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Research Article

A Critical Review of Researches on Straight Anchorages in Reinforced Concrete Structure

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Abstract

This paper reviews the researches on anchorages with straight end of main bars in reinforced concrete structure. The theoretical approaches by Tepfers, Cairns, Cairns and Jones and Morita and Fujii and the plastic theory by Nielsen have been studied.Also, those empirical equations for calculating the bond strength are reviewed which are from Orangun, Darwin and Btayneh. The anchorages of main reinforcement at simple supports are important in the design of reinforced concrete and the most significant characteristics of are their generally short lengths and the presence of transverse pressure from the support reactions. Published work in this area is rather limited. The only major research is that by Danish authors, working in the field of plasticity and works by Rathkjen, Jensen, Ghaghei and Regan are considered in this review.This review gives comments on types of bond failure, analyses of behaviour adjacent to a bar, analysis of the surrounds to a bar, treatments of the influences of transverse pressure and transverse reinforcements on the anchorage capacity of the main bar in structural concrete members. Two important characteristics of conditions at simple supports are that anchorage lengths are short and that the support reaction produces compressions perpendicular to the plane of the bars. Because of the first of these the treatment of bond strength must take account of the ratio of the anchorage length to the bar diameter. In relation to the second, there is ample evidence showing that transverse pressure can increase bond strength. It is concluded that the type of test specimen and the way in which loads and reactions are applied to it can materially affect the results of bond tests.

Keywords: *Bond stress, bond failure, bond strength, slip, splitting of cover, concrete strength, concrete cover, anchorage length, bar diameter, rib areas ,transverse reinforcement and transverse pressure*

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1. Introduction

The bond resistance is understood to be influenced by the bearing of the ribs on the surrounding concrete which the thickness of the concrete cover to a bar. It is also reasonable to expect influences from transverse reinforcement crossing the surface at which failure occurs and from transverse pressure acting at a support. The bearing of the ribs on the surrounding concrete produces outward radial forces and, for normal ratios of cover to bar size, bond failure involves splitting of the concrete cover.

Bond failure of a ribbed bar involves local movement between the bar and the concrete immediately around it. This can occur either at a surface, which includes the outer faces of wedges of concrete trapped in front of the ribs and continues at the steel/concrete interface (Fig1.a) or at a cylindrical surface defined by the outer perimeters of the ribs (Fig.1.b). In both cases, the longitudinal movement, or slip, is accompanied by a radial movement needed to clear the unevenness in the concrete fracture surfaces. In the case of Fig.(1.a) the rib projections must also be cleared. In both cases, the stresses at the failure surface are a compression and a shear and their resultant is a compression which is inclined to the bar axis with its longitudinal component providing the bond.

The stress field in the concrete outside the local failure surfaces has to maintain equilibrium between the forces at this surface and the wider stress field in the member. The tensile stresses involved can produce splitting, which may cause an immediate failure, if the compression on the local surface is thereby released. In other cases, the remaining uncracked concrete, or transverse reinforcement, or transverse pressure, may provide enough resistance to the outward forces for bond stresses to be increased or redistributed. Bond failure may then occur at a higher load or may be avoided.

If the surrounds provide sufficient resistance, splitting may be avoided, but bond failure may still occur if the deformability of the surrounds is sufficient to accommodate the dilation of failure at the local failure surface.

A view of failure, which partially contradicts the above is presented by Batayneh(Batayneh,1993) and implied by others. According to this there are only two types of failure. In one, the wedge type of local failure surface

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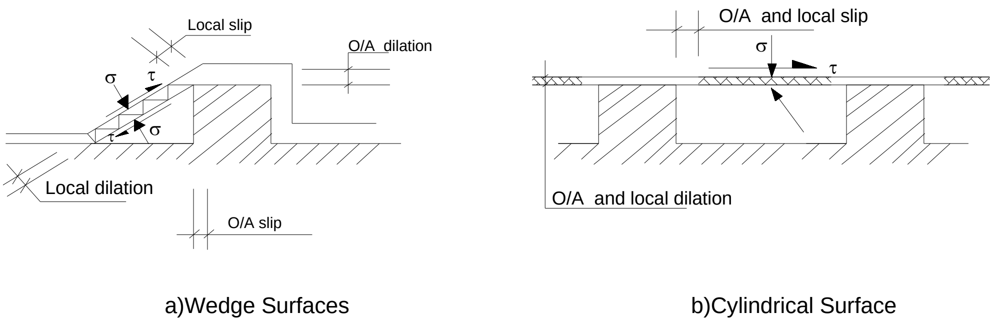


Fig.1 Local movements at failure

produces splitting in the surrounds and the bond capacity increases with increasing resistance in the surrounds. In the other, which occurs when this resistance is above a critical level, there is no splitting, the local failure surface is of the cylindrical type, and the bond capacity is independent of any further increase in restraint.

The above description of the characteristics of bond failure is written without reference to the development, or anchorage length. If the length is considered, there are issues of the distribution of bond along it. In some cases, this can be determined externally, e.g. by the rate of change of main steel force along a beam.In other cases, notably in pull-out situations, and in lap splices, the distribution is a function of the bond-slip relationship and, at least until splitting or cylindrical failure commences, the bond stresses are greatest at the loaded ends of anchorages. For a non-uniform distribution of bond, the average bond stress at failure is practically certain to increase with a reduction of the ratio l_b/φ , until this ratio is small enough for the bond along l_b to be more-or-less uniform.

Another general influence on bond resistance is the "top-bar" effect.It seems clear that, for concrete mixes that exhibit significant settlement after casting, bond strengths are lower for top bars than for bottom bars. High strength concretes exhibit very little settlement and appear to give equal strengths for top and bottom bars. At present however most codes of practice and many papers assume constant ratios of 60% to 70% for the bond resistances of top as compared with bottom bars. The EC2(2004) approach of limiting the bars, for which bond is reduced, to those at more than a specified depth below the top surface seems more realistic than ACI 318's(2005)definition of a top bar, which relates only to the height above the bottom.

The factors affecting bond resistance are numerous and any general analysis treating all of them would be extremely complex.The most complete of the analyses reviewed is the upper-bound plastic theory, developed by Hess(Hess ,1984), Andreasen(Andreasen ,1989) and Nielsen(Nielsen ,1999), but even this makes many assumptions, including those of the plasticity of the concrete in tension and of the polar-symmetry of the stress distribution around bars. There are also considerable simplifications in the processes of obtaining solutions. At the opposite extreme, BS8110(2005) ignores almost all of the potentially infuential parameters to produce a very simple design method. The other theories and empirical formulations. They also make some comparisons between treatments of particular parameters.

The rest of following sections are reviews of those approaches and theories about the circumstances of bond and anchorage in reinforced concrete by researchers.

2. Straight Anchorages without Transverse Pressure

2.1 Theoretical approaches

2.1.1 Lower and upper-bound expressions for cracking resistance by Tepfers(Tepfers 1973, 1979)

Tepfers idealized the concrete around a bar as a hollow cylinder, in which circumferential tension balances the outward radial components of the forces from the bar,and considered two limiting cases.In one the concrete is treated as elastic and unless the cover is very small, the maximum ring tension resistance corresponds to a situation in which there is already local radial cracking close to the bar.The radial compression is assumed to be at 45^0 to the bar.This gives the following expression for the bond stress causing cracking to reach the outside of the ring (the surface of a member):

$$f_{bc} = f_{ct}.\frac{c_d + \varphi/2}{1.664\varphi} \tag{1}$$

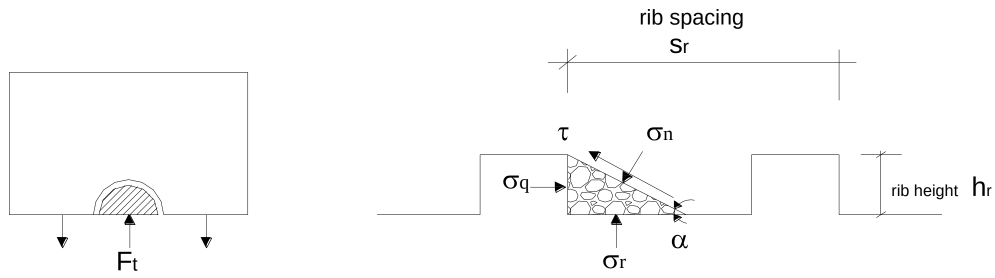
Where c_d is the minimum cover, and f_{ct} is the tensile strength of the concrete taken as $0.5\sqrt{f_c}$. The other solution is that of plastic theory with uniform ring tension from the bar outward. Again, with the assumption that the radial compression is at 45^o:

$$f_{bc} = f_{ct}.c_d/\varphi \tag{2}$$

Tepfers showed that test results for short bond lengths ($l_b = 3.13\varphi$) fell between the two. In his thesis (Tepfers,1973). Tepfers treats the ultimate strength of laps by considering various patterns of splitting.

He analyses two conditions for local bond capacity. One is that in which a radial crack first penetrates the minimum cover at a bond stress f_{bc}. The other is that at which one of six patterns of splitting cracks leaves a bar or bars free to move away from the core of a member

In his analysis of laps he uses the full $f_{ct} = \left(0.5\sqrt{f_c}\right)$ and equation (2) on the splitting surface if it cracks just at failure. He uses eqn.(1) for the resistances of parts of the surface that crack before failure but at which the cracks do



a) Forces across splitting crack b) Stresses on a concrete wedge

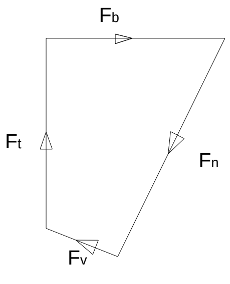
Fig. 2 Forces and stresses in the failure model by Cairns

not widen so much that bond resistance is lost. Eq. (1) can be written as

$$\frac{f_{bc}}{\sqrt{f_c}} = 0.15(1 + 2\frac{c_d}{\varphi}) \qquad (3)$$

2.1.2Cairns/ Cairns and Jones(1975,1979,1995,1995)

Cairns's thesis(Cairns,1975) which is the basis of (Cairns,1979) was published just 3 years after that by Tepfers(Tepfers 1973). Like Tepfers, Cairns recognized that in bond failures by splitting of the cover, wedges of concrete can frequently be found adhering to the bars at the loaded end faces of ribs and that the forces on the outer faces of these wedges were responsible for the splitting. Unlike Tepfers, who considered only the compression normal to the faces of the wedges, Cairns also took account of the shear on these faces. Thus in the Cairns approach the model of failure is one in which sliding occurs on the faces of the wedges under the actions of shear and normal stresses the magnitudes of which depend on the resistance to tension across the splitting surface -see Fig. 2. The polygon of forces governing the equilibrium of the wedge to one side of the splitting crack is shown in Fig.3.



F_b = bond force

F_n = force from σ_n

F_v = force from τ

F_t = force shown in Fig.2.16

Fig. 3 Polygon of forces on a wedge

In Cairns's solution the transverse force F_t is treated as a known quantity and the ultimate stresses on the surface of the wedge are governed by the Mohr-Coulomb failure criterion

$$\tau = f_{co} + \sigma_n \tan\theta \qquad (4)$$

where f_{co} is the apparent cohesive strength of the concrete , and θ is its angle of internal friction.

In terms of the bond stress the result is

$$f_{bu} = 2\left[\frac{F_t}{\varphi l_b}.\frac{h_r}{I} + f_{co}.f_R\right]\cot\alpha \qquad (5)$$

in which f_R is the relative rib area = $A_r / \pi\phi\, s_r$, where A_r is the projected area of a rib ($\approx \pi\varphi\, h_r$ for an annular rib), α is the angle between the outer face of a wedge and the axis of the bar and

$$I = \int_{-\pi/2}^{\pi/2} h_r \cos\mu\, d\mu \qquad (6)$$

where μ is the angle shown in Fig.3.

