International Journal of Current Engineering and Technology
E-ISSN 2277 – 4106, P-ISSN 2347 - 5161

Available at http://inpressco.com/category/ijcet

Research Article

A Comparison between the Treatments of Punching by EC2 and the Critical Shear Crack Theory (CSCT) - Slabs without Shear Reinforcement

Rizgar Amin Agha[Ȧ*]

[Ȧ]Faculty of Engineering-University of Sulaimani, Kurdistan region of Iraq

Accepted 19 Oct 2014, Available online 25 Oct 2014, **Vol.4, No.5 (Oct 2014)**

Abstract

This paper treats the punching shear capacity for Critical Shear Crack Theory (CSCT) and EC2 equation considering connection between supporting columns and slabs without shear reinforcement. It shows the effects of individual parameters as found from CSCT calculations with the corresponding results from EC2, to demonstrate the differences between the two methods. It treats the conditions simulated in most tests where a load or reaction acts through a column or plate at the centre of the slab and is balanced by reactions or loads near the edges of the slab. In general, the predictions by EC2 are more conservative compared to those by CSCT. The increasing of compressive strength causes in increasing of punching shear capacity in both methods and the rate of increasing in shear capacity is decreases with increasing the compressive strength. The providing of flexural reinforcement in the perimeter area around the support enhanced the punching shear capacity and it increased with increasing of the reinforcement ratio. the stiffness of the slab where investigated and shows that the stiffer slab produces lesser rotation and then higher shear resistance. It was found that the CSCT approach is a complex method to be used in a design practice; however the overestimations in cases of smaller size aggregate and lower stiffness in EC2 can justify can justify the need for a new and more complex equation to consider these parameters.

Keywords: *Punching shear strength, size factor, aggregate size, flexural reinforcement, concrete strength, steel strength, slab rotation and slenderness*

Introduction

According to EC2 (Eurocode 2(2004), characteristic punching resistances are given by

$$V_{Rk,c} = v_{Rk,c} u_1 d \le v_{Rk,c} u_0 d \qquad (1)$$

where $v_{Rk,c} = 0.18k(100\rho_1 . f_{ck})^{1/3}$, $k = 1 + \sqrt{200/d} \le 2.0$, u_1 is the length of a perimeter constructed to obtain the minimum length without coming closer to a column than 2d from it
$u_1 = 2(c_1 + c_2) + 4d$ for rectangular columns with side lengths c_1 and c_2, and $u_1 = \pi(c + 4d)$ for a circular column of diameter c. ρ_1 is the ratio of flexural tension reinforcement determined as $\sqrt{\rho_{1x}\rho_{1y}}$ calculated for the orthogonal directions of the reinforcement and for widths equal to those of the column plus 3*d* to either side. *P* ≤ *0.02* for calculation purposes. is the *d* mean effective depth of the reinforcement $= (d_x + d_y)/2$, f_{ck} is the characteristic cylinder compression strength of the concrete $v_{Rk,\max} = 0.24(1 - f_{ck}/250) f_{ck}$, u_0 is the length of the perimeter of the column.The above definition ignores a minimum value given for $v_{Rk,c}$, which is of no practical significance for normal reinforced concrete slabs.

The critical shear crack theory (CSCT)(Muttoni 2008) is a fairly recent approach to punching that is the basis of the new fib model code 2010. The CSCT is developed by Muttoni in 1985. The theory is presented in a mechanical model for failure criteria in evaluating the shear strength. According to this theory the aggregate interlocks contribute in carrying the shear forces, and then when cracks occur as a result of rotation the shear strength reduces. Unlike EC2and other statutory codes of practice, the CSCT does not provide explicit expressions for punching resistances, but obtains them by the numerical simultaneous solution of equations relating slab rotation to applied load and shear capacity to slab rotation. This makes it difficult to see the effects of individual parameters. The basic CSCT equations are given in (Muttoni A.2008), as

$$V = \frac{0.75 b_0 d \sqrt{f_c}}{1 + 15\psi d \sqrt{16 + d_g}} \qquad (2)$$

$$V_R = \left[\frac{\psi\, d\, E}{1.5 r_s f_y}\right]^{2/3} V_{flex} \qquad (3)$$

*Corresponding author: **Rizgar Amin Agha** (BSc, MSc, PhD)

where

$$V_{flex} = 2\pi m_R r_s / (r_q - r_c) \quad (4)$$

b_o is the length of a perimeter constructed to obtain the minimum length without coming closer to the columns than 0.5d from it, $\psi$is the rotation of the slab outside the critical shear crack (in radians), d_g is the maximum size of the aggregate. It has been proposed that d_g should be taken as 0 for high strength concrete, to allow for the smoother fracture surfaces developed in high strength (and lightweight aggregate) concretes, E_s is the modulus of elasticity of the reinforcement, here taken as 200 GPa, f_y is the yield or 0.2% proof stress of the reinforcement, V_{flex} is the yield-line flexural capacity of the slab as given by eqn.(4) and m_R is the plastic moment of resistance at a yield line (averaged for the length of the line) and all the radii r_c, r_q and r_s are as shown in Fig.(1) for a circular slab on a circular column. For rectangular columns with c_1 and $c_2 \le 3d$, r_c is taken as $r_c = (c_1 + c_2)/\pi$. If the length of a side of a rectangular column is greater than $3d$, a value of $3d$ is substituted for it in calculating b_0 and presumably also V_{flex} .

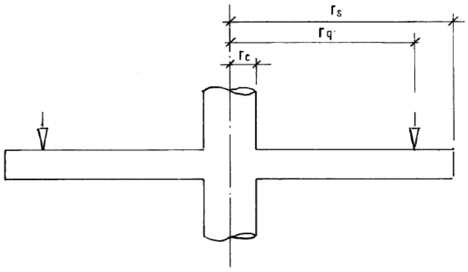


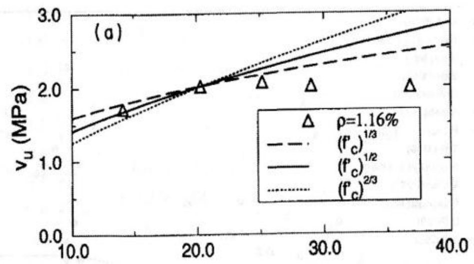
Fig. 1 The main dimensions of a flat slab supported by a circular column

There are extensive experimental works have been done on the punching in flat slabs, in which numbers of empirical expressions are proposed to predict the punching shear strength. Some of these expressions are become the base of most of building codes of design. To obtain more accurate predictions there are still efforts are contributing in producing other approaches or modifying these expressions by including new influential parameters.

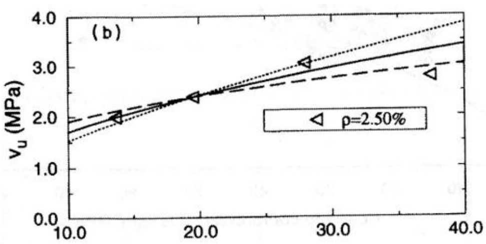
There is a great discrepancy between codes of practice in the definitions of punching shear strength of slabs without shear reinforcement. Some codes consider only the influence of compressive strength of concrete or with the flexural reinforcement ratio and arrangement of reinforcing steel, others account for size effect, column dimensions and position and aggregate size. Even they differed in considering the mathematical function of these parameters. The main parameters concerned by researchers and codes of practice are reviewed below:

(a) The compressive strength of concrete

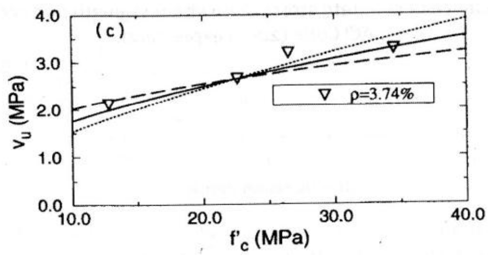
The compressive strength of concrete could be the main parameter which related to the shear strength by all researchers and codes of practice but in different functions of square root or cubic root or not considering any root dependence by earlier researches. Moe (Moe J.1961) used square root as the tensile strength was generally assumed to be proportional to $\sqrt{f_c}$. Mitchell, Cook and Dilger (Mitchell et al. 2005) studied the influence of the square and cubic root using data from literature but they did not achieve the clear conclusion of whether square or cubic root is the best function as shown in Fig.(2).



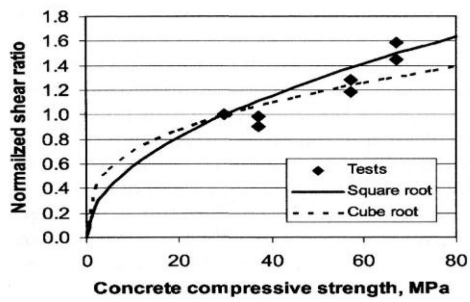
(a)



(b)



(c)



(d)

Fig. 2 a,b and c are Effect of concrete strength on shear strength (tests by Elstner et al., figure by Mitchell et al. .d- Comparison of square root and cube root functions with test results reported by Ghannoum and McHarg et al., figure by Mitchell et al. . (Duplicate from Lantsogh (Lantsoght E.2009))

Also, others concluded through experimental works that the punching shear capacity is proportional to the cube root of concrete compressive like Elstner and Hognestad (Elstner et al.1956), Regan (Regan 1986) and Gardner and Shao (Gardner N.J. et al.1990). EC2 considers a cubic root while the CSCT uses square root.

