$  to  $t = +\infty$ . However in real time signal processing our signals do not exist for an infinite time and hence only those signals which start from a finite time and end at a finite time are of use to us. Another disadvantage of using the Fourier analysis is that the Fourier components cannot be calculated at just one point i.e. at time  $t = t_0$  we cannot determine which all frequencies are present. Similarly in the frequency domain one particular

frequency exists for how many time instants also cannot be answered. The spectrum that we get is for the entire length of the signal. Thus the spectrum or the time signal lacks localization (complete information). Therefore the wave  $\sin(\omega t + \Theta)$  lacks localization in the time domain because it runs from  $-\infty$  to  $+\infty$ . Hence to avoid this localization we have to modify the Basis function  $e^{j\omega t}$ . So we take only a section of the wave to decompose the signal and this section is called a wavelet.

In our project we have made use of the 'Haar' wavelet also known as 'db1' belonging to the Daubechies family of wavelets. The Basis function  $\phi(t)$  and the wavelet function  $\Psi(t)$  for the Haar wavelet are  $\phi(t)$  is the amplitude scaling function while  $\Psi(t)$  is the time scaling function.

$\phi(t)$  basically gives the information at one level while  $\Psi(t)$  gives the additional information between two levels. The entire signal is decomposed using these two functions.

The signal is decomposed into two components – approximations (terms involving  $\phi(t)$ ) and details (terms involving  $\Psi(t)$ ) by means of passing it through a filter bank which comprises of a low pass and a high pass filter at the very basic level. Such decompositions can be made at multiple levels in order to reach closer and closer to the details of the signal as illustrated in the block diagram.