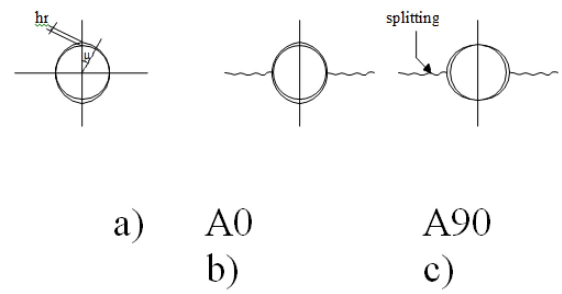


Fig. 4 Terminology for crescent shaped ribs

It should be noted that for equal values of f_R , h_r / I =0.5 for annular ribs.For crescent-shaped ribs it is less for the orientation A0 and more for the orientation A90 of Fig.4.

Cairns and Jones(Cairns and Jones 1995-1)refer to splitting and non-splitting components of bond resistance, which correspond respectively to the first and second terms in the bracket of equation (5).The first depends on the tensile resistance available across the splitting plane while the second is independent of it. In terms of Fig 3, the splitting component is the length BC and the non-splitting component is CD and can be seen to be the larger (typically about $0.7 f_{bu}$, in their tests). In analyzing test results the angle α was taken as $45^{\circ} - \theta/2$. From tests of mortar representing the concrete of a wedge, the concrete resistance was taken as $f_{co} = 0.29 f_{cm}$ and θ was 32° . The tensile resistance at the splitting plane was equated to the force in the stirrups. The first tests by Cairns(Cairns 1979) were of laps of compression reinforcement in columns and subsequent tests by Cairns and Jones(Cairns and Jones1995,1-2) were of tension laps. Both types of tests involve factors outside the scope of this thesis- the forces on splitting planes produced by laps and also end-bearing in the case of column bars. Only the tests of tension laps are considered here and they are summarized very briefly.

Comparisons between bond strengths in specimens which were precracked on the splitting plane and ones that were not precracked showed an average reduction of bond strength due to precracking of only 8% and thus supported the assumption that the tensile resistance across the splitting plane at the stage of bond failure could reasonably be taken as that of the links alone. For bars with crescent- shaped ribs comparisons between specimens with all four bars in the AO and A90 orientations showed that the bond strengths obtained with the A90 orientation were the higher by 10% to 15%. Results for bars with annular ribs giving the same f_R were intermediate. This confirms the effect predicted by the term I in eqn. (6).

Results for the otherwise similar specimens with the thicknesses equal to 320 or 100mm were practically identical, which supports eqn.(5)'s including a single term F_t for the splitting resistance, and the second term's independence of cover. These results are actually a little surprising as the anchorages of the links in the 100mm specimens would seem to be significantly poorer than those in the thicker ones. Where bars with different rib areas were tested in otherwise similar specimens, increasing f_R had a significant influence on the ultimate bond strength. Fig.4, which is adapted from reference (Cairns and Jones 1995-2)shows bond strengths, averaged from tests with AO and A90 orientations where relevant, plotted against f_R .

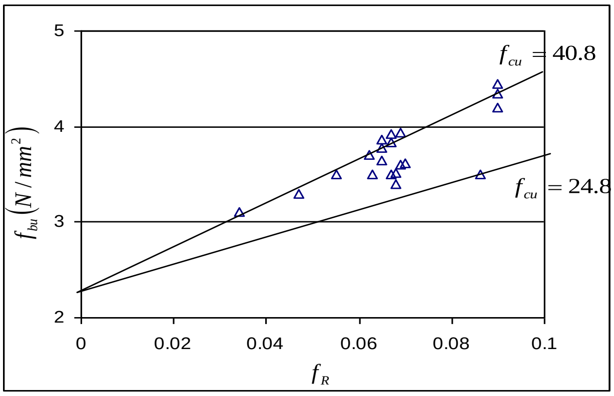


Fig. 5 Cairns and Jones – influence of relative rib area on bond strength

The cube strength of the concrete varied from 24.8 to 40.8 (N/mm^2), but only two points are for specimens with $f_{cu} < 33\,N/mm^2$.The lines in the figure are drawn from the value of f_{bu} at f_R =0 obtained from a regression analysis by Cairns and Jones and have slopes as given by eqn.(5) for $f_{cu} = 24.8$ and 40.8 N/mm^2.The correlation between eqn. (6) and the test results can be seen to be good.

In general terms the approach of Cairns seems to give a good account of the behaviour involved in bond failures, in the circumstances which it considers.It does however have limitations. The most obvious of these is that it does not consider the resistance to splitting that can be provided by concrete. Although this appears to have been small in the tests, the similarity of the results for specimens with and without precracking really only shows that bond strengths were similar in both cases.It is possible that the resistances from the concrete and the transverse reinforcement were equal , that the contribution of the steel was negligible until the concrete cracked and that after cracking the contribution of the concrete was negligible. Even if the concrete contribution was negligible in these tests, it is clearly significant in cases (e.g.slabs) where there is likely to be no transverse reinforcement,so the concrete resistance has to be evaluated. A simple horizontal split is also unlikely in the slabs, and at beam corners. Another limitation is that the approach does not cover the type of failure in which the slip surface is cylindrical. The assumption that the angle α between the wedges and the bar axis is $45^\circ - \varphi/2$ is correct if the radial stress (σ_r in Fig.1) is zero. This may not influence f_{bu} greatly at low values of σ_r, but, as the restraint increases, the angle obtained from the Mohr-Coulomb criterion is(Nielsen1999)

$$\alpha = arc\cot\frac{1}{2\sqrt{k}}\left[(k-1)+(k+1)\sqrt{1+\sqrt{k}\,\frac{2\sigma_r}{f_R\,\nu\,f_c}}\right] \tag{7}$$

where $k = \dfrac{1+\sin\theta}{1-\sin\theta}$ and $\nu\,f_c$ =effective compressive strength of concrete and θ is the angle of internal friction. As σ_r increases $\cot\alpha$ increases until the end of the wedge at one rib reaches to the next rib, and shearing occurs on a cylindrical surface.

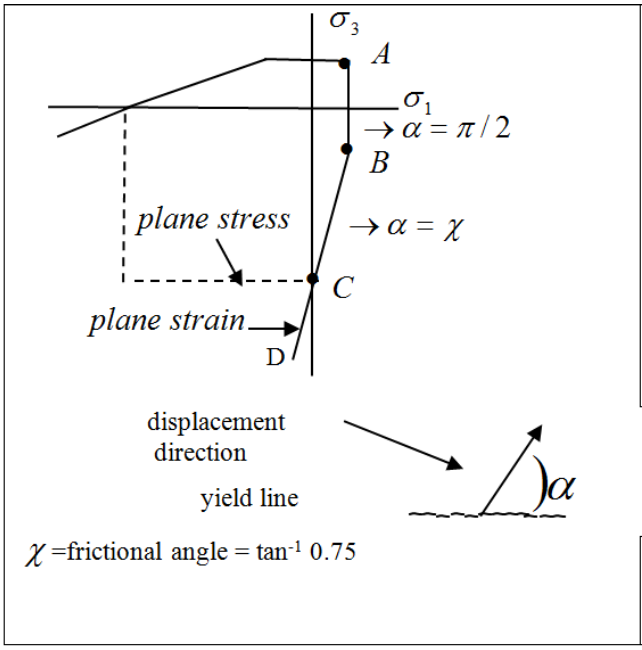
Although in the later papers, the stress in stirrups crossing the splitting surface is taken as 80 (N/mm^2) on the basis of the strain measurements and this produces a good correlation with experimental values of f_{bu} (Cairns and Jones1995-2), in the first paper it was taken as the yield stress ($f_y = 390\,N/mm^2$) and also gave good agreement with the results of tests.

2.2 A plastic theory approach by Nielsen (Nielsen 1999)

- Introduction

A plastic theory approach to problems of bond in reinforced concrete has been developed at the Technical University of Denmark, notably in thesis by Hess(Hess 1984) and Andreasen(Andreasen 1989). Its most recent and complete presentation is given in the current edition of Nielsen's book (Nielsen 1999),to which this review refers.

Nielsen's approach is an upper bound one in which collapse mechanisms are postulated and the equation of the internal work, done in their yield lines, and the



Posn. on yield locus	α	Work per unit area and unit displacement
AB	$\pi/2$	$\rho\ f_c$
B	$\chi : \pi/2$	$\frac{1}{2} f_c(\lambda - \mu \sin\alpha)$
BD	χ	$\frac{1}{2}\nu f_c(1-\mu\sin\alpha)$

Coordinates: A $(\rho f_c, \rho f_c)$; B $(\rho f_c, k\rho f_c - \nu f_c)$; C $(0, -\nu f_c)$

Constants: $\lambda = \nu - (k-1)\rho$; $\mu = \nu - (k+1)\rho$; $k = (1+\sin\chi)/(1-\sin\chi)$

Fig. 6 Yield locus, displacement directions and internal work

external work, done by the loading (bar force), gives estimates of ultimate loads. The best estimate of an ultimate load is the lowest one among those for different mechanisms, each of which has been refined in terms of its geometry to give a minimum capacity.

Concrete is treated as a rigid perfectly plastic material with the plane strain failure criterion and associated flow rule shown in Fig.7.The imperfect plasticity of concrete is allowed for by empirical effectiveness factors, such that the plastic compressive and tensile strengths are taken as:

$$f_{cp} = \nu\ f_c = 1.8\sqrt{f_c} \quad \text{and} \quad f_{cp} = \rho\ f_c = 0.6\sqrt{\frac{\varphi}{l_b}}\sqrt{f_c}$$

Each mechanism considered comprises two parts- a local one in the immediate vicinity of the bar and another in the surrounding concrete. The work done in the local mechanism is a function of the concrete properties, the bar size and the rib geometry. The work done in the surroundsdepends on the concrete properties and the covers and bar spacing. The two work components are linked by the need for compatibility of deformations between them. When the surrounding concrete contains transverse reinforcement crossing yield lines, or is acted on transverse pressure against which work is done, the effects of these are included in the work of the second mechanism. The deformations in the local mechanism are polar-symmetric, while those in the second mechanism are generally not so. The compatibility of deformations between the two is thus in most cases established only in the direction of the movement of the surrounding concrete.

- Local mechanisms

The bar geometry used in Nielsen's analyses is one of simple annular ribs perpendicular to the longitudinal axis as shown in Fig.7.

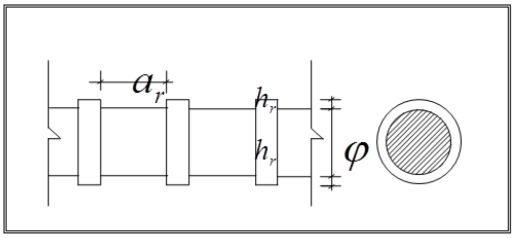


Fig. 7 Geometry of a deformed bar

Local failure takes the form of sliding, accompanied by dilation, on the faces of truncated conical surfaces between the core diameter (φ) of the bar and the external diameter $(\varphi + 2h_r)$ of the ribs. The angle γ between these surfaces and the axis of the bar can range from a maximum of, 26.6^o with wedges running down from the ribs and reaching the core before the next ribs, to a minimum of *0*^{o}for which the wedge shape is transformed to a cylinder of diameter $(\varphi + 2h_r)$.