Sacramento P.V.P.et al.(Sacramento 2012) studied the influence of the compression concrete strength on the punching strength of flat slab using results from literature. They observed a good correlation when in comparing the calculated shear strength by EC2 and test results on the base of a function proportional to cubic root of compressive strength of concrete as shown in Fig.(3) .

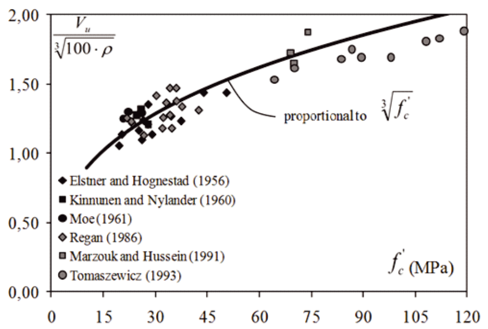


Fig. 3 Influence of the compression concrete strength on the punching resistance of flat slabs. (Duplicate figure by Sacramento P.V.P.,et al.)

(b)The flexural reinforcement ratio and arrangement of reinforcing steel

The flexural reinforcement ratio (ρ) defines the steel area (A_s) as a fraction of concrete area $(A_c = bd_{eff})$, excluding the protective concrete cover. The influence from flexural reinforcement in terms of reinforcement ratio has been taken into account by some of the code provisions and researchers. They consider that the flexural reinforcement will enhance the punching shear strength around the column perimeter when it raises the compression zone, however they are differed significantly in terms of their functions and distribution of steel bars. The earlier tests by Elstner and Hognestad showed there is no influence from compression reinforcement on the ultimate shearing strength of slabs. Also, Hawkins's (Hawkins et al 1979) theoretical analysis shows that steel ratio does not clearly affect the punching shear strength. However, the experimental work and studies of most researchers showed that the increasing of flexural reinforcement within a perimeter of punching area does increase the punching shear strength of slabs, like Regan(Regan 1974) ,Rankin and Long(Rankin et al.1987)found the increasing of punching shear strength with a function of $\sqrt[4]{100\rho}$, Shehata and Regan(Shehata et al. 1989) and Alexander and Simmonds(Alexander et al.1992) found that the concentration of flexural reinforcement near the column or loaded could improves the shear capacity ,Regan and Braestrup (Regan et al. 1985) and Sherif and Dilger (Sherif et al.1989) accounts for it as a function of $\sqrt[3]{100\rho}$,Gardner and Shao (Gardner et al.1996) accounts for the reinforcement ratio multiplied by yielding stress in term of $\sqrt[3]{\rho f_y}$.

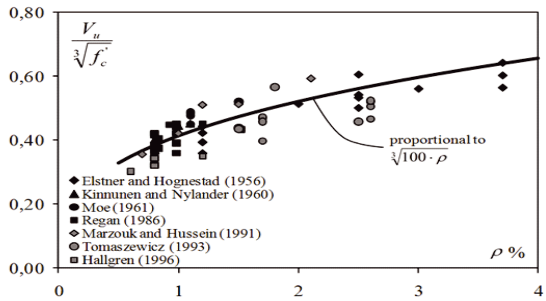
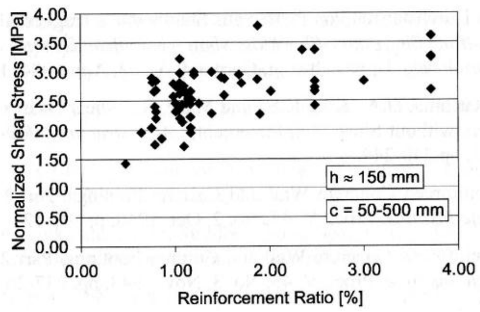
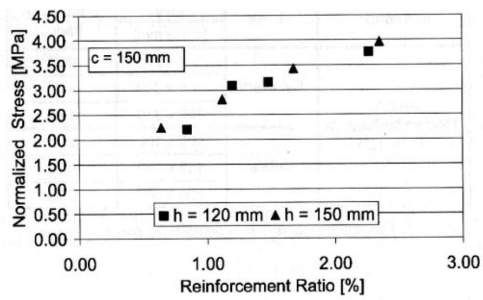


Fig. 4 Influence of the flexural reinforcement ratio on the punching strength of flat slabs, (Figure by Sacramento P.V.P. et al.)

Dilger, Birkle and Mitchell studied the influence of the reinforcement ratio on the ultimate shearing strength. They carried out a comparison of test results from literature and concluded that with an increase in flexural reinforcement ratio the punching shear strength is increased as shown in Fig. (5)



(a)



(b)

Fig.5 a- Failure load vs. flexural reinforcement ratio (h ≈ 150 mm ≈ 6 in), b-from Dilgeret al. (2005)

Guandalini, Burdet and Muttoni(Guandalini et al. 2009) carried out a test series on the punching behavior of slabs with varying low flexural reinforcement ratios and without transverse reinforcement The results are compared with design codes and to the critical shear crack theory. The predictions by EC2 and CSCT are agreed with the test results while those by ACI 318-08 are less conservative for thick slab and for low reinforcement ratios than the test results as shown in Fig.(6).

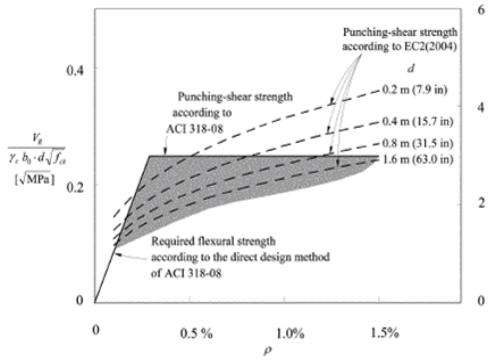


Fig. 6 Influence of flexural reinforcement ratio on punching shear strength according to ACI 318-08 and EN 1992-1-1 ($f_{ck} = 30N/mm^2$, $f_{yk} = 414N/mm^2$, $c.d = 1.0$, $l/d = 25$), Figure by Guandalini et al 2009).

The influence from flexural reinforcement ratio is not accounted by ACI 318-08 but it is considered by EC2 and MC90 as a function of $\sqrt[3]{100\rho}$ but in a complex function in CSCT which is accounted within a plastic moment of resistance at a yield line where $m_R = \rho f_y d^2 \left[1 - \frac{\rho}{2}\frac{f_y}{f_c}\right]$.

The influence of the percentage of this ratio is observed by some researchers, Kinnunen and Nylander (Kinnunen et al. 1960) tested numbers of slabs found the increasing of reinforcement ratio from 0.8% to 2.1% causes in increasing of punching shear strength by 95%. Also, Marzouk and Hussain (Marzouk et al.1991) found 63% of punching shear strength increasing when the ratio raised from 0.6% to 2.4%

(c) Size effect

Size effect is one of the parameters which accounted by researchers and most of codes of practice and concluded that the nominal shear stress in flat slabs decreases with increasing thickness of the slab. This factor had been recognized first by Graf (Graf 1939) who observed that the shear capacity of a 500mm tick slab is about half the punching shear strength of a 150mm thick slab. Regan and Braestrup (Regan et al.1985) estimated the reduction in shear strength by a function of $\sqrt[3]{1/d}$. Tests by Li (Li 2000) (200,300 and 500mm thick), Urban et al.(Urban et al.2013) (218,268 and 318 mm thick) and Birkle (Birkle 2004) on thicker slabs were to investigate the seize effect showed the tendency of decreasing the punching shear strength when the thickness of slabs increased. However tests for the first two authors had short shear span and possibly influencing the failure surface and thus the failure load.

ACI code does not recognize the size effect influence but it states that. But EC2 and CEB-FIP MC90 consider the influence of size effect to be estimated by $k = 1 + \sqrt{200/d}$ in which k is limited to the maximum of 2.0 by EC2 but not by CEB-FIP MC90. This limitation is to reduce the increase in shear strength when slab thickness is less than 200mm. However, there are a few experimental tests on punching shear strength in thick flat slabs support in understanding the decreasing in shear resistance when the thickness of slabs is increased but the recommendations by codes of practice seem to be taken from relatively small thickness slabs and beams or solid slabs.

Fig.(7) shows a parametric shear strength predictions for different thickness of slabs by numbers of codes of practice against the ratio of shear strength for a specified thickness to shear strength for 200mm thickness.