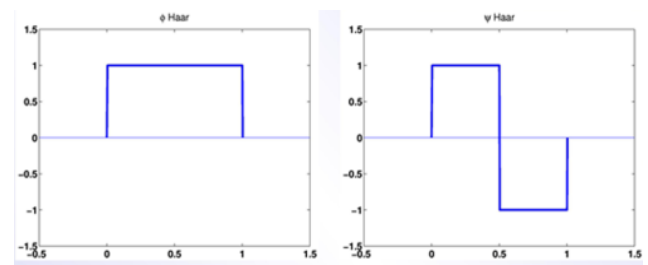


Fig. 1 Basis Function and Wavelet Function for Haar wavelet

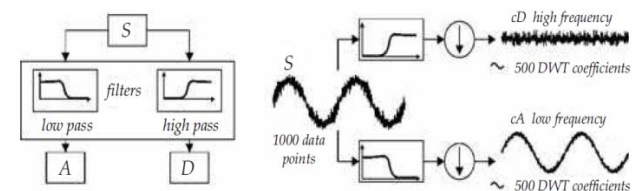


Fig. 2 Low and High Pass filtering in wavelet decomposition

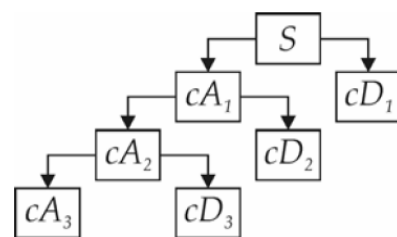
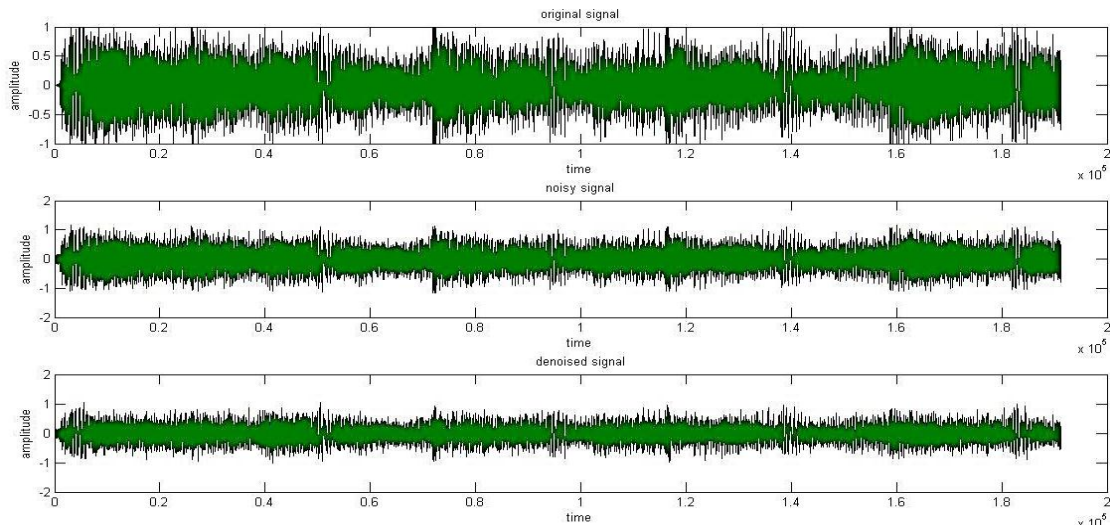
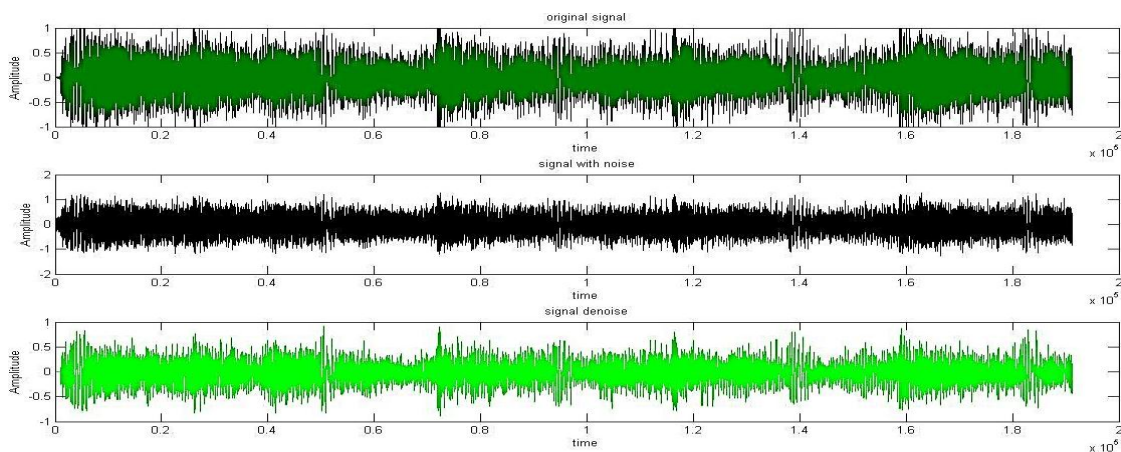


Fig. 3 Multilevel decomposition

We have made use of 15 levels to decompose the signal. Now the signal which is decomposed is filtered by setting a particular threshold value for the noise which is added.



**Fig. 4** Fourier Transformation method



**Fig. 5** Wavelet method

This threshold value decides what will be the level of noise in the signal. We have made use of soft threshold in our algorithm. The coefficients of the decomposed wavelet are threshold for this value. This process eliminates the noise from the signal. The signal is reconstructed by using the inverse process. This gives us the de-noised output.

*2.3.1 Algorithm of Wavelet method*

- (1) Read the original signal 'x' using the WAVREAD command
- (2) Add white Gaussian noise to get the noisy signal 'y'. keep the SNR moderately high
- (3) Specify the level of decomposition as 15
- (4) Decompose the noisy signal 'y' using the WPDEC command. Select the wavelet as 'Haar' or 'db1'
- (5) Store the coefficients of this decomposition
- (6) Now based on the standard deviation of the Gaussian distribution and the tuning parameter decide the threshold for the signal using the WPBMPEN command
- (8) Reconstruct the signal after soft threshold using the threshold value calculated. This is the de-noised output 'z'

- (9) Plot the signals 'x','y' and 'z' and compare the outputs
- (10) Draw inferences

**3. Implementation**

*Outputs*

(A) *Fourier Transforms method*

It is shown in fig.4

(B) *Wavelet method*

It is shown in fig.5

**4. Comparison**

(1) *Fourier Transforms*

- Easier to analyze.
- In the reduction of noise components some of the signal components are also made zero. This decreases the overall amplitude of the signal.

- Fourier is localized only in frequency domain.
- As compared to the Wavelet method the efficiency of Fourier is less

(2) *Wavelet Methods*

- Wavelet is a new tool used for real time signal processing as compared to Fourier.
- Localized in time as well as frequency.
- The thresholding allows only the removal of noise components without affecting some of the signal components. Wavelets give a better representation using Multi resolution analysis, with balanced resolution at any time and frequency.

Therefore wavelets are better than Fourier analysis.

**Conclusion**

In this project we have analyzed and compared two

algorithms to de-noise audio signals. We explore the use of Fourier to de-noise signals which is a more classical method of de-noising. In the second algorithm we have used Wavelets to de-noise signals. This method was more efficient and gave more flexibility as many types of wavelets can be used. The disadvantages of Fourier can be overcome with the use of wavelets in this application specifically. The quality of the output is not very close from the original signal and in order to improve the quality we need other techniques as well along with a de-noising algorithm.

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