Fig.8 gives the results of analyses of these failure shapes in terms of the internal work$W_{i,1}$ done per unit length of bar for a unit displacement at an angle $\alpha = (\gamma + \chi)$ to the bar axis, where χ is the internal friction angle taken as arc tan 0.75=36.8°. There are two expressions for $W_{i,1}$, one for $\gamma \geq \gamma_0$ and the other for $\gamma < \gamma_0$, where $\gamma_0 = h_r / a_r$. For three particular values of γ, the figure also gives $\tan\alpha$ and the values of $f_{bu0}/\nu\ f_c$. Where f_{bu0} is the bond resistance obtained by equating internal ($W_{i,1}$) and external work $(W_e = P\cos\alpha)$. This is the hypothetical bond stress obtained if no work is done in the second part of the failure mechanism.

In realistic cases $W_e = W_{i,1} + W_{i,2}$ and a more meaningful idea of the results in terms of calculated bond resistances

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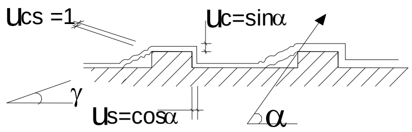
<td>
</td>
<td>$$W_{i,1}=\frac{\pi}{2}\nu\, f_c(\varphi+h_r)\left(\frac{el}{a_r}\right)\frac{1-\sin\chi}{\sin\gamma}$$
$$\underline{1.}\ \ \gamma=\gamma_{\max}=45^\circ-\frac{\chi}{2}=26.6^\circ$$
$$\tan\alpha=\tan\left(45^0+\frac{\chi}{2}\right)=2.0$$
$$\frac{f_{bu0}}{\nu\, f_c}=\frac{\varphi+h_r}{\varphi}\cdot\frac{h_r}{a}$$
$$\underline{1a}\quad \gamma=17.6^\circ\ ,\ \tan\alpha=1.4$$
$$\frac{f_{bu0}}{\nu\, f_c}=1.14\frac{\varphi+h_r}{\varphi}\cdot h_r$$</td>
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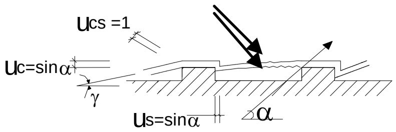
</td>
<td>$$W_{i,1}=\frac{\pi}{2}\nu\, f_c[\varphi+2e-a\tan\gamma]l\cdot\frac{(1-\sin\gamma)}{\cos\gamma}$$
$$\underline{2.}\quad \gamma=0\quad ,\quad \tan\alpha=0.25\frac{\varphi+2h_r}{\varphi}$$</td>
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<tr>
<td>$u_{cs}=1$ unit relative displacement between bar and surrounding concrete
$u_c=\sin\alpha$ outward displacement of concrete
$u_s=\cos\alpha$ longitudinal displacement of steel
$\alpha=$ angle between u_{cs} and bar axis.</td>
<td>$W_{i,1}$ =work done in local mechanism per unit length of bar.
$W_{i,1}\,.\,l$ =total interal work in a length in the local mechanism.

1,1a,2 – cases corresponding to given values of γ</td>
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Fig. 8 Displacements at failure and internal work in local mechanisms

can be obtained from considering the very simple case of a central bar anchored in a concrete cylinder. The yield lines in the cylinder are radial and each segment between adjacent lines is displaced outward a distance equal to $\sin\alpha$.

The resulting tangential strain at a radius r is $\sin\alpha/r$ and the work done in the outer part of the mechanism is $2\pi l\rho\, f_c\sin\alpha.c$, where c is the concrete cover to the bar. The shape of the local failure surface changes with γ decreasing from 26.6° to zero as c/φ increases. Fig.10 shows f_{bu}/f_c as a function of c/φ for the particular circumstances considered. The continuous heavy line represents the exact solution, while the bi-linear approximation has been obtained considering failure shapes 1a and 2 of Fig.9.

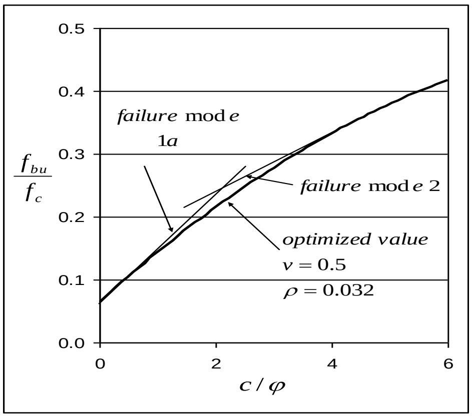


Fig. 9 Relationships between f_{bu}/f_c and c/φ by Nielsen

$$f_{bu1a}=1.14\nu\ f_c\frac{\varphi+h_r}{\varphi}.\frac{h_r}{a}+2.8\frac{c}{\varphi}.\rho\ f_c \tag{11}$$

$$f_{bu2}=.25\frac{\varphi+2h_r}{\varphi}.\nu\ f_c+1.5\frac{c}{\varphi}.\rho\ f_c \tag{12}$$

The correct value for $\frac{c}{\varphi}$. =0 comes from failure shape 1and is

$$f_{bu1}=\nu\ f_c\frac{\varphi+h_r}{\varphi}\frac{h_r}{a} \tag{13}$$

However the slight non-conservatism at c/φ =0 is of no practical consequence.

- Failure mechanisms in the surrounds

Fig.10 shows the principal mechanisms treated by Nielsen. A mechanism similar to the V-notch failure pattern of Morita and Fujii was also considered but rejected in favour of the cover bending mechanism of Fig.10d. In each of the mechanisms, for a unit relative displacement between the bar and the core concrete, the bar's displacement normal to the yield line is sin a , and the displacement of the outer concrete relative to the core is 2sin a. Each mechanism except the face splitting of Fig.10c is defined geometrically by a single parameter - β or x - and the internal work $W_{i,2}$ in the surrounds can be minimised with respect to the relevant parameter. Due to the possible variations not only of β and x but also of the covers and the form of the local mechanism, solutions for ultimate bond stresses are numerically complex and Nielsen uses numerical approximations to obtain the final results given in Fig.11.

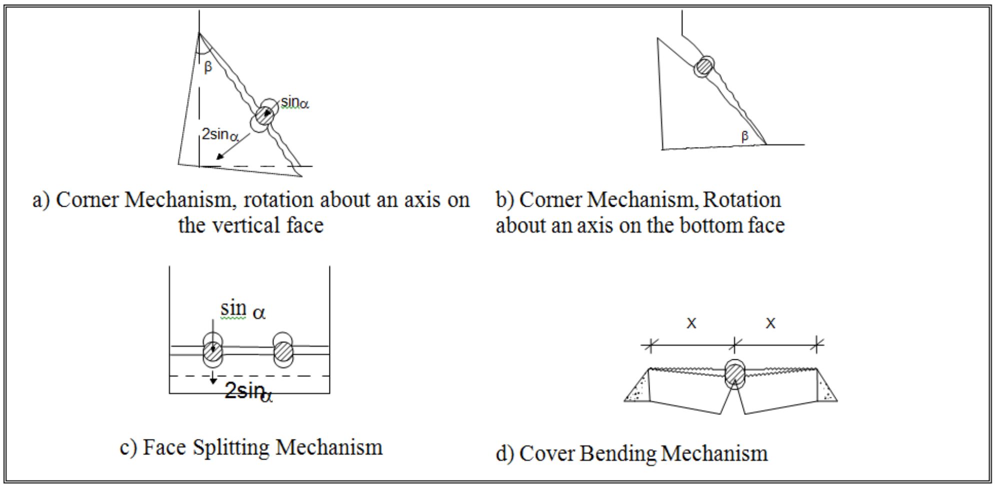
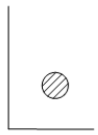


Fig.10 Failure mechanisms in surrounds

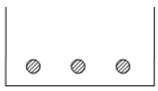


Corner Failure

$$\frac{f_{bu}}{\sqrt{f_c}} = 0.23 + 0.31\sqrt{\frac{\varphi}{l_b}}\left(3.2\frac{c_{\min}}{\varphi} + \frac{c_{\max}}{\varphi}\right) \qquad (14)$$

Cover Bending Failure

$$\frac{f_{bu}}{\sqrt{f_c}} = \min\left[\begin{array}{l} 0.22 + 0.54\left(6\frac{c}{\varphi} + 2\right)\sqrt{\frac{\varphi}{l_b}} \\ 0.50 + 0.29\left(6\frac{c}{\varphi} + 2\right)\sqrt{\frac{\varphi}{l_b}} \end{array}\right. \qquad (15)$$



Face Splitting Failure

$$\frac{f_{bu}}{\sqrt{f_c}} = \min\left[\begin{array}{l} 0.22 + 0.54\sqrt{\frac{\varphi}{l_b}\left(\frac{b}{n\varphi} - 1\right)} \\ 0.50 + 0.29\sqrt{\frac{\varphi}{l_b}}\left(\frac{b}{n\varphi} - 1\right) \end{array}\right. \qquad (16)$$

Note: for $f_c \succ 75N/mm^2$, $f_{bu}/\sqrt{f_c}$ should be multiplied by $(2 - f_c/75)$

Fig. 11 Final results for different failure mechanisms

In each of the expressions the first term derives from the work in the local mechanism and should vary slightly with the bar deformations. The second term comes from the work done in the surrounds and includes a term $\sqrt{\varphi/l_b}$ originating in the expression for ρ .The lowest strength corresponding to any of the mechanisms is decisive. Nielsen's summary of solutions includes further examples corresponding to cover bending failures for beams with finite widths and various numbers of bars.

In reference(Nielsen 1999) the results obtained from the expressions in Fig.12 are compared with tests of contact splices, for which the same expressions are applicable. The results of the comparisons are summarised in table 2 and can be seen to be generally satisfactory, although the data for cover bending failures are very limited.

Nielsen notes that the factor ρ can obviously not be extended to very small values of (l_b/φ) and suggests a lower limit of 5 or 6. He also notes there is a size effect with the ratios of actual to calculated strengths being about 15% too high when $\varphi = 10mm$ and 15% too low when $\varphi = 35.8mm$. Nielsen gives two treatments of anchorages with transverse reinforcement. In the first, which is more likely to be relevant to anchorages at supports, the maximum resistance is assumed to be reached with the concrete resistance still essentially intact, while the transverse reinforcement is only lightly stressed. In this case, the increment of the ultimate bond stress for a bar in the corner bar of a stirrup is $50\sum A_{st}/\varphi l_b$ where A_{st} is the area of one leg of a stirrup and the summation is for the stirrups in the length l_b. In the second treatment it is assumed that the transverse reinforcement can develop its yield stress but that by this stage the resistance provided by the concrete at its yield lines has been lost along with cover. The main bars are held against the core concrete by the stirrups. The local actions are as illustrated by Fig.12 and are modelled by simple truss systems such as that shown

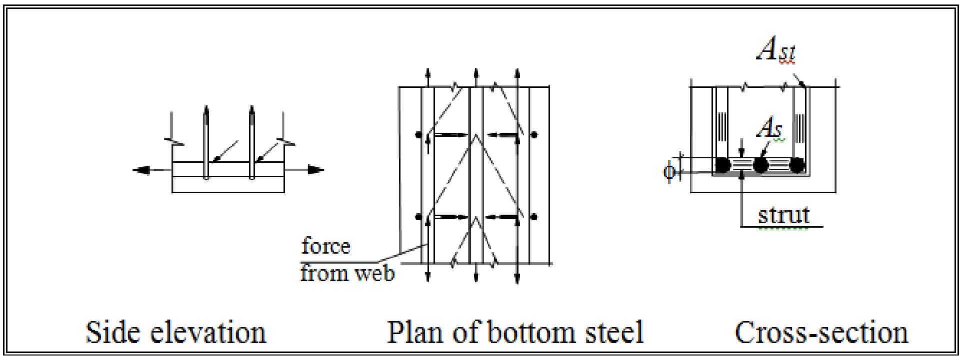


Fig. 12 Truss model for yielding stirrups

Table 1 Results of comparisons by Darwin *et al*.