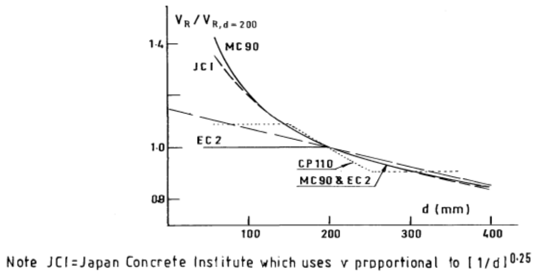


Fig.7 Parametric shear strength predictions for different slab thickness by some codes of practice

(d) Aggregate size and type

The influence from aggregate size and types come from the thought that when cracks occur in a web there is amount of shear cannot be transferred between the cracked sides of concrete. So, the idea of larger aggregate can sustain larger load before cracks gives a higher shear capacity of the section. **S**hioya et al.(Shioya 1989) conducted an extensive experimental work on beams. They found that there is a reduction in shear capacity at failure when the member depth increased and maximum aggregate size decreased. ammadi and Regan (Hamadi et al. 1980) tested beams with normal and lightweight concrete and they found the importance of roughness and size of aggregate in the interlock strength. Muttoni et al. (Muttoni 2008) used the conclusion in (Regan et al. 1985) and (Sherif et al.1986) in his paper, for their accounts for the size of aggregate in the critical cracks in CSCT. As the transferring of shear forces in the critical cracks is linked to the roughness of the concrete. Therefore a factor of $\psi\, d/(d_{g0} + d_g)$ is included into their shear strength formula. Where d_g the maximum is aggregate size and d_{g0} is a reference equal to 16mm.

Parameters to be considered in the comparison

In order to carry out reasonable parametric comparisons between the prediction of CSCT and EC2, some parameters have been considered for a point load application on a slab supported on a 300x300mm square column. The parameters are as following:

- Column size, which can be represented by u_o/d and thence by $u_1/d = u_o/d + 4\pi d$ and $b_o/d = u_o/d + \pi d$ (except where, for a rectangular column, a side length exceeds 3d).

Table 1 The ratio of V_{CSCT}/V_{EC2} for the given data and $f_y = 400N/mm^2$

$f_y = 400N/mm^2$	$r_s = 8d, r_s = r_q$			$r_s = 12d, r_s = r_q$			$r_s = 8d, r_s = 1.25r_q$			$r_s = 12d, r_s = 1.25r_q$		
$d = 500mm$, d_g =0 ρ as given	0.005	0.01	0.02	0.005	0.01	0.02	0.005	0.01	0.02	0.005	0.01	0.02
$f_{ck}(N/mm^2)$												
20	0.53	0.47	0.37	0.46	0.47	0.37	0.46	0.47	0.37	0.52	0.47	0.37
40	0.51	0.52	0.42	0.43	0.48	0.42	0.43	0.48	0.42	0.49	0.52	0.42
60	0.48	0.54	0.45	0.41	0.47	0.44	0.56	0.56	0.45	0.47	0.53	0.45
90	0.46	0.54	0.48	0.39	0.45	0.48	0.53	0.59	0.48	0.44	0.52	0.48
$d = 500mm$, d_g =10												
20	0.63	0.64	0.53	0.53	0.59	0.51	0.53	0.59	0.51	0.61	0.64	0.51
40	0.59	0.66	0.57	0.50	0.58	0.57	0.50	0.58	0.57	0.57	0.64	0.57
60	0.57	0.65	0.61	0.49	0.56	0.60	0.65	0.72	0.61	0.55	0.63	0.61
90	0.55	0.63	0.65	0.47	0.54	0.53	0.62	0.71	0.60	0.53	0.61	0.65
$d = 500mm$, d_g =20												
20	0.69	0.74	0.61	0.60	0.65	0.61	0.60	0.65	0.61	0.67	0.73	0.61
40	0.66	0.73	0.68	0.57	0.64	0.66	0.57	0.64	0.66	0.64	0.71	0.68
60	0.64	0.72	0.73	0.55	0.62	0.67	0.72	0.81	0.73	0.62	0.70	0.73
90	0.61	0.70	0.74	0.53	0.60	0.53	0.70	0.79	0.78	0.59	0.79	0.78
$d = 500mm$, d_g =30												
20	0.74	0.79	0.68	0.65	0.70	0.68	0.65	0.70	0.68	0.72	0.79	0.68
40	0.71	0.79	0.77	0.62	0.68	0.72	0.62	0.69	0.72	0.69	0.77	0.77
60	0.69	0.77	0.81	0.60	0.67	0.73	0.78	0.86	0.82	0.67	0.75	0.79
90	0.67	0.75	0.80	0.57	0.68	0.53	0.76	0.84	0.88	0.65	0.84	0.88
$d = 250mm$, d_g =0												
20	0.60	0.67	0.61	0.52	0.64	0.57	0.69	0.75	0.61	0.58	0.67	0.61
40	0.57	0.65	0.40	0.50	0.54	0.60	0.65	0.65	0.40	0.70	0.79	0.81
60	0.55	0.63	0.69	0.47	0.53	0.60	0.63	0.71	0.73	0.61	0.69	0.44
90	0.53	0.60	0.68	0.46	0.52	0.55	0.61	0.69	0.76	0.58	0.67	0.75
$d = 250mm$, d_g =10												
20	0.71	0.77	0.77	0.61	0.64	0.69	0.80	0.88	0.99	0.69	0.79	0.75
40	0.68	0.75	0.46	0.58	0.65	0.70	0.79	0.75	0.46	0.65	0.72	0.43
60	0.66	0.74	0.79	0.56	0.62	0.69	0.74	0.83	0.88	0.63	0.71	0.77
90	0.64	0.72	0.76	0.53	0.60	0.65	0.72	0.81	0.88	0.62	0.69	0.76
$d = 250mm$, d_g =20												
20	0.79	0.84	0.84	0.68	0.73	0.75	0.89	0.97	0.99	0.77	0.86	0.82
40	0.75	0.83	0.50	0.65	0.70	0.76	0.86	0.83	0.50	0.74	0.81	0.47
60	0.74	0.85	0.75	0.64	0.69	0.76	0.83	0.92	0.96	0.71	0.79	0.85
90	0.71	0.83	0.85	0.61`	0.67	0.73	0.81	0.89	0.96	0.69	0.89	0.96
$d = 250mm$, d_g =30												
20	0.85	0.90	0.88	0.74	0.79	0.79	0.95	1.03	0.99	0.83	0.93	0.87
40	0.82	0.89	0.53	0.72	0.79	0.81	0.93	0.89	0.53	0.80	0.86	0.53
60	0.81	0.88	0.92	0.68	0.76	0.82	0.91	0.98	1.01	0.77	0.85	0.90
90	0.76	0.86	0.92	0.68	0.74	0.81	0.85	0.96	1.01	0.74	0.96	1.01
$d = 150mm$, d_g =0												
20	0.70	0.76	0.78	0.59	0.65	0.68	0.81	0.91	0.87	0.68	0.79	0.76
40	0.67	0.75	0.80	0.56	0.63	0.68	0.77	0.75	0.80	0.78	0.91	0.74
60	0.65	0.73	0.80	0.53	0.61	0.68	0.74	0.83	0.91	0.71	0.79	1.05
90	0.62	0.71	0.76	0.52	0.59	0.67	0.72	0.81	0.88	0.69	0.78	0.86
$d = 150mm$, d_g =10												
20	0.83	0.89	0.89	0.70	0.76	0.78	0.95	1.05	0.98	0.79	0.87	0.87
40	0.79	0.87	0.91	0.66	0.75	0.80	0.84	0.87	0.91	0.75	0.84	0.68
60	0.76	0.85	0.91	0.65	0.73	0.80	0.88	0.97	1.02	0.74	0.82	0.88

90	0.74	0.83	0.88	0.57	0.70	0.92	0.84	0.95	1.02	0.71	0.81	0.87
$d = 150mm$, d_g =20												
20	0.92	0.97	0.96	0.78	0.84	0.85	1.05	1.14	1.05	0.89	0.94	0.93
40	0.88	0.96	0.99	0.73	0.83	0.89	1.01	0.96	0.99	0.85	0.92	0.75
60	0.86	0.94	1.00	0.65	0.83	0.87	0.98	1.07	1.10	0.82	0.93	0.96
90	0.79	0.93	0.97	0.57	0.79	0.87	0.96	1.05	1.11	0.73	1.05	1.11
$d = 150mm$, d_g =30												
20	0.99	1.04	1.01	0.85	0.85	0.91	1.13	1.18	1.10	0.95	1.01	0.99
40	0.96	1.03	1.05	0.73	0.89	0.89	1.10	1.03	1.05	0.92	1.00	1.05
60	0.90	1.02	1.06	0.65	0.87	0.93	1.06	1.14	1.17	0.83	0.98	1.03
90	0.79	1.02	1.03	0.57	0.85	0.92	1.02	1.13	1.17	0.73	1.13	1.17

Table 2 The ratio of V_{CSCT}/V_{EC2} for the given data and $f_y = 500N/mm^2$

$f_y = 500N/mm^2$	$r_s = 8d, r_s = r_q$			$r_s = 12d, r_s = r_q$			$r_s = 8d, r_s = 1.25r_q$			$r_s = 12d, r_s = 1.25r_q$		
$d = 500mm$, d_g =0 ρ as given	0.005	0.01	0.02	0.005	0.01	0.02	0.005	0.01	0.02	0.005	0.01	0.02
$f_{ck}(N/mm^2)$												
20	0.55	0.47	0.37	0.51	0.47	0.37	0.59	0.47	0.37	0.53	0.47	0.37
40	0.52	0.52	0.42	0.45	0.50	0.42	0.59	0.52	0.42	0.64	0.66	0.52
60	0.50	0.56	0.45	0.42	0.49	0.45	0.59	0.56	0.45	0.56	0.63	0.51
90	0.48	0.55	0.48	0.41	0.48	0.48	0.56	0.60	0.48	0.53	0.61	0.54
$d = 500mm$, d_g =10												
20	0.65	0.64	0.51	0.60	0.64	0.51	0.72	0.64	0.51	0.63	0.64	0.51
40	0.61	0.68	0.57	0.53	0.60	0.57	0.70	0.72	0.57	0.59	0.66	0.57
60	0.59	0.67	0.61	0.51	0.58	0.61	0.67	0.74	0.61	0.58	0.65	0.61
90	0.57	0.65	0.65	0.48	0.56	0.65	0.64	0.73	0.65	0.55	0.63	0.65
$d = 500mm$, d_g =20												
20	0.72	0.75	0.61	0.66	0.71	0.61	0.80	0.76	0.61	0.70	0.74	0.61
40	0.68	0.75	0.68	0.59	0.66	0.68	0.77	0.83	0.68	0.67	0.73	0.68
60	0.66	0.74	0.73	0.57	0.64	0.69	0.74	0.83	0.73	0.64	0.72	0.73
90	0.64	0.72	0.78	0.55	0.62	0.78	0.73	0.81	0.78	0.62	0.69	0.78
$d = 500mm$, d_g =30												
20	0.77	0.81	0.68	0.72	0.77	0.68	0.87	0.86	0.68	0.75	0.79	0.68
40	0.74	0.81	0.77	0.64	0.71	0.75	0.83	0.90	0.77	0.72	0.79	0.77
60	0.72	0.80	0.80	0.62	0.70	0.74	0.81	0.89	0.82	0.70	0.77	0.82
90	0.69	0.77	0.88	0.59	0.68	0.88	0.79	0.87	0.88	0.68	0.75	0.88
$d = 250mm$, d_g =0												
20	0.62	0.68	0.61	0.58	0.63	0.61	0.72	0.75	0.61	0.60	0.66	0.61
40	0.50	0.67	0.39	0.50	0.56	0.37	0.68	0.75	0.39	0.57	0.65	0.39
60	0.57	0.65	0.70	0.49	0.55	0.61	0.66	0.74	0.73	0.55	0.62	0.69
90	0.55	0.62	0.78	0.46	0.54	0.78	0.63	0.72	0.78	0.53	0.61	0.59
$d = 250mm$, d_g =10												
20	0.73	0.79	0.77	0.68	0.73	0.73	0.80	0.88	0.77	0.71	0.77	0.75
40	0.70	0.77	0.48	0.60	0.67	0.42	0.80	0.87	0.54	0.68	0.75	0.63
60	0.68	0.76	0.81	0.58	0.65	0.71	0.77	0.85	0.89	0.66	0.73	0.79
90	0.66	0.74	0.98	0.56	0.52	0.98	0.75	0.83	0.98	0.64	0.72	0.98
$d = 250mm$, d_g =20												
20	0.81	0.86	0.84	0.76	0.78	0.79	0.92	0.96	0.87	0.79	0.83	0.82
40	0.78	0.85	0.52	0.60	0.62	0.36	0.89	0.95	0.54	0.75	0.83	0.70
60	0.82	0.84	0.89	0.59	0.64	0.69	0.86	0.94	0.98	0.74	0.81	0.86
90	0.74	0.82	1.11	0.57	0.63	0.67	0.84	0.92	1.11	0.71	0.80	1.11
$d = 250mm$, d_g =30												
20	0.88	0.91	0.89	0.83	0.86	0.84	0.98	1.01	0.94	0.85	0.89	0.87

40	0.85	0.91	0.55	0.66	0.68	0.39	0.96	1.01	0.59	0.83	0.89	0.53
60	0.83	0.91	0.94	0.64	0.70	0.74	0.93	1.01	1.03	0.80	0.88	0.92
90	0.80	0.89	1.20	0.62	0.69	0.72	0.91	1.00	1.20	0.77	0.86	1.20
$d=150mm, d_g$ =0												
20	0.73	0.78	0.79	0.66	0.72	0.73	0.83	0.90	0.87	0.70	0.75	0.76
40	0.69	0.77	0.84	0.58	0.65	0.72	0.80	0.87	0.91	0.66	0.74	0.80
60	0.67	0.75	0.82	0.56	0.64	0.70	0.77	0.86	0.94	0.64	0.71	0.79
90	0.65	0.73	1.07	0.54	0.61	0.93	0.74	0.84	1.12	0.62	0.70	1.03
$d=150mm, d_g$ =10												
20	0.85	0.91	0.90	0.78	0.85	0.83	0.97	1.01	0.98	0.82	0.87	0.87
40	0.82	0.90	0.95	0.69	0.76	0.83	0.93	1.01	1.03	0.79	0.87	0.90
60	0.79	0.87	0.93	0.67	0.75	0.80	0.92	1.00	1.05	0.76	0.84	0.90
90	0.77	0.86	1.20	0.61	0.73	1.09	0.89	0.98	1.31	0.74	0.83	1.17
$d=150mm, d_g$ =20												
20	0.95	0.99	0.97	0.86	0.91	0.90	1.07	1.10	1.05	0.89	0.96	0.94
40	0.91	0.99	1.07	0.77	0.85	0.90	1.04	1.10	1.12	0.87	0.95	0.98
60	0.96	0.97	1.02	0.70	0.84	0.88	1.02	1.10	1.13	0.85	0.94	0.99
90	0.85	0.96	1.29	0.61	0.83	1.15	0.98	1.08	1.93	0.78	0.92	1.26
$d=150mm, d_g$ =30												
20	1.02	1.06	1.01	0.91	0.98	0.96	1.15	1.15	1.11	0.99	1.02	0.99
40	0.99	1.06	1.09	0.78	0.92	0.96	1.13	1.18	1.17	0.95	1.03	1.05
60	0.96	1.05	1.09	0.70	0.90	0.94	1.10	1.18	1.19	0.89	1.01	1.05
90	0.85	1.03	1.35	0.61	0.88	1.22	1.06	1.16	2.05	0.78	0.99	1.32

Table 3 The ratio of V_{CSCT}/V_{EC2} for the given data and $f_y = 600N/mm^2$

$f_y = 600N/mm^2$	$r_s = 8d, r_s = r_q$			$r_s = 12d, r_s = r_q$			$r_s = 8d, r_s = 1.25r_q$			$r_s = 12d, r_s = 1.25r_q$		
$d=500mm, d_g$ =0 ρ as given	0.005	0.01	0.02	0.005	0.01	0.02	0.005	0.01	0.02	0.005	0.01	0.02
$f_{ck}(N/mm^2)$												
20	0.56	0.47	0.37	0.55	0.47	0.37	0.59	0.47	0.37	0.55	0.47	0.37
40	0.54	0.52	0.33	0.53	0.52	0.42	0.61	0.52	0.37	0.66	0.66	0.52
60	0.52	0.56	0.45	0.51	0.56	0.45	0.59	0.56	0.45	0.58	0.64	0.51
90	0.50	0.56	0.48	0.49	0.56	0.48	0.57	0.60	0.48	0.55	0.63	0.54
$d=500mm, d_g$ =10												
20	0.66	0.64	0.51	0.66	0.64	0.51	0.74	0.64	0.51	0.65	0.64	0.51
40	0.63	0.69	0.57	0.62	0.68	0.57	0.71	0.72	0.57	0.61	0.67	0.57
60	0.60	0.69	0.61	0.59	0.68	0.61	0.69	0.76	0.61	0.59	0.67	0.61
90	0.59	0.67	0.65	0.58	0.67	0.65	0.66	0.75	0.65	0.57	0.65	0.65
$d=500mm, d_g$ =20												
20	0.73	0.76	0.61	0.73	0.75	0.61	0.82	0.76	0.61	0.71	0.75	0.61
40	0.70	0.77	0.68	0.69	0.76	0.48	0.79	0.84	0.68	0.68	0.75	0.68
60	0.68	0.77	0.73	0.68	0.76	0.59	0.77	0.85	0.73	0.66	0.74	0.73
90	0.66	0.74	0.78	0.65	0.74	0.62	0.75	0.81	0.78	0.64	0.72	0.78
$d=500mm, d_g$ =30												
20	0.79	0.82	0.68	0.78	0.82	0.68	0.88	0.86	0.68	0.77	0.81	0.68
40	0.76	0.83	0.77	0.76	0.82	0.77	0.85	0.92	0.77	0.74	0.81	0.77
60	0.74	0.82	0.82	0.73	0.81	0.78	0.83	0.93	0.82	0.71	0.80	0.82
90	0.65	0.79	0.88	0.71	0.79	0.88	0.81	0.90	0.88	0.70	0.78	0.88
$d=250mm, d_g$ =0												
20	0.64	0.70	0.61	0.62	0.68	0.61	0.73	0.76	0.61	0.62	0.67	0.61
40	0.61	0.68	0.39	0.60	0.67	0.38	0.70	0.77	0.30	0.75	0.84	0.88
60	0.59	0.66	0.71	0.58	0.66	0.66	0.68	0.76	0.73	0.65	0.73	0.46
90	0.57	0.65	0.72	0.56	0.63	0.78	0.65	0.74	0.78	0.63	0.72	0.89