Statistical results	257 tests(1)		290 tests(2)		200 tests(3)	
	Orangun	Darwin	Orangun	Darwin	Orangun	Darwin
Mean	1.095	1.126	1.079	1.112	1.054	1.073
C.O.V.	0.233	0.167	0.235	0.172	0.202	0.154
Min.	0.634	0.642	0.634	0.642	0.678	0.715
Max.	2.854	1.802	2.854	1.802	1.974	1.656

Table 2 Summary of comparisons of actual and predicted bond strengths (Nielsen)

Failure mode	$\frac{l_b}{\varphi}$	$\frac{c}{\varphi}$	$\varphi(mm)$	$f_c\left(\frac{N}{mm^2}\right)$	No. of tests	Mean Test/calc	Coeff. of variation
Corner	6-80	0.6-4.96	9.5-35.8	18-100	100	1.00	12.7%
Cover bending	6-39	0-1.56	10-35.8	17-43	14	1.11	14.0%
Face splitting	5-6	-	8-35.8	19-52	41	1.02	13.7%

The strut widths are equal to the bar diameters and

$$f_{bu}/\nu\, f_c = \frac{1}{\pi}\sqrt{\psi(1-\psi)} \le \frac{0.5}{\pi}$$

where υ is the effectiveness factor for the concrete in the struts and

$$\psi = \sum A_{st} f_{yw} / \varphi l_b . \upsilon f_c$$

and the summation is for the stirrups in the length l_b.

2.3 Empirical equations for bond strength

2.3.1 Orangun *et al*.(Orangun *et al*.1977)

Orangun *et al*.developed an empirical equation for bond strength based on non-linear regression analysis of published results of tests of bottom- cast bars. Most of the tests were of splices but some development length tests were included.

$$f_{bu} = (0.1 + 0.25\frac{c_d}{\varphi} + 4.15\frac{\varphi}{l_b}).\sqrt{f_c} \text{ (SI Units)} \qquad (8)$$

f_{bu} is the approximate mean value of the ultimate bond stress, c_d is the governing concrete cover which is taken as the least of the clear bottom cover , the clear side cover and half the clear bar spacing, l_b is the length of the anchorage or lap splice and f_c is the concrete cylinder strength.

Where transverse reinforcement is present the bond stress can be increased by $0.024(A_{st} f_{yt} / s_t\varphi)\sqrt{f_c} \le 0.25\sqrt{f_c}$, where A_{st} is the area of one leg of a stirrup. This expression was derived from tests with two bars or splices and stirrups with two legs. If it is extended to other cases as $0.024(n_{st} A_{st} f_{yt} / n s_t\varphi)\sqrt{f_c} \le 0.25\sqrt{f_c}$,where n_{st} = number of stirrup legs in a section and n= number of main bars, it should be recognized that the increase is likely to be concentrated to the bars in the corners of stirrups. Eqn. (8) was derived for bottom-cast bars and a 25-30% reduction was suggested for top-cast reinforcement, although the test series reviewed by Orangun showed average reductions of only 12% to 13%.

The empirical equation has an upper limit of $c_d \le 3\varphi$ as it was thought that for large c_d/φ ratios failure would be by shearing rather than splitting and most of the data used in developing the equation was for $c_d \le 3\varphi$. The equation becomes conservative for large side covers and bar spacings, and Orangun shows a stepped relationship between experimental and calculated strengths and the ratio $c_s/c_b\varphi$. $f_{bu,test} / f_{bu,calc}$ =1.06 for $c_s/c_b\varphi < 0.12mm^{-1}$, 1.21for $0.12 \le c_s/c_b\varphi \le 0.24mm^{-1}$ and 1.64 for $c_s/c_b\varphi > 0.24mm^{-1}$.

Orangun's expression contains a numerically significant term in (φ/l_b).Codes of practice do not involve this parameter but are presumably concerned primarily with anchorages long enough to develop yield in the bars. An influence from $(\varphi/l_b)$is present in some of the ultimate load cases treated by Tepfers, but not in most of them. Test data does generally support (φ/l_b) being influential but in some work by Baldwin and Clark(Baldwin and Clark,1994) with (l_b/φ) from 2.5 to 30 the mean ultimate bond stress was practically independent of (φ/l_b) while the characteristic value actually decreased for short anchorage lengths. The arrangements used for these tests were however unusual and failures by conical fracturing of the concrete or by flexuraltension would seem more probable than bond failures as shown in Fig.(13).

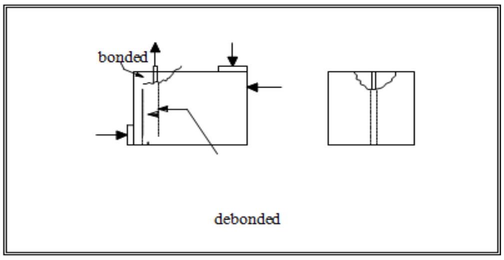


Fig. 13 Test arrangement and failure mode for specimen with $l_b/\varphi = 2.5$ by Baldwin and Clark

2.3.2 Darwin *et al*.(Darwin *et al*.1992)

Darwin *et al*.developed an empirical equation to represent bond strength in straight bars and splices not confined by transverse reinforcement or transverse pressure.

$$f_{bu}/\sqrt{f_c} = 0.176(c_m/\varphi + 0.5)(0.92 + 0.08c_M/c_m) + 6.23(\varphi/l_b) \quad (9)$$

where : c_M =greater of $c_s^{'}$ and c_b , c_m = lesser of $c_s^{'}$ and c_b , $c_s^{'}$ =the lesser of $(s/2)$ and c_s , s = clear spacing between bars, c_s = side concrete cover and c_b = bottom concrete cover

This expression is an update of Orangun *et al*.s' equation with a more rational treatment of the effect of the greater cover .It seems to be the most comprehensive empirical equation, as it accounts for the effects of concrete strength, both concrete covers, bar spacing and bond length. The equation was derived for bottom-cast bars and would presumably need a reduction co-efficient if applied to top-cast bars.

Darwin *et al* compared predictions by this equation and results from the literature as summarized in table (1) and eqn.(9) was found to be more accurate than other equations. The sources of data were those used by Orangun *et al*. paper and four series published after 1975(Darwin *et al*.1992). The results obtained are summarised below in terms of $f_{bu,test} / f_{bu,calc}$.

It is unclear whether Orangun's treatment of widely spaced bars was taken into account in the comparison, but Darwin's expression appears to be an improvement on Orangun's and his treatment of wide spacings (large c_M/c_m) is preferable to Orangun's step function.

Since (Darwin *et al*.1992)Darwin and various co-authors have published a number of different equations for bond strength, but eqn. (9) appears to be the only one developed specifically for bars in normal strength concrete without transverse reinforcement. One of the more recent equations is given in section 2.3 and includes an allowance for the effect of transverse reinforcement. There is also a paper by Canbay and Frosch(Canbay and Frosch,2005) which follows the work of Orangun and Darwin and seeks to develop " the calculation of bond strength based on a physical model of tension cracking of concrete in the lap-splicedregion". It could potentially be applied to anchorages and it is in their context that it is described here.

The approach is derived for bars in a single layer and it is initially assumed that bond stresses are uniformly distributed along anchorages and that the splitting component of the force from a bar isequal to the bond force multiplied by $\tan 36^\circ$.

Splitting is assumed to occur simultaneously at all the bars,either with a horizontal crack across the width of the member or with a vertical crack under each bar. The forces resisting splitting are calculated as the products of thes tensile strength of the concrete, taken as $0.5\sqrt{f_c}$ and the areas ofthe splitting surfaces. Equating these resistances to the forces from the bars, the ultimate bond stress can be found as the lesser of two values. For a case where there are n bars of diameter φ at a spacing s . The bond stress for horizontal splitting depends on f_{ct} ,n, c_s/φ and s/φ and that for vertical splitting depends on f_{ct} and c_b/φ .

To obtain agreement with test results the actual covers and bar spacings are replaced by effective values, e.g $c_{b,eff}/l = 0.77\sqrt{c_b\varphi} \le c_b$. The bond stress is multiplied by a factor $l_{b,eff}/l = 9.5\sqrt{\varphi/l_b}/\sqrt[4]{f_c} \le 1.0$ and the equation for the bond strength for vertical splitting is modified by a term in C_b, C_b, S and n to allow for the actual cracking beingof a V-notch type rather purely vertical. The final agreement with test results for splices is good but not significantly better than that for one of these by Darwin *et al*. Canbay and Frosch quote coefficients of variation of $f_{bu,test} / f_{bu,calc}$ of 12% for their method and 12.7% for the Darwin equation. The method is extended to treat splices confined by stirrups through the addition of the stirrup forces to the splitting resistances, which are based on the stirrup stresses being about 60N/mm^2.

The degree to which the method can be said to represent physical modelling is limited. It does not consider corner splitting, it assumes smiultaneous anchorage failure of all the bars and it relies on numerous empirical expressions. The expressions for effective covers and bar spacings are considerably more complicated to use than simple limits on these quantities and the final expression treating V-notch failures is both complicated and somewhat ambiguous as it mixes the use of actual and effective cover dimensions. The method is simplified in a second paper (Canbay and Frosch,2006) but this is written very much in the context of ACI 318.

2.3.3 Morita and Fujii(Morita and Fujii,1982)

Morita and Fujii proposed the following equation for the bond strength of top – cast bars :

$$\frac{f_{bu}}{\sqrt{f_c}} = 0.0962 b_i + 0.134 \qquad (10)$$

For bottom bars the bond stresses of eqn.(10) are multiplied by 1.22.

b_i is a parameter for evaluating the influence of covers and bar spacings and is the smallest of b_{si} , b_{ci} and b_{vi} as shown in Fig.14. They compared predictions of this equation with test data from various sources and it showed better correlation than Orangun's expression in most cases.It may be noted that the corner splitting failure mode related to b_{ci} is not referred to by Orangun.They do not mention any influence of (l_b/φ). Their own tests were in the range $20 \le l_b/\varphi \le 26$ and it is not clear what variation of (l_b/φ) was present in the other data they considered. The effect of transverse reinforcement can be taken into account by adding $7.87kA_{st}/s_t\varphi$ to the right side eqn. (10).

3. Straight Anchorages with Transverse Pressure

3.1 pull-out tests by Untrauer and Henry (Untrauer and Henry, 1965)

Untrauer and Henry made pull-out tests on bars embedded centrally in 152mm cubes which were subjected to transverse pressure in one direction. A leather pad 1.6mm thick was placed between the cube and the plate taking the reaction to the bar force but no measures seem to have been taken to reduce friction between the concrete and the plates applying the transverse pressure.

The maximum transverse pressure applied was 16.4 N/mm^2 and the final empirical equation derived for bond strength was

$$f_{bu} = \left(1.49 + 0.45\sqrt{p}\right)\sqrt{f_c} \quad \text{(in SI units)} \qquad (17)$$

where P is the transverse pressure.

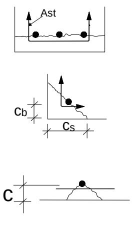
Its correlation with the test results is shown in Fig.15.

$b_{si} = \dfrac{b}{n\varphi} - 1$

n= no of bars in line

n_{st}= no of vertical stirrup legs

$k = \dfrac{n_{st}}{n}$



$k = \sqrt{2}$

$b_{ci} = \sqrt{2}\left(\dfrac{c_s + c_b}{\varphi} + 1\right) - 1$

$b_{vi} = \sqrt{3}\left(\dfrac{2c_{\min}}{\varphi} + 1\right)$ $\qquad k = 0$

Fig. 14 Failure patterns of anchored bars(Morita and Fujii)

3.2 Robins and Standish (Robins and Standish.1982,1984)

Robins and Standish investigated the influence of lateral pressure on the bond strengths of plain and deformed bars in lightweight aggregate concrete.