$d = 250mm, d_g$ =10												
20	0.75	0.80	0.77	0.74	0.78	0.73	0.84	0.89	0.77	0.73	0.77	0.75
40	0.72	0.79	0.48	0.71	0.79	0.45	0.82	0.89	0.50	0.70	0.76	0.46
60	0.70	0.77	0.83	0.69	0.77	0.79	0.79	0.88	0.91	0.68	0.76	0.80
90	0.76	0.85	0.98	0.67	0.76	0.98	0.77	0.85	0.98	0.66	0.74	0.98
$d = 250mm, d_g$ =20												
20	0.83	0.87	0.84	0.82	0.86	0.86	0.94	0.97	0.87	0.80	0.85	0.82
40	0.81	0.87	0.52	0.79	0.86	0.40	0.91	0.97	0.57	0.78	0.85	0.50
60	0.78	0.86	0.90	0.77	0.85	0.75	0.89	0.96	0.99	0.76	0.83	0.88
90	0.76	0.84	1.11	0.75	0.82	0.76	0.86	0.95	1.11	0.74	0.67	1.11
$d = 250mm, d_g$ =30												
20	0.90	0.93	0.89	1.01	0.91	0.90	1.00	1.02	0.94	0.95	0.90	0.87
40	0.87	0.93	0.55	0.86	0.92	0.53	0.98	1.04	0.59	0.85	0.91	0.53
60	0.85	0.93	0.96	0.84	0.91	0.93	0.96	1.03	1.05	0.83	0.90	0.94
90	0.76	0.91	1.20	0.81	0.90	1.20	0.93	1.01	1.20	0.80	0.88	1.20
$d = 150mm, d_g$ =0												
20	0.74	0.80	0.63	0.72	0.76	0.73	0.85	0.91	0.88	0.71	0.76	0.76
40	0.71	0.78	0.84	0.68	0.77	0.80	0.82	0.90	0.93	0.86	0.95	1.01
60	0.70	0.77	0.83	0.67	0.75	0.80	0.79	0.88	0.95	0.76	0.84	0.93
90	0.67	0.75	1.12	0.65	0.73	1.03	0.77	0.86	1.12	0.74	0.83	1.28
$d = 150mm, d_g$ =10												
20	0.87	0.91	0.73	0.83	0.89	0.83	0.99	1.02	0.98	0.84	0.89	0.87
40	0.84	0.91	0.95	0.82	0.89	0.92	0.96	1.04	1.04	0.81	0.88	0.91
60	0.82	0.90	0.95	0.79	0.88	0.93	0.94	1.03	1.06	0.78	0.87	0.92
90	0.79	0.88	1.33	0.77	0.86	1.21	0.92	1.01	1.33	0.76	0.85	1.32
$d = 150mm, d_g$ =20												
20	0.97	1.00	0.80	0.94	0.98	0.89	1.09	1.12	1.05	0.93	0.97	0.94
40	0.94	1.01	1.02	0.89	0.98	1.00	1.07	1.13	1.12	0.90	0.97	0.99
60	0.91	0.99	1.04	0.88	0.97	1.01	1.04	1.12	1.15	0.88	0.96	1.01
90	0.89	0.99	1.41	0.85	0.95	1.32	1.02	1.12	1.45	0.83	0.95	1.41
$d = 150mm, d_g$ =30												
20	1.04	1.07	0.85	1.02	1.05	0.96	1.17	1.18	1.10	1.00	1.03	0.99
40	1.01	1.08	1.08	0.99	1.04	1.06	1.15	1.20	1.19	0.97	1.05	1.05
60	0.99	1.08	1.10	0.96	1.05	1.07	1.12	1.20	1.21	0.94	1.04	1.07
90	1.39	1.06	1.50	0.85	1.03	1.39	1.10	1.19	1.53	1.02	1.02	1.47

Table 4 The ratio of V_{CSCT}/V_{EC2} for the given data and $f_y = 800N/mm^2$

$f_y = 800N/mm^2$	$r_s = 8d, r_s = r_q$			$r_s = 12d, r_s = r_q$			$r_s = 8d, r_s = 1.25r_q$			$r_s = 12d, r_s = 1.25r_q$		
$d = 500mm, d_g$ =0	0.005	0.01	0.02	0.005	0.01	0.02	0.005	0.01	0.02	0.005	0.01	0.02
$f_{ck}(N/mm^2)$ ρ as given												
20	0.58	0.47	0.37	0.59	0.47	0.37	0.59	0.47	0.37	0.56	0.47	0.37
40	0.57	0.52	0.42	0.61	0.52	0.42	0.63	0.52	0.42	0.55	0.52	0.42
60	0.55	0.56	0.45	0.60	0.56	0.45	0.62	0.56	0.45	0.53	0.56	0.45
90	0.53	0.59	0.48	0.58	0.60	0.48	0.60	0.60	0.50	0.51	0.57	0.48
$d = 500mm, d_g$ =10												
20	0.68	0.64	0.51	0.74	0.64	0.51	0.76	0.64	0.51	0.67	0.64	0.51
40	0.66	0.71	0.57	0.73	0.72	0.57	0.75	0.72	0.57	0.64	0.69	0.57
60	0.64	0.71	0.61	0.70	0.76	0.61	0.73	0.77	0.61	0.62	0.69	0.61
90	0.61	0.70	0.65	0.67	0.76	0.65	0.70	0.78	0.65	0.60	0.68	0.65
$d = 500mm, d_g$ =20												
20	0.76	0.76	0.61	0.82	0.76	0.61	0.85	0.76	0.61	0.74	0.75	0.61
40	0.73	0.79	0.68	0.80	0.83	0.68	0.83	0.86	0.68	0.71	0.78	0.68