They carried out cube and semi-beam pull-out tests on 8,12 and 16mm plain and deformed bars, but only the results for deformed bars are considered here. Concrete cube strengths were varied between 18 to 45*N/mm*^{2} and the maximum transverse pressure applied was 24*N/mm*^{2}. The bond lengths were 100mm in all series but one with 8mm bars in which l_b was 50 or 150mm. The transverse pressure, which was applied first, was held constant throughout a test. It was generally applied by steel plates bearing directly on the concrete but in one series MGA pads were inserted between the steel and concrete.

The test results showed that the influence of the lateral pressure on bond resistance was similar for both types of specimens. Where the transversepressure was applied directly by steel platens, the bond strength was increased with increasing transverse pressure up to (0.3 f_{cu}). At this level of pressure the mode of failure changed from splitting to shearing of the matrix between the lugs of the bars and the bond strength was constant for higher pressures. Robins and Standish found that Tepfers's equation gave generally satisfactory predictions of bond strengths when *p=0*. The bond stresses at which shearing failures occurred were predicted by Frandon's expression

$$f_{btr} = 1.8\sqrt{f_{cu}} \qquad (18)$$

Reasonable predictions of bond strengths were obtained by linear interpolation between the Tepfers values when *p=0*and eqn.(18) for $p \ge 0.3 f_{cu}$. Where the transverse pressure acted through MGA pads it was found that the pressure did not increase the bond resistance, from which it was concluded that the favourable effect of *p* was dependent on an accompanying frictional restraint perpendicular to the applied pressure.

3.3 Nagatomo and Kaku(Nagatomo and Kaku.2005)

Nagatomo and Kaku made tests with the arrangements shown in a simplified form in Fig.2.33.

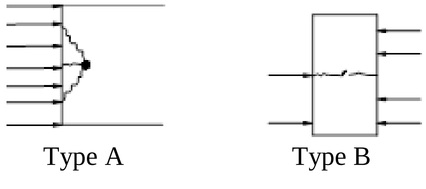


Fig. 16 Nagatomo and Kakus' test arrangements

The bar diameter was 22.2 mm and the bond length was 155mm. The concrete cylinder strength was low (15.5 to 18.8 *N/mm*^{2}). In type A tests with a cover equal to 1.08φ there was a marked increase of bond resistance with increasing transverse pressure with the V-notch form of failure shown. When the cover was increased to 1.98φ the failures were by shearing was very small. Rather oddly most of results for 1.98φ with transverse pressure show strengths lower than those for 1.08φ. In type B specimens, where the minimum cover was 3.1φ, the failures were by splitting as sketched in Fig.16 and there was no real increase of bond strength.

The test results, together with those of Robins and Standish and Navaratnarajah and Speare were used to develop an extension of Morita and Fujji's approach to cover the influence of transverse pressure. The result is the equation

$$f_{bu} = (1 + k_1 \frac{p}{f_c}) f_{buo} \le f_{b\max} \tag{19}$$

Where: $k_1 = a_1.a_2$,
$a_1 = 4.55 - 1.75 c_b / \varphi \quad \ge 0$,
$a_2 = 0.327 c_s / \varphi \le 1.0$, $p \le 0.3 f_c$

and f_{buo} is the bond strength when $p = 0$ from Morita and Fujii .

Nagatomo and Kaku used top-bar values of f_{buo} from Morita and Fujii, inspite of the bars in their own tests and those by Robins and Standish being either vertical or bottom-cast, for which Morita and Fujii recommend a 22% increase of the calculated bond stresses. They report a mean ratio of experimental to calculated strength of 1.19 with a standard devation of 0.22 for 163 test results, but it is not clear how the results from Navaratnarajah and Speare have been interpreted nor how numerical results have been obtained from reference(Nagatomo and Kaku.2005) as Robins and Standish do not tabulate any data.

3.4 Batayneh(Batayneh.1993,1999)

Batayneh's main series of tests was modified pull-out tests on bars at the corners of specimens. The details varied to some extent and it is not clear which specimens each version applied to. Each specimen contained four bars and it was possible to obtain two independent test results, one for a top-cast and the other for a bottom-cast bar.

The variables in this group of tests were the bar size(8,12 and 16*mm*), the bonded length (10φ or 15φ), the concrete strength (f_{cu} for 100*mm* cubes from 18.5 to 45.8 N/mm^2), the transverse pressure (0, 0.2 f_{cu} and 0.4 f_{cu}, where f_{cu} was the intended cube strength), the cover ($c_b = c_s = \varphi, 2\varphi, 3\varphi$ and in a few cases 5φ) and the bar position during casting. Smaller numbers of tests were made with pairs of bars at each corner, spaced either horizontally or vertically.

As reported by Batayneh the failures were by corner splitting for small covers and transverse pressures, but by shearing (pull-out without splitting) at higher covers and pressures. For splitting failures the bond strength increased with increasing cover and increasing pressure, but for shearing it was solely dependent on the concrete strength. Resistances for top-cast bars were lower than bottom –cast ones in ratios between 0.5 and 1.0. Batayneh gives two expressions for the ultimate bond strengths of bottom-cast bars. The first is based on the Tepfers cracked elastic solution for f_{bu} when $p = 0$, with the CEB(CEB.1993)value $f_{ct} = 0.25 f_{cu}^{2/3}$ replacing $0.5\sqrt{f_c}$. The increase of f_{bu} with p was found empirically and an upper limit corresponding to shearing was applied.

$$f_{bu} = 0.25 f_{cu}^{2/3} \left(1 + 0.6 \frac{c}{\varphi}\right)\left(1 + 0.833 \frac{p}{f_{cu}^{2/3}}\right) \le f_{cu}^{2/3} \tag{20}$$

The second is based on Eligehausen's expression for the bond strength with $p=0$ and gives

$$f_{bu} = \left(0.39 f_{cu}^{2/3} \sqrt{\frac{c}{\varphi}}\right)\left[1 + 0.764 \frac{p}{f_{cu}^{2/3}}\right] \tag{21}$$

Comparisons with the test results for single bottom-cast bars gave following results:

Mean $f_{bu,test} / f_{bu,calc}$

- eqn.(20) 1.06 , eqn. (21) 1.03

Std,dev. of $f_{bu,test} / f_{bu,calc}$

- eqn.(20) 0.10 , eqn. (21) 0.15

For top-cast bars the following reduction coefficients were proposed.

c / φ	1.0	2.0	3.0
$f_{bu,top} / f_{bu,btm}$	0.67	0.76	0.85

Batayneh also made beam tests.The test arrangements differed from the other beam and beam-end tests that the transverse pressure was applied at the start of a test and then held constant by reducing the force from the jack over the support as the main load was increased to failure.

Seen from the side faces, the cracking at the supports, that produced anchorage failures, was a combination of a horizontal crack along the bar and short inclined cracks above it. The results of the beam tests that ended in anchorage failures are given in the table (3), which includes a comparison between the experimental ultimate bond stresses ($f_{bu,test}$) and values calculated by eqn.(21) ($f_{bu,calc}$)

Table 3 Results of beam tests by Batayneh

Beam ref.	$f_{cu,100}$ (N/mm^2)	p_u (N/mm^2)	$(f_{bu,test})$ (N/mm^2)	$\frac{f_{bu,test}}{f_{bu,calc}}$
01	17.4	0.62	3.33	1.15
21	27.1	1.13	6.10	1.53
11	27.2	5.00	7.74	1.46
02.1	25.3	3.26	6.93	1.53
02.2	22.8	3.26	6.60	1.53

With the exception of beam 01, where the transverse pressure was very low, the values of $(f_{bu,test} / f_{bu,calc})$ are much higher than those from the block tests, and there is clearly a problem here related to test arrangements.

3.5 Cairns and Jones(Cairns and Jones.1996)

The work by Cairns and Jones, reviewed in section 2.1.2, is complemented by a paper (27), which treats the influence on bond of an external compression normal to the splitting surface. The test specimens were generally similar to those in Fig.2.19, but with the transverse reinforcement reduced to two 6mm diameter links of plain round mild steel and transverse compression applied to the 400x225*mm*surfaces. The concrete cube strength varied from 27.0 to 39.9*N/mm*^{2} with a mean of 31.3 N/mm^2.

A regression analysis of the test results yielded the expression

$$\delta f_{bu} = 15.5x10^{-6} P \quad \text{(units } N \text{ and } mm\text{)} \tag{22}$$

where δf_{bu} is the increment of bond stress produced by the application of a lateral compression force equal to P. In terms of lateral pressure

$$\delta f_{bu} = 1.4\, p \tag{23}$$

In reference (Cairns and Jones.1996) the influence of transverse pressure is examined in terms of the basic theory. With F_t adjusted for the forces from laps rather than individual bars, it appears that the angle between the normal forces on the inclined faces of the wedges and the bar axes was 72° compared with the 61° of the original theory.

3.6 Rathkjen(Rathkjen,1972)

Information on Rathkjen's tests has been obtained primarily from Andreasen(Andreasen,1989), who gives full details of three series of tests without stirrups in the anchorage zone and a few tests with stirrups. More restricted data on some other series has been obtained from Hess(Hess,1984).The details of the tests without stirrups given by Andreasen are tabulated below. All the main bars were of Danish kam steel ($f_y = 560 N/mm^2$, $f_R \approx 0.123$) and concrete cylinder strengths varied from 18 to 33 N/mm^2.

Table 4 Data for Rathkjen's beams without transverse reinforcement

No. of main bars	No. of specimens	φ (mm)	c_s/φ	c_b/φ	l_b/φ
1	18	14	4.5	1.4	8.6
2	13	14	2.1	1.4	8.6
3	16	10	3.2	2.0	12.0

The results from these series are shown in Fig.17 where $f_{bu}/\sqrt{f_c}$ is plotted against $p_u/\sqrt{f_c}$. The figure shows definite increases of f_{bu} with p_u and there are no real signs of an upper limit on $f_{bu}/\sqrt{f_c}$.

3.7 Jensen (Jensen,1982)

Jensen studied the anchorage of deformed bars at supports. He tested a large number of eccentrically loaded pull-out specimens with two 16mm diameter bars with and without stirrups.The bars were $Ks900$ (Swedish kam steel)with $f_y \approx 900 N/mm^2$ and $f_R = 0.138$.Concrete cylinder strengths varied from 10 to 45 N/mm^2. Almost all specimens had the same $c_b/\varphi = 1.5$ and $c_s/\varphi = 3.13$, while l_b/φ was 8.0, 12.0 or 16.0. From a regression analysis of his data, Jensen derived the following expression for the ultimate bond stress of an anchorage without stirrups:

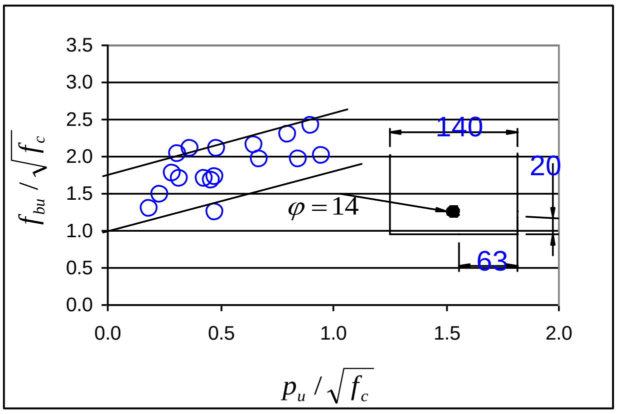
$$f_{bu} = 1.52 + 2.85\frac{R_u}{2F_u} + 2.15 f_{ct} \tag{24}$$

in which : f_{bu} is the average bond stress at failure N/mm^2, R_u is the reaction (kN), F_u is the ultimate tensile force in one bar (kN) and *fct*is tensile strength of concrete N/mm^2. The predicted values of bond strength from eqn.24 are satisfactory for l_b/φ =8 and 12 ,but, as Jensen observed in his conclusions, the strengths of specimens with $l_b/\varphi = 16.0$ are overestimated ashown in table 2.7.The lack of significant variations of side and bottom covers , as well as the bar size , are also likely to limit the applicability of eqn.24. The expression does not take any account of effects from c_b/φ , c_s/φ or l_b/φ and a further problem is with the generality of the term $R_u/2F_u$ .It could be re-expressed as $(b/n\pi\varphi)(p/f_b)$ where n is the number of bars. However$n$and φ were constant and b was almost constant in all the tests.