60	0.71	0.79	0.73	0.78	0.85	0.73	0.80	0.87	0.73	0.69	0.77	0.73
90	0.69	0.78	0.78	0.76	0.84	0.78	0.78	0.87	0.78	0.67	0.75	0.78
$d = 500mm$, d_g =30												
20	0.82	0.83	0.68	0.89	0.86	0.68	0.91	0.86	0.68	0.79	0.81	0.68
40	0.79	0.96	0.77	0.85	0.91	0.77	0.89	0.93	0.77	0.77	0.83	0.77
60	0.78	0.85	0.82	0.84	0.92	0.81	0.87	0.93	0.82	0.75	0.83	0.82
90	0.75	0.83	0.88	0.82	0.91	0.88	0.85	0.93	0.88	0.76	0.81	0.88
$d = 250mm$, d_g =0												
20	0.66	0.70	0.61	0.72	0.74	0.61	0.75	0.76	0.49	0.64	0.68	0.61
40	0.64	0.71	0.39	0.69	0.76	0.39	0.73	0.79	0.39	0.78	0.87	0.87
60	0.62	0.70	0.73	0.68	0.75	0.70	0.71	0.79	0.73	0.68	0.77	0.47
90	0.60	0.68	0.78	0.65	0.74	0.78	0.57	0.77	0.78	0.66	0.74	0.89
$d = 250mm$, d_g =10												
20	0.78	0.81	0.75	0.83	0.87	0.77	0.87	0.90	0.57	0.75	0.79	0.74
40	0.75	0.82	0.50	0.82	0.88	0.47	0.86	0.92	0.50	0.73	0.79	0.47
60	0.73	0.81	0.85	0.79	0.87	0.83	0.83	0.91	0.92	0.71	0.78	0.83
90	0.71	0.79	0.98	0.77	0.85	0.98	0.81	0.89	0.98	0.68	0.77	0.98
$d = 250mm$, d_g =20												
20	0.86	0.88	0.82	0.93	0.94	0.85	0.97	0.98	0.87	0.83	0.86	0.80
40	0.84	0.89	0.56	0.90	0.96	0.52	0.95	1.00	0.56	0.81	0.87	0.51
60	0.82	0.89	0.92	0.88	0.96	0.91	0.93	1.00	1.01	0.79	0.87	0.96
90	0.80	0.88	1.12	0.86	0.95	1.11	0.90	0.98	1.11	0.77	0.85	1.11
$d = 250mm$, d_g =30												
20	0.92	0.94	0.87	0.99	1.00	0.94	1.04	1.04	0.79	0.90	0.91	0.85
40	0.91	0.96	0.59	0.98	1.02	0.56	1.02	1.06	0.60	0.88	0.93	0.54
60	0.89	0.96	0.98	0.95	1.03	0.98	1.00	1.06	1.07	0.86	0.93	1.01
90	0.87	0.95	1.20	0.94	1.01	1.20	0.98	1.06	1.20	0.85	0.92	1.01
$d = 150mm$, d_g =0												
20	0.77	0.81	0.76	0.82	0.85	0.81	0.88	0.91	0.85	0.73	0.78	0.75
40	0.75	0.82	0.84	0.79	0.85	0.84	0.85	0.92	0.94	0.89	0.99	1.02
60	0.72	0.81	0.86	0.77	0.86	0.86	0.83	0.91	0.98	0.79	0.88	0.96
90	0.70	0.79	1.12	0.75	0.84	1.11	0.80	0.90	1.12	0.77	0.86	1.28
$d = 150mm$, d_g =10												
20	0.90	0.93	0.87	0.96	0.98	0.92	1.02	1.04	0.96	0.86	0.89	0.84
40	0.87	0.94	1.07	0.93	1.00	0.98	1.00	1.06	1.06	0.85	0.88	0.93
60	0.86	0.93	0.97	0.92	1.01	0.99	0.98	1.06	1.09	0.82	0.90	0.94
90	0.83	0.92	1.33	0.89	0.98	1.30	0.96	1.04	1.33	0.79	0.88	1.33
$d = 150mm$, d_g =20												
20	1.00	1.02	0.94	1.07	1.07	0.99	1.12	1.13	1.03	0.97	0.98	0.91
40	0.97	1.04	1.16	1.04	1.09	1.07	1.11	1.16	1.14	0.94	1.00	1.01
60	0.95	1.03	1.06	1.02	1.10	1.08	1.09	1.16	1.17	0.92	1.00	1.10
90	0.93	1.01	1.45	0.99	1.10	1.41	1.06	1.15	1.45	0.89	0.99	1.45
$d = 150mm$, d_g =30												
20	1.07	1.09	0.99	1.14	1.14	1.04	1.21	1.20	0.91	1.04	1.05	0.97
40	1.05	1.12	1.23	1.12	1.17	1.13	1.20	1.24	1.19	1.02	1.08	1.07
60	1.04	1.11	1.12	1.10	1.17	1.14	1.17	1.24	1.23	0.99	1.07	1.17
90	1.26	1.33	1.51	1.07	1.16	1.49	1.15	1.23	1.53	1.02	1.06	1.53

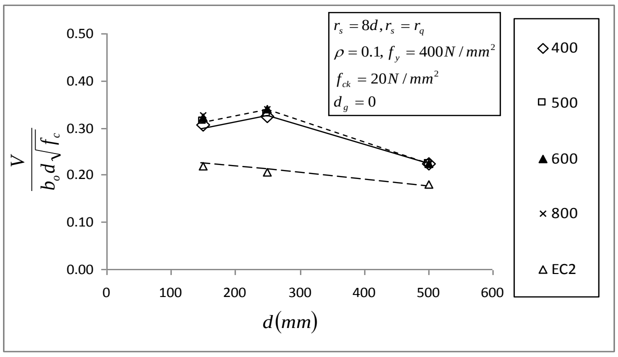
- Ratio of reinforcement: it's probably worth looking at two cases, one in which ρ is constant throughout the slab, and another where ρ in the central half of the width is $4/3\rho_{avg}$ and ρ in the other quarters is $2/3\rho_{avg}$. $2/3\rho_{avg}$ considering 0.005, 0.01, 0.02.
- Concrete strength from, considering $f_{ck} = 20MPa$ to $f_{ck} = 90MPa$
- Slab effective depth *d*, considering 150, 250 and 500 mm.
- Maximum size of aggregate, considering d_g = 0, 10, 20 and 32 mm.
- Slab diameter r_s represented by r_s / d, considering 8 and 12.
- Ratio r_q / r_s, considering1.0 and 1.25.
- f_y, considering 400, 500, 600 and 800 MPa.

The shear strength predictions by CSCT and EC2 have been calculated and the ratio of V_{CSCT}/V_{EC2} is shown in

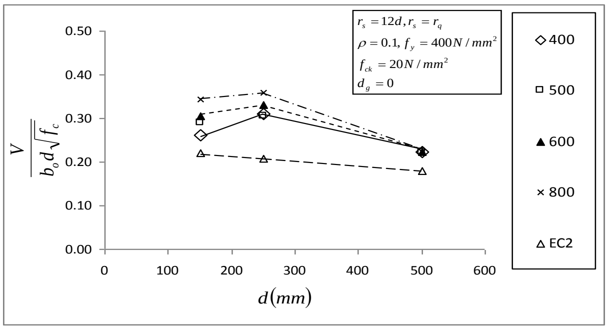
Table 1 to 4 for $f_y = 400N/mm^2 - 800N/mm^2$ respectively. Also, the results for $f_y = 400N/mm^2$ are plotted and shown in Figs.9-12. They demonstrate the influence of the slab thickness, concrete strength, steel strength and the stiffness of the slab on the shear strength in terms of $V/b_o d\sqrt{f_c}$.

Fig.(9) shows the influence of the effective thickness of slab influences on punching shear . CSCT allows the increasing of shear strength with increasing of the thickness up to 250mm in contrast of EC2 which its increasing is precisely limited to 200mm. The results agree with the conclusions by Regan et al.(Regan et al. 1985), Li(Li 2000), Urban et al.(Urban et al2013) and Birkle (Birkle 2004)when the seize effect showed the tendency of decreasing the punching shear strength when the thickness of slabs increased.

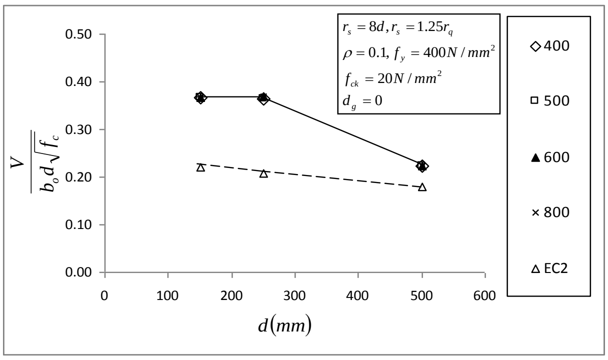
The increasing of the flexural reinforcement shows an increase of the shear strength in both methods and CSCT predictions are higher than those by EC2 in a small differences at low reinforcement as shown in Fig.(10 d).



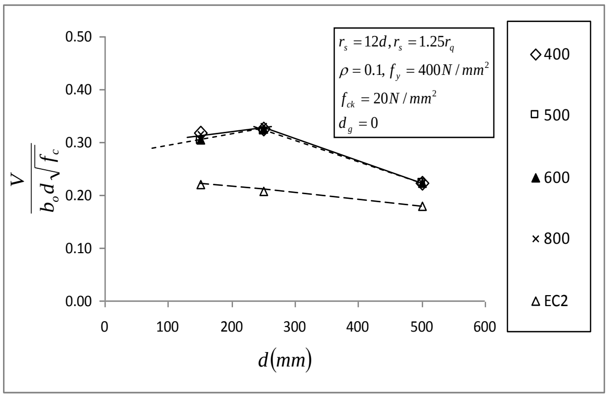
(a)



(b)



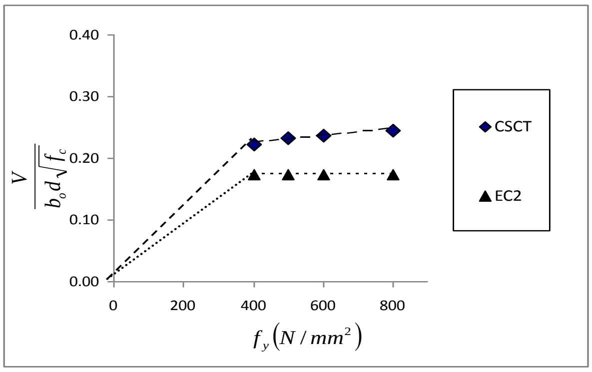
(c)



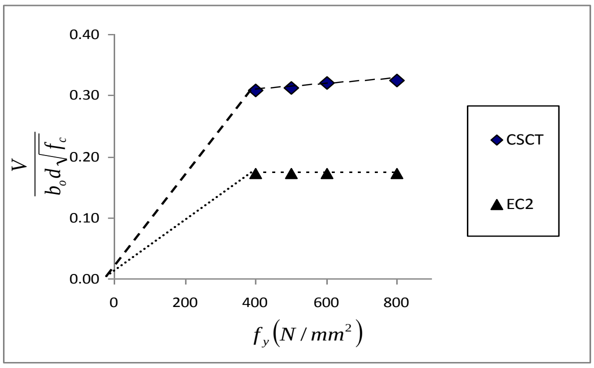
(d)

Fig. 9 Comparison of punching shear strength according to EC2 and CSCT showing the thickness of slab

This is due to the consideration of the slab rotation in CSCT solution where an increase in flexural reinforcement reduces the slab rotation. A slight increase with increasing yield strength of reinforcement is predicted by CSCT in shear strength for all ρ values as shown in Fig.(10 a-c) The influence of aggregate size is obvious as the shear capacity increases with increasing of aggregate size and this agree with Shioya et al.(Shioya 1989) conclusions .The influence is more crucial for all steel strength where the aggregate size are 20 and 30mm and the predictions by both methods are very close to each other.