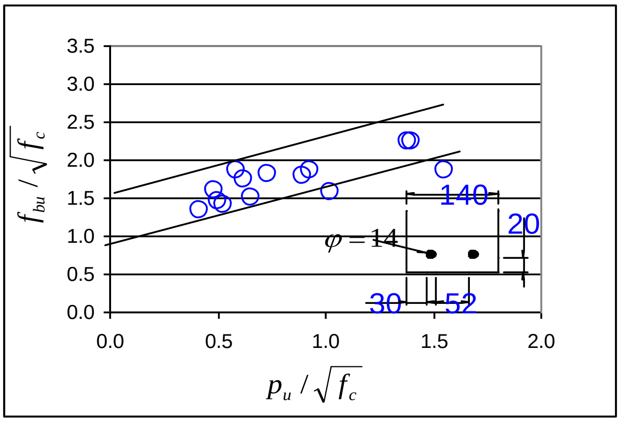
Table 5 Results of eqn.(24) for Jensen's tests

$\frac{l_b}{\varphi}$	Average $\frac{f_{bu.test}}{f_{bu.cal.}}$	Std.dev. of $\frac{f_{bu.test}}{f_{bu.cal.}}$
8.0	1.01	0.09
12.0	1.00	0.09
16.0	0.83	0.10

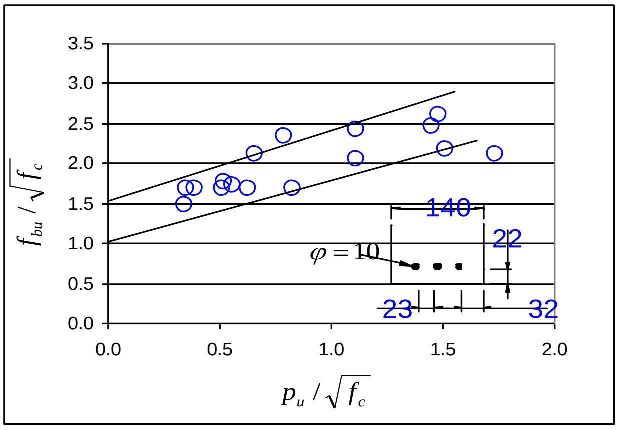
The relationships between $f_{bu}/\sqrt{f_c}$ and $p_u/\sqrt{f_c}$ in Fig. (18) show that with $p/f_b \approx 0.6$ (the highest ratio for $l_b/\varphi = 12$ and 16,there is really very little difference in f_{bu} for different bond length.



(a) $l_b/\varphi = 8.6$



(b) l_b/φ =8.6

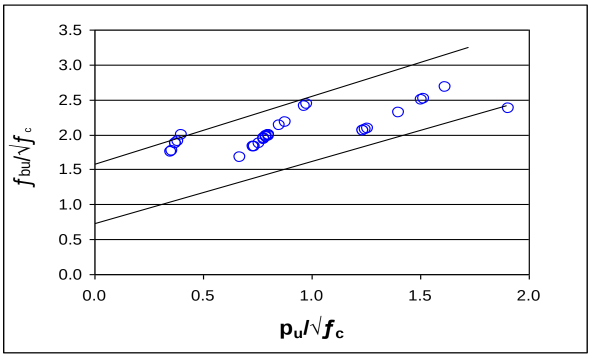


(c) l_b/φ =12.0

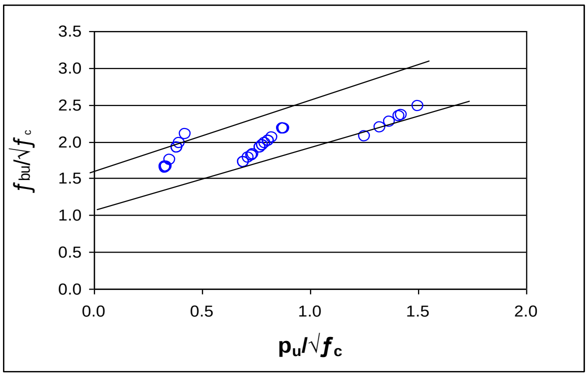
Fig. 17 Relationships between $f_{bu}/\sqrt{f_c}$ and $p_u/\sqrt{f_c}$ for Rathkjen's tests

3.8 Ghaghei(Ghaghei,1990)

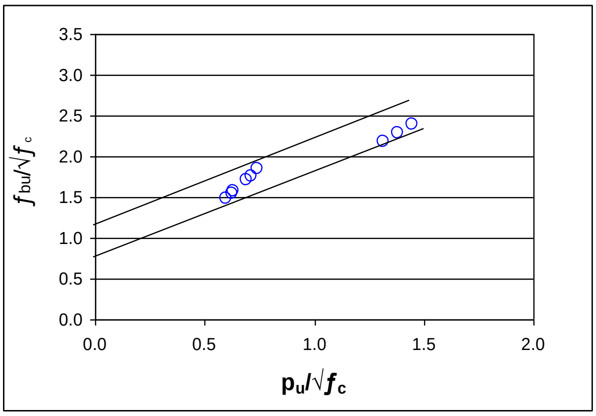
Ghaghei tested 9 beams, similar in form to Rathkjen's. Six of the tests resulted in anchorage failures. Details of these beams are shown in Fig.19, where it can be seen that one difference from Rathkjen's specimens was that the beams



(a) $l_b/\varphi = 8.0$



(b) l_b/φ =12.0



(c) l_b/φ =16.0

Fig. 18 Relationships between $f_{bu}/\sqrt{f_c}$ and $p_u/\sqrt{f_c}$ for Jensen's tests

were extended beyond the plates at the supports.This was done to avoid the risk of horizontal shear failures immediately above the bars. The data and test results for the beams that failed at their anchorages are given in table 6. The failures were all by combinations of side and corner modes of splitting. In beam 1, prior to complete failure, a wedge of concrete with a base length about 40mm at the soffit and its apex at Fig(19) Ghaghei's typical test arrangements point A of Fig.19 was pulled away from the anchorage. The result from this beam is not used

Table 6 Data and results for tests by Ghaghei

Beam no.	f_{cu} $\left(\frac{N}{mm^2}\right)$	c_b (mm)	a (mm)	l_b (mm)	V_u (kN)	p_u $\left(\frac{N}{mm^2}\right)$	f_{bu} $\left(\frac{N}{mm^2}\right)$	$\frac{p_u}{\sqrt{f_{cu}}}$	$\frac{f_{bu}}{\sqrt{f_{cu}}}$
1(1)	44.6	25	465	200	140	4.66	6.59	0.70	0.99
4	44.4	50	465	200	225	7.50	11.31	1.13	1.70
6	41.2	50	465	100	100	6.67	9.33	1.04	1.45
7	41.0	50	382.5	100	160	10.67	13.46	1.67	2.10
8	37.0	50	700	100	75	5.00	10.24	0.82	1.68
9(2)	36.7	50	465	100	110	7.33	10.31	1.21	1.70

Table 7 Details and results of tests by Regan

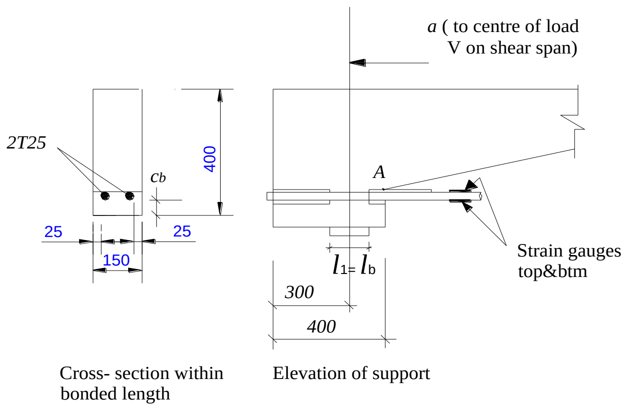
Slab no.	f_{cu} $\left(\frac{N}{mm^2}\right)$	$\frac{a_{eff}}{d}$ (1)	Details of support	V_u (kN)	F_s (kN)	bars	$\frac{f_{bu}}{\sqrt{f_{cu}}}$ (3)	$\frac{p_u}{\sqrt{f_{cu}}}$ (3)
6	47.4	3.82	Full width steel plate	62.5	94.6	all	1.46	0.12
7	46.9	6.76	Full width steel plate	32.0	84.0	all	1.31	0.06
8	50.4	3.82	Full width Fabreeka on steel plate	50.0	65.4	edge centre	0.98 1.46	0.08 0.09
9	44.0	3.82	2 steel plates 150mmwide centred under outer bars	57.0	99.3 81.7	edge centre	1.59 1.31	0.18 0
10	52.1	3.82	3 steel plates 100 mm wide centred under 3 bars	75.0	107.8	all	1.58	0.23

Notes: 1- a_{eff} = effective shear span (M/V) from centre of support to centre of application of V.

2- Fabreeka is a fabric reinforced rubber bearing material- the pads in test 8 were 25mm thick.

3- While V_u is the maximum shear force applied to the slab, the values of f_{bu} and p_u correspond to the maximum forces for individual bars in cases where there were significant differences between the edge and centre bars.

4- $f_{bu} = F_s / \pi\varphi\, l_b$ where l_b =150 mm and F_s = force in a bar, just outside the support = $\varepsilon_s E_s A_s$, where ε_s = measured strain



Fig, 19 Ghaghei's typical test arrangements point A

Note. 1) Excluded from Fig.19and from all later comparisons, 2) beam with a layer of soft fibreboard between soffit and support plate, 3) length of support plate $=l_b$, 4) $f_y = 495 N/mm^2$

The results except that for beam 1 are plotted in Fig. 20. They show an increase in f_{bu} as p_u as increases, no apparent difference in the f_{bu} : p_u relationship between beam 4 with $l_b = 200mm$ and beams 6-9 with $l_b = 100mm$ and a slightly higher strength for beam 9 with fibre – board at the supports than for the otherwise similar beam 6 without the fibreboard.

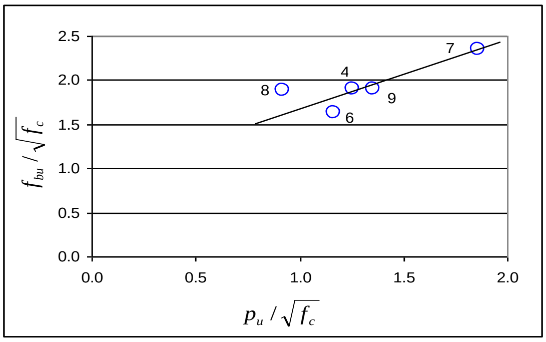


Fig. 20 Relationship between $f_{bu}/\sqrt{f_c}$ and $p_u/\sqrt{f_c}$ for Ghaghei's tests

3.9 Regan (Regan.1997,1-2)

Regan tested a series of slab strips in 5 of which there was no bond within the shear spans. Details of the specimens are shown in Fig.21. The only intentional variables in this group of tests were the shear span/effective depth ratio and the details of the supports, which are given in the table (7), along with the test results. In slabs 6,and 10 the edge and corner bar forces were practically equal up to failure. In slab 8 they were nearly equal up to about 90% of the ultimate load and, beyond this stage, the forces in the outer bars declined while that in the centre bar increased rapidly up to failure, from which it appears that the transverse tension produced by the rubber pad's dilation had more influence in the corner split mode of failure than in the V-notch type of failure around the centre bar (see Fig.21).