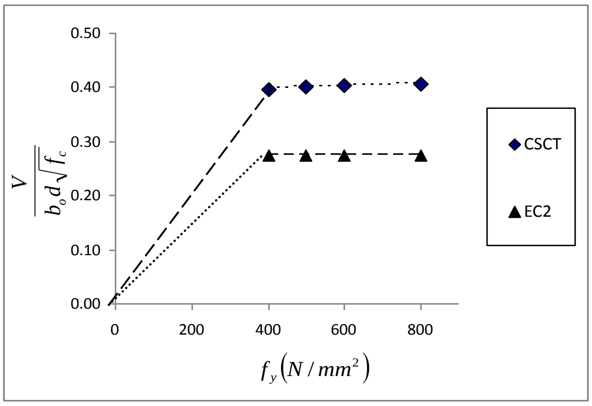
(a) $\rho = 0.005$



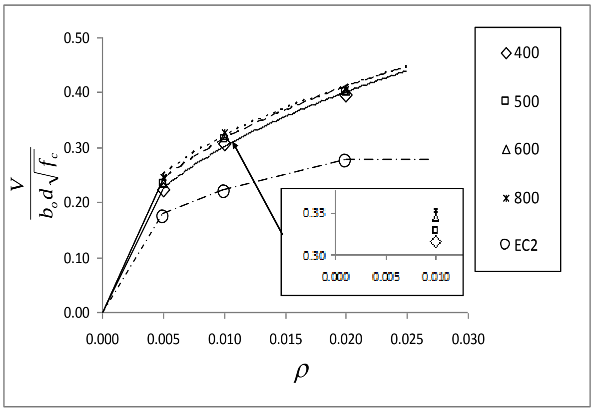
(b) $\rho = 0.01$

The influence of slenderness of the slab on punching is represented by the ratio of r_s/d s shown in Fig.(11). This effect is not considered by EC2 but according to CSCT the slender slab shows lower shear strength.

The influence of compressive strength in Fig.(12)shows a trend of decreasing the shear strength with increasing the compressive strength for both methods. Also, there are higher shear strength for thinner slabs in all compressive strength.

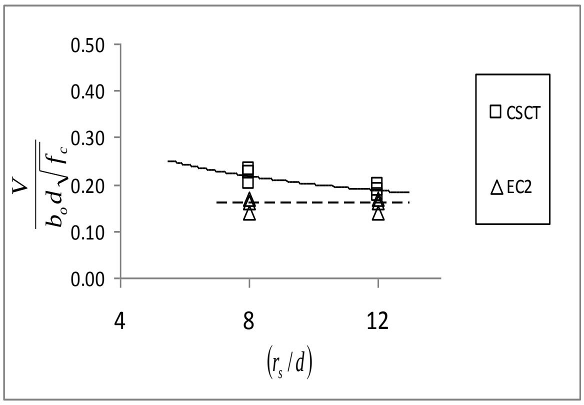


(c) $\rho = 0.02$

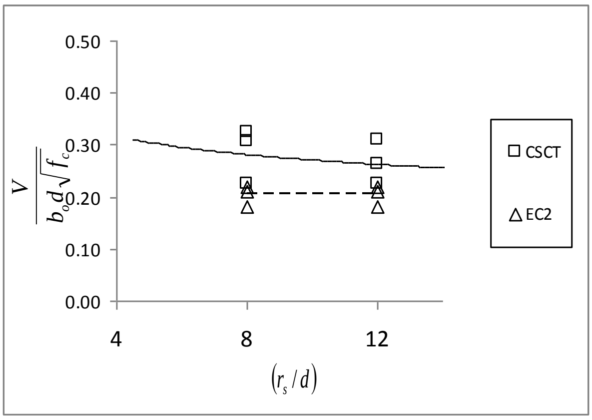


(d) $r_s = 8d, r_s = r_q, d = 150mm \,\&\, d_g = 0$

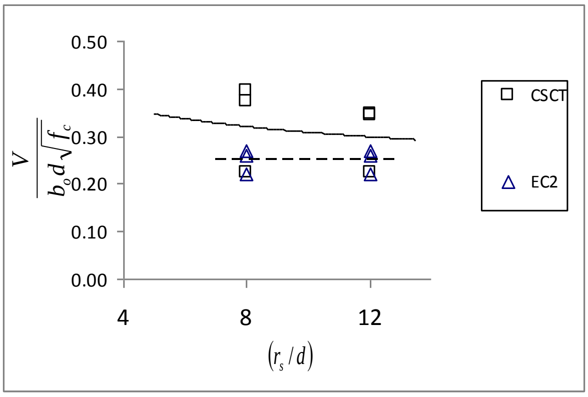
Fig. 10 Comparison of punching shear strength according to EC2 and CSCT showing the yield strength of $r_s = 8d, r_s = r_q, f_c = 20MPa, \rho = 0.01, d = 150mm \,\&\, d_g = 0$ (except fig.d)