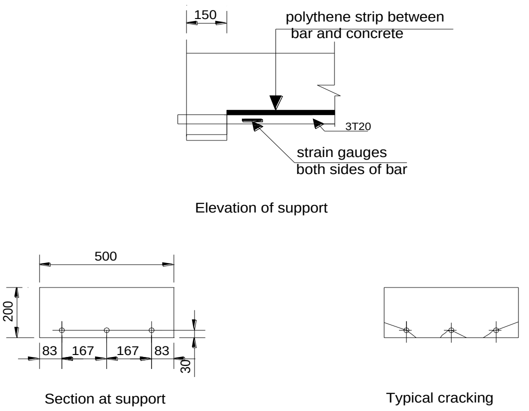


Fig. 21 Test arrangements for Regan's slabs

In slab 9 with the forces in the three bars almost equal up to the maximum load, in the process of the initial failure the load decreased, the edge bar forces increased slightly while the centre bar force decreased considerably. When the load was brought up, the centre bar force increased very slightly while the edge bar forces increased significantly until complete failure occurred at a load just below the previous maximum.

The results in terms of $f_{bu}/\sqrt{f_{cu}}$ and $p_u/\sqrt{f_{cu}}$ are plotted in Fig.22.The point for the edge bars with rubber pad support is clearly well below the others. For the others a reasonable fit is obtained with line A:

$$f_{buA} = f_{bu0} + 1.2p_u \tag{25}$$

Where the value f_{bu0} for the bond strength with $p=0$ is taken as $1.3\sqrt{f_{cu}}$ from the result for the centre bar of slab 9, and f_{bu0} is assumed to be equal for all the bars since c_b/φ is constant and $c_s/\varphi = s/2\varphi$. This is the solution proposed by Regan.

The point for slab 9's centre bar may however be misleading as the plates under the edge bars could have provided some restraint to the centre bar, which might in any case have a higher value of f_{bu0} than the edge bars due to the differences in the patterns of splitting at the two locations. Line B in Fig. 22 represents

$$f_{buB} = f_{bu0} + 2p_u \tag{26}$$

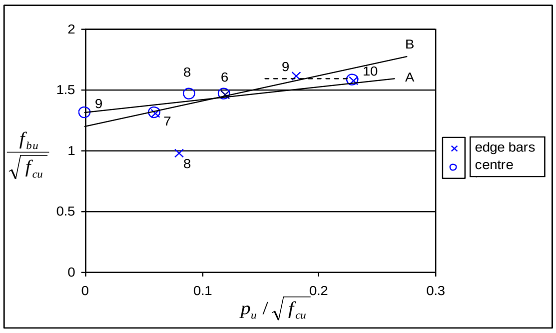
with f_{bu0} reduced to $1.2\sqrt{f_{cu}}$. Line B can be seen to fit most of the points rather better than line A. The exception is that for slab 10. For this slab it is possible that the reduction of the plate width was unfavourable with p_u taken as the actual local bearing pressure. If the average pressure R/b were used as p_u the point for slab 10 would move to the position indicated by the horizontal arrow in the figure. Fig(22) Relationship between the $f_{bu}/\sqrt{f_{cu}}$ and $p_u/\sqrt{f_{cu}}$ forl_b/φ =7.5 in tests by Regan

The test series is too small to be in any way conclusive, but it does raise three interesting points:

1- the influence of different materials at bearings may depend on the mode of failure at the bar considered.
2- the average bearing pressure over the width $(c_s + \varphi + s/2)$ or $(s+\varphi)$ associated with a bar may be a better parameter for representing the influence of transverse pressure than the local pressure at a bar.
3- where the conditions differ for different bars in an anchorage the total anchorage capacity can be less than the sum of the individual bars' anchorage capacities.

3.10 Nielsen (Nielsen,1999)

Nielsen's use of upper-bound plastic theory, extends to cover anchorages at supports. The modes of local failure possible remain unchanged. In type 1 local wedges are formed in front of the bar's ribs and in type 2 the failure surface is a cylinder with a diameter equal to the external diameter of the ribs. The relevant equations for work done in the (local) mechanism remain as in Fig.23, while the work done in the surrounding concrete is modified. When an anchorage is subject to transverse (vertical pressure), the displacements at yield lines cause work to be done against the pressure, i.e. they reduce the external work done. This negative external work is numerically equivalent to extra positive internal work. Fig.23 illustrates the situation for corner mechanisms with centres of rotation on the side faces of a beam.



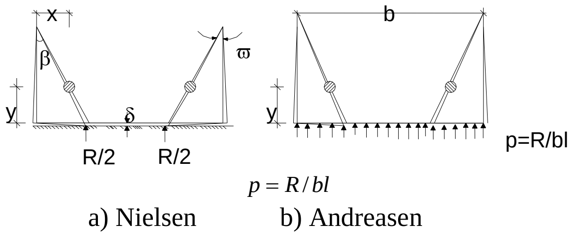


Fig. 23 Corner mechanisms with centres of rotation on the side face of a beam for Nielsen and Andreasen

Fig.23a shows Nielsen's approach. Both the parts of the beam bounded by the yield lines and the support are assumed to be rigid, with the result that the entire reaction acts at the points at the bottom ends of the yield lines, and the work done to the transverse pressure is $W_p = \omega(x + y\tan\beta)R$. Part b of the figure shows the approach of Andreasen, which was also used by Nielsen in the earlier edition of his book and is implicit in most other treatments of transverse pressure. The difference between the two approaches is considerable. In Fig.23a work is done by the whole reaction R and the displacement involved is the maximum (δ). In Fig.23b only a part $[2(x + y\tan\beta)/b]R$ of the reaction is involved and the average displacement is $\delta/2$.

In Nielsen's analysis the failure modes considered are the same as for anchorages without transverse pressure, but for corner failures the results cannot be simplified to a single equation expressed in term of $c_{\min}$ and $c_{\max}$, as the position of the transverse forces must be considered, as must the influence of the pressure on β.

The final results of Nielsen's analysis are essentially as follows:

- Corner Failures

For local mechanisms of type 1a

$$\frac{f_{bu}}{f_c} = 0.12\nu + 0.45\left[B\rho + \frac{b}{\varphi}C\frac{p}{f_c}\right] \tag{27}$$

For local mechanisms of type 2

$$\frac{f_{bu}}{f_c} = 0.28\nu + 0.24\left[B\rho + \frac{b}{\varphi}C\frac{p}{f_c}\right] \tag{28}$$

The local mechanism is generally of type 2 if p is significant.

In eqns. (27) and (28)

For rotation about an axis on the side face

$$B = \left(\frac{\sin\beta}{x/\varphi}\right)\left[\frac{x/\varphi}{\sin\beta} + \frac{y/\varphi}{\cos\beta}\right]^2 - 2 \tag{29}$$

$$C = \sin\beta\left[1 + \frac{y}{x}\tan\beta\right] \tag{30}$$

For rotation about an axis on the bottom face

$$B = \left(\frac{\cos\beta}{y/\varphi}\right)\left[\frac{x/\varphi}{\sin\beta} + \frac{y/\varphi}{\cos\beta}\right]^2 - 2 \tag{31}$$

$$C = \cos\beta\left[\frac{x}{y} + \tan\beta\right] \tag{32}$$

For low transverse pressures, with rotation about an axis on the side face

$$\tan\beta \approx \left[\frac{x/y}{2 + 2.8(y/x)}\right]^{1/3} \tag{33}$$

With rotation about an axis on the bottom face

$$\tan\beta \approx \left[\frac{2 + 2.8\,(x/y)}{(y/x)}\right]^{1/3} \tag{34}$$

For high transverse pressures

$$\sin\beta \approx \sqrt{\frac{\rho\,x}{b.p/f_c}} \tag{35}$$

For any particular case, for example with rotation about an axis on the side face, equations (33) and (35) provide two values for β, with that from(35) depending to some extent on the value adopted for *p*. The corresponding two values of B and C are given by equations (29) and (30) and can be substituted into equations (27) and (28) giving a total of 4 relationships between f_{bu} and *p*. For any value of *p*, the lowest value of f_{bu} is the solution. The position of the axis of rotation is always on the side face if $x > y$. With $x < y$ the axis is on the bottom face for low transverse pressure and solutions dependent on equations (29),(30) and (31) are critical. However at higher pressures the position of the axis moves to the side face.

- Cover Bending and Face Splitting Failures

Where a cover bending or face splitting failure is critical when *p*=0, the type of failure changes to a corner one as *p* increases. This can be treated by determining the ultimate bond stress f_{buo} for *p*=0 from Fig.10 and constructing the initial part of the f_{bu} against *p* relationship as:

$$f_{bu} = f_{buo} + \frac{1}{\pi}.\frac{b}{n\varphi}.\tan\alpha.p \tag{36}$$

where *n* is the number of bars in a horizontal layer and tan α is 1.4 when the local failure mechanism is of type 1a or 0.75 when it is of type 2. This line intersects that for the corner mechanism at a relatively small value of *p*.

The above summary clearly does not cover all possible cases of anchorage at supports. One further situation, that is discussed by Nielsen, is that where there are three bars in a single layer, for which it would appear that the influence of a transverse reaction should be concentrated to the two corner bars. The treatment finally proposed isthat the corner bar expressions should be applied with b/φ replaced by $2b/n\varphi$ and all three bars should be assumed to develop the same bond resistance.

Reference(Nielsen,1999)) includes a limited comparison with the results of tests by Rathkjen(Rathkjen,1972), see section 3.6. For beams with two or three bars in one layer, the correlations between actual and predicted anchorage strengths are good with discrepancies generally no more than 10%. For beams with single central bars, the trend of the results is predicted satisfactorily but deviations of individual results go up to about 25% , on the unsafe side.

The applicability of all of the above is subject to an upper limit dependent on the inclined compression in the web concrete, see Fig.24 . With the web stress limited to $f_{cw} \le f_c$, the maximum value of *p* is given by:

$$p \le \frac{f_c}{1+\left(\frac{T}{R}\right)^2} \quad (37)$$

where *T* is the force in the main steel.

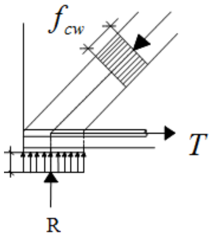


Fig. 24 The limitation of support pressure by concrete web compression

It may be noted that the value of f_c for the limiting web concrete stress is higher than that permitted by EC2's section 6.54 on nodes which gives design limit of $0.85(1-f_{ck}/250)f_{cd}$.