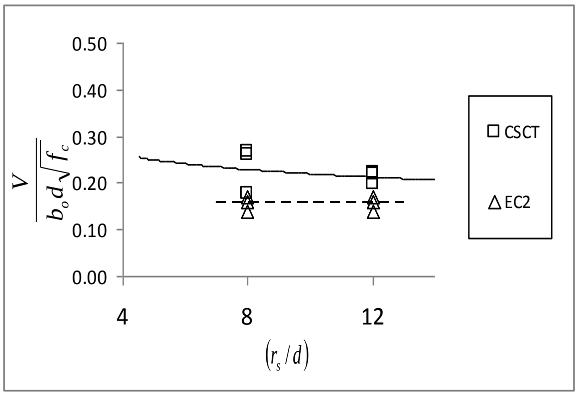
(a) $\rho = 0.005, r_s = r_q$



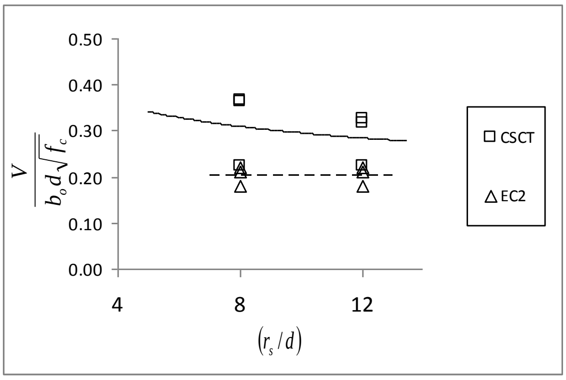
(b) $\rho = 0.01, r_s = r_q$



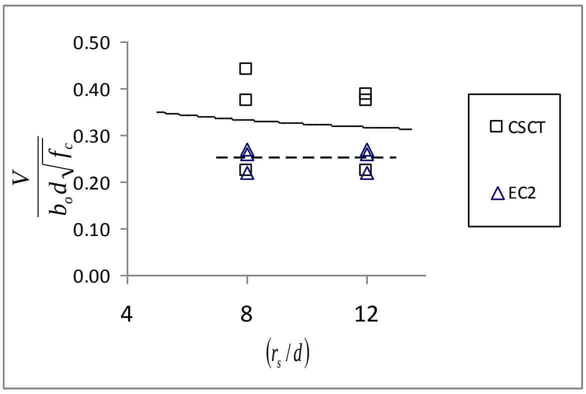
(c) $\rho = 0.02, r_s = r_q$



(d) $\rho = 0.005, r_s = 1.25 r_q$

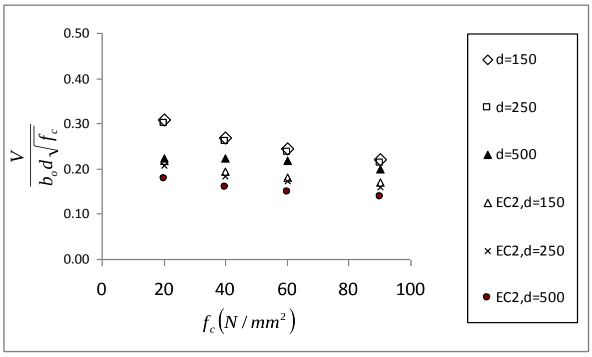


(e) $\rho = 0.01, r_s = 1.25 r_q$

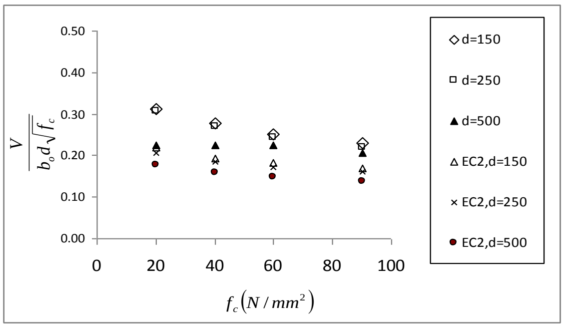


(f) $\rho = 0.02, r_s = 1.25 r_q$

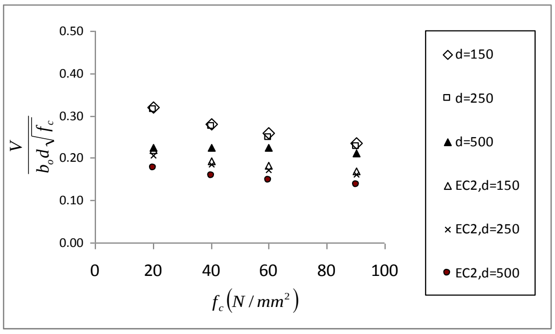
Fig. 11 Comparison of punching shear strength according to EC2 and CSCT showing the slenderness of slab with $f_c = 20MPa, f_y = 400MPa, d_g = 0$



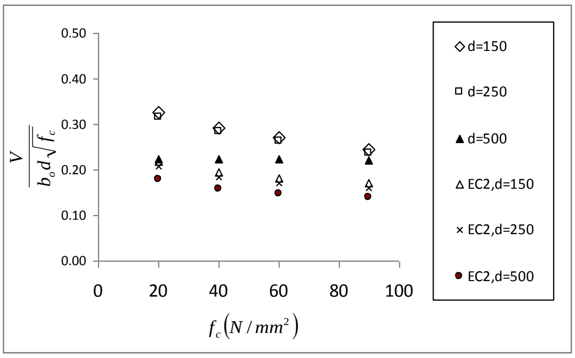
(a) $f_y = 400MPa$



(b) $f_y = 500MPa$



(c) $f_y = 600MPa$



(d) $f_y = 800MPa$

Fig. 12 Comparison of punching shear strength according to EC2 and CSCT showing the concrete strength for specimens with $r_s = 8d, r_s = r_q, f_y = 400MPa, \rho = 0.01 \& d_g = 0$

Conclusion

There are some significant conclusions could be drawn from this comparative parametric analysis for CSCT and EC2 equations. The increasing of compressive strength causes in increasing of punching shear capacity in both methods and the rate of increasing in shear capacity is decreases with increasing the compressive strength.

The increasing of slab thickness causes in increasing the shear capacity up to 200mm in EC2 while CSCT showed the increase up to 250mm and then the shear strength decreases as the thickness increased beyond these values. The increasing ratio for punching shear capacity from increasing the flexural reinforcement showed significant effects on increasing the punching shear capacity. The increasing of punching capacity from $\rho =$ 0.5% to 2% by CSCT and EC2 are 1.77 and 1.59 respectively.

The solution of the equations from CSCT is a quiet complex method for design by engineers. This study aided in evaluation of the CSCT in general and particularly to highlight combinations of variables, for which its results deviate significantly from EC2, i.e. the combinations for which test results are critical. The parametric results are more conservative by EC2, the reason is basically due to considering some parameters in CSCT but not in EC2 like aggregate size and rotation of the slab, the limits for the influence of size effect and the influence from the slenderness of slab presented in r_s / d ratio.

The differences between CSCT and EC2 results can justify the need for a new and more complex approach than EC2's to involve the parameters like aggregate size, slab slenderness and slab rotation but to be approved by experimental works.

References

Muttoni A.(2008), Punching shear strength of reinforced concrete slabs without transverse reinforcement, *ACI Structural Journal*, V105 No 4, pp 440-450.

Eurocode 2(2004), Design of concrete structures, Part 1-1, General rules and rules for buildings, *EN1992-1-1*, CEN, Brussels.

Moe J.(1961), Shearing strength of reinforced concrete slabs and footings under concentrated loads, Development Dept. Bulletin D47. *Portland Cement Association*, Skokie, Illinois.

Mitchell D., Cook W.D., and Dilger W.(2005), Effects of size, geometry and material properties on punching shear resistance.*SP-232,Ed.Polak,M.A.,ACI*.Farmington Hill,MI,pp.39- 56

Elstner R. and Hognestad E.(1956), Shearing strength of reinforced concrete slabs. *ACI Journal*.V.28, No.1,pp.29-58.

Regan P.E.(1986), Symmetric punching of reinforced concrete slabs, *Magazine of Concrete Research*, Vol. 38, No 136, pp 115-128

Gardner N.J. and Shao X.(1996), Punching shear of continuous flat reinforced concrete slabs. *ACI Journal*.V.93.No.2,March-April,pp 218-288.

Lantsoght E.(2009), Literature Review of Punching Shear in reinforced concrete slabs. *Report No.09-10,* Georgia Institute of Technology.85p

Sacramento P.V.P., Ferreira M.P., Oliveira D.R.C. and Melo G.S.S.A.(2012), Punching strength of reinforced concrete flat slabs without shear reinforcement, *Revista Ibracon De Estruturas E Materials*, Vol.5, No.5.

Hawkins N.M. and Mitchell D.(1979), Progressive collapse of flat plate structures. *ACI Journal, Proceedings*.Vol.76, No.7, July,pp.775-808

Regan P.E.(1974), Design of punching Shear, *Structural Engineer (London)* .V.52. No.6,pp.197-207

Rankin G.I.B. and Long A.E.(1987), Predicting the punching strength of conventional slab- column specimens. *Proceedings of the Institution of Civil Engineers (London)*.V.82, Part 1, April, pp.327-346.

Shehata I.M. and Regan P.(1989), Punching in reinforced concrete slabs. *ASCE Journal of the Structural Division*.V.115,ST7, pp.1726-1740

Alexander S.D.B and Simmonds S.H.(1992), Tests of column-flat plate connections. *ACI Structural Journal*. Vol.89.No. 5,pp.495-502.

Egan P.E. and Braestrup M.W.(1985), Punching shear in reinforced concrete: A state of Art Report *Bulletin D'information* Comite, *Euro-International de Beton*, No.168. Luzzanne, 232 pp

Sherif A.G. and Dilger W.H.(1989), Critical Review of the CSA A23.3-94 Punching shear strength Provisions for inter Columns. *Canadian Journal of Civil Engineering*.V.23, No.5, pp.998-1101.

Guandalini S.,Burdet O.L. and Muttoni A.(2009), Punching tests of slabs with low reinforcement ratios. *ACI Structural Journal*,pp.87-95.

Kinnunen S. and Nylander H.(1960), Punching of concrete slabs without shear reinforcement. *Transactions of the Royal Institute of Technology*. Stockholm. No.158, 112pp

Marzouk H.M. and Hussein A.(1991), Punching Shear analysis of reinforced High-strength concrete slabs. *Candian Journal of Civil engineering*.V.18, No.6, pp.954-963.

Graf O.(1939), Strength tests of thick reinforced concrete slabs supported on all sides under concentrated loads. *Deutscher Ausschuss fu Eisenbeton*. No.88, Berlin. Germany

Li K.K.L.(2000), Influence of size on punching shear strength of concrete slabs, *M Eng Thesis, Dept. of Civil Engineering and applied Mechanics* .Mc Gill University, Monteral, pp7

Urban T., Goldyn M., Krakowski J. and Krawczyk L.(2013), Experimental investigation on punching shear behaviour of thick reinforced concrete slabs, Archives of Civil Engineering, Lodz University of Technology, LIX, 2.

Birkle G.(2004), Punching of flat slabs, The influence of slab thickness and stud layout, *PhD thesis, Dept. Of Civil Eng*, University of Calgary.

Shioya T.(1989), Shear properties of large reinforced concrete members. Special reports of Institutes of Technology, Shimizu Corporation.No.25

Hamadi Y. D. and Regan P. E.(1980), Behavior of normal and lightweight aggregate beams with shear cracks, The Structural Engineer, Volume 58B/No. 4.