3.11 Cleland *et al*.(Cleland *et al*.2001)

Cleland *et al*. studied the influence of end anchorage on the strength of the end support regions of slab bridges. The research was motivated by the many instances in which existing bridges without deterioration had failed to meet the assessment criteria of BD 44/95(BD 44/95.1995). With safety factors removed, the basic capacity equations of the BD, for slabs without shear reinforcement are:

$$M_u = \sum A_s f_y . d\left[1-\frac{0.84\sum A_s f_y}{f_{cu} bd}\right] \quad (38)$$

$$V_{u1} = 0.24\left(\frac{550}{d}\right)^{0.25}\left(\frac{100\sum A_s}{bd}\right)^{0.33} f_{cu}{}^{0.33} bd \quad (39)$$

The development of the full M_u of eqn.(38) is dependent on there being a full anchorage length from the section, at which the moment is taken , for the bars considered in $\sum A_s$. In eqn. (39) the term $\sum A_s$ is the area of the longitudinal tension reinforcement continuing at least an effective depth beyond the section considered, except that at supports the full area may be used provided it has an anchorage equivalent to 12 times the bar size beyond the centre line of the support, with no bend beginning before the centre line. Where this is not the case, the effective size of a tension bar may be taken as 1/12 th of the anchorage beyond the centre line of the support.

For a section at a distance $a_v < 3d$ from the face of a support, the shear resistance can be increased to

$$V_{u2} = \frac{3d}{a_v}.V_c \le \min\begin{Bmatrix} 0.92\sqrt{f_{cu}} \quad bd \\ 7(N/mm^2)bd \end{Bmatrix} \quad (40)`$$

but only if there is an anchorage equivalent to 20 times the bar size (beyond the centre line of the support).

Difficulties with assessments have often arisen where the anchorages at supports are less than 12φ , or less than 20φ if an enhanced shear resistance is required.

Cleland *et al*. made 47 tests on 24 simply supported slab strips with a cross-section $bxh = 380x225mm$. The tension reinforcement was of 12mm plain round bars with an average yield strength of 325*N/mm*^{2}and the ratio of reinforcement was varied from 0.7 to 1.9%. Transverse horizontal reinforcement was provided by ten 8mm bars at 76mm centres at each end. The concrete cube strength was generally between 28 and 38*N/mm*^{2}, but there were three specimens with $f_{cu} = 60\,N/mm^2$. The anchorages (l'_b) beyond the centres of the supports were:

For $l'_b/\varphi =$ 3 or 6 straight

For $l'_b/\varphi = 9$ or 9.5 non-standard 90^o bend

For $l'_b/\varphi = 12$ standard 90^o bend

For $l'_b/\varphi = 20$ standard 90^o bend +long tail

The experimental ultimate strengths were compared with resistances calculated by a modified version of BD 44/95, in which the basic shear resistance was taken as V_c even when l'_b was less than 12φ, while $(3d/a_v)V_c$ was used only when $l'_b = 20\varphi$. The predictions made in this way were better than those from an unmodified BD44/95, but the resulting mean and standard deviation of experimental/calculated strength were 1.49 and 0.67 showing little correlation between actual and predicted resistances. The authors proposed a more logical approach with the capacity of a bar's anchorage beyond the outmost crack calculated on the basis of an ultimate bond stress $f_{bu} = 0.64\sqrt{f_{cu}}$ in N/mm^2, which was derived from measurements of bars strains in the anchorage zones of seven slabs.

$$F_{su} = \frac{4f_{bu}}{\varphi}\left(l'_b + \frac{1}{2} \text{ support length}\right) \quad (41)$$

$$\le F_y \quad if \quad a_v/d \le 1.0$$

$$F_{su} = \frac{4f_{bu}}{\varphi}\left(l'_b + \frac{1}{2} \text{ support}) \; +0.3d\right) \quad (42)$$

$$\le F_y \quad if \quad a_v/d > 1.0$$

For $F_{su} < F_y$ the lever arm(*z*) is calculated from a cracked elastic sectional analysis and the ultimate moment $M_u = \sum F.z$, This equation replaces eqn.2.62.

The final expression for shear strength is

$$V_c = 0.27\left(\frac{F_{su}}{F_y}.\frac{100\sum A_s}{bd}.f_{cu}\right)^{0.33}\left(\frac{500}{d}\right)^{0.25}\frac{3d}{a_v} \quad (43)$$

with the last term applying only where $a < 3d$. For no stated reason the constants 0.24 and 550 of BD 44/95 have been replaced by the 0.27 and 500 of BS5400 even though $3d/a_v$ is retained rather than BS5400's $2d/a_v$.

The approach produced a significant improvement over the previous one, with the mean test strength / calculated strength reduced to 1.18 and the standard deviation reduced to 0.21. There was also a improvement in the prediction of failure modes.The calculated strengths are however about 20% too high to be characteristic resistances (for which mean-1.64 std.dev ≥ 1.0) . The main source of the improvement is equations 41 and 42 which have a clear physical basis. If the less justifiable eqn. 43 were omitted and all the predictions were based on eqns.41 and 42 and $M_u = \sum F_{su} z$, the results from the comparison would change very little. The only obvious problem with eqns. 41 and 42is that, although the increase of f_{bu} above the unfactored BS value of $0.39\sqrt{f_{cu}}$ does make an allowance for the effect of transverse pressure at a support, the allowance is independent of the magnitude of the pressure. It is also independent of the form of the bar-straight or hooked-which may or may not be appropriate

4. Commentary

Analyses of behaviour adjacent to a bar

Only the Danish plastic theory and the papers of Cairns and Jones(Cairns and Jones.1976,1979,19951-2,1996) consider the actions adjacent to bars in detail.

In practical terms both assume that the deformation of bars can be modelled as annular ribs and that both the bond stresses and the radial compression can be treated as uniform around the circumference. These may be adequate assumptions in practice, but those relating to stress distributions do not seem physically realistic in some cases,e.g. those illustrated in Fig.25.The relationships between the bond (or shear) stresses and the outward compressions are functions of the geometry of the failure surface, the failure criterion for the concrete and the restraint from the surrounds.

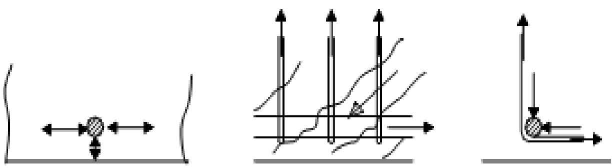


Fig. 25 Cases of non-polar symmetric restraints

In the plastic theory approach, the geometry of the failure surface is also a function of the restraint from the surrounds. With low restraints, the surface is of the wedge type and the maximum inclination of the wedge faces to the bar axis is 37^o (arc tan 0.75), which follows from the failure criterion adopted for the concrete. As the restraint increases, this angle decreases and the failure surface eventually transforms to the cylindrical one. Coinciding upper and lower bound solutions bound, restraint is defined in terms of the work done in the surrounds, per unit slip of the bar. For the lower bound, restraint is defined by the radial pressure per unit slip. The solutions define the local stresses and the direction of the relative displacement between the bar and the concrete outside the failure surface.

In Cairns and Jones, only the wedge type of failure is considered and the wedge angle is taken to be constant, this allows the stress at failure to be defined directly from the local failure criterion and the restraint available from the surrounds expressed as a stress or force.

Tepfers (Tepfers1973) makes the simpler assumption that the compression resultants are not only polar-symmetric, but are at 45^o to the bar axis, which makes the outward pressure on the surrounds equal to the bond stress. No local failure criterion is applied. The other methods reviewed do not consider either the local stresses or the local displacements, but treat bond failure directly in terms of the relationships between ultimate bond stresses and the characteristics of the surrounds.

It is only in the plastic theory approach and that of Cairns and Jones that any influence of the geometry of the deformations of bars can be taken into account in a rational manner. For the wedge type of failure the relevant dimensions can be expressed in terms of the relative rib area which is equal to the projected area of ribs per unit length of bar divided by the nominal surface area of the bar per unit length. (f_R = approx h_r / s_r for annular ribs). For the cylindrical type of surface the relevant parameter is the area of the fracture surface divided by the nominal surface area, which in practice varies much less than the relative rib area.

Analyses of the surrounds to a bar

In all the methods considered, where an analysis of the surrounds is made, it is simplified to a 2-dimensional treatment of a plane, perpendicular to the bar's axis. This is a significant simplification, as stresses in the third direction, parallel to the bar, could influence results. It also means that the stress field treated does not really coincide with that used in the rest of the analysis of a member and that no distinction is made between conditions in a simple pull-out test and those in a beam subjected to bending and shear. The loading from the local surface around a bar follows from the assumption of polar-symmetry, which has already been criticized. Within the overall limitations above, the methods of analysis vary. In the plastic theory approach, the analyses are upper bound treatments of possible failure mechanisms. The parts of the section between yield lines are treated as rigid bodies and deformations are concentrated to the yield lines. They are expressed in terms of deformations per unit slip of bar. This allows a wide range of the failure modes involving both translations and rotations to be analysed and the three modes most likely to be critical are those shown in Fig.26.

Where the surrounds are plain concrete, it is assumed to be stressed to its plastic limit at all yield lines. Transverse pressure can be taken into account by treating the negative work done against the transverse loading as being equivalent to positive work in the surrounds. The treatments of this by Andreasen and Nielsen differ in their assumptions about the positions at which the transverse forces act. Andreasen assumes support reactions to be uniformly distributed over the areas of supports, whereas Nielsen takes account of the deformations involved in the failure mechanisms considered. The difference between the two approaches is illustrated by Fig.27. It seems likely that actual behavior will lie between the two extremes and depend on the stiffnesses of supports and the ability of the cover to resist the moments implied by Nielsen's approach.

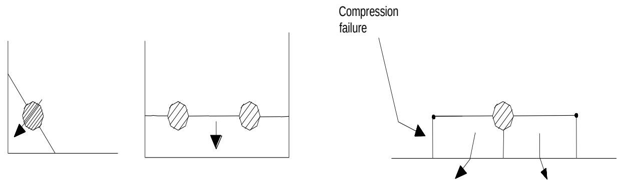


Fig. 26 Movements in surrounds at failure according to Nielsen

Treatment of the influence of transverse reinforcement involves some difficulties. Andreasen assumes that both the concrete and the transverse reinforcement develop plastic stresses at yield lines. Nielsen considers two possibilities either the concrete resistance is lost and the steel yields or the concrete acts at its plastic resistance while the steel develops a relatively low stress compatible with the retention of the concrete's strength. The last of these possibilities seems most likely to be realistic for simple supports.

Tepfers provides two analyses of the surrounds. The first considers the cracking of a minimal surround- a hollow cylinder with an internal diameter equal to the bar size and an external diameter equal to that of the bar plus twice its minimum cover. The second considers a variety of failure surfaces at fixed angles in the beam's cross-section.

The first analysis is used to provide results for the bond stress at which radial cracking penetrates the minimum cover. Two calculations are made. In one the concrete is treated as a brittle elastic material, and in this the maximum bond stress is normally achieved in a state after the first cracking has progressed some distance from the bar. In the other the concrete is treated as behaving plastically and $f_{bc} = 2c_d f_{ct} / \varphi$. The average of the two values of the bond stress is treated as that required to crack the minimum cover. These cracking stresses would be the ultimate bond stresses for bars embedded in cylinders of diameter $\varphi + 2c_d$.

The second analysis considers bond failures in realistic cross-sections. It was developed for the lap splices but can be applied to anchorages, in which case the failure surfaces considered are as shown in Fig.28together with the tensions in the concrete at them. The distribution of bond stresses along an anchorage may be constant, but may also be as determined by bond-slip theory either for the whole length or for a part length away from the loaded end at which there remains a residual bond stress after cracking.

The possibilities here are numerous, not only in respect of the distribution of bond along an anchorage, which can introduce an influence from l_b / φ , but also in terms of the stresses in the transverse reinforcement, which may be limited by the need for compatibility with the deformation of uncracked concrete, but can in other cases reach yield values. There are even cases in which all the relevant transverse reinforcement yields while the concrete tensions fall to zero. These variations together with the six failure modes to be considered make the method unsuitable for practical use unless very considerablesimplifications can be made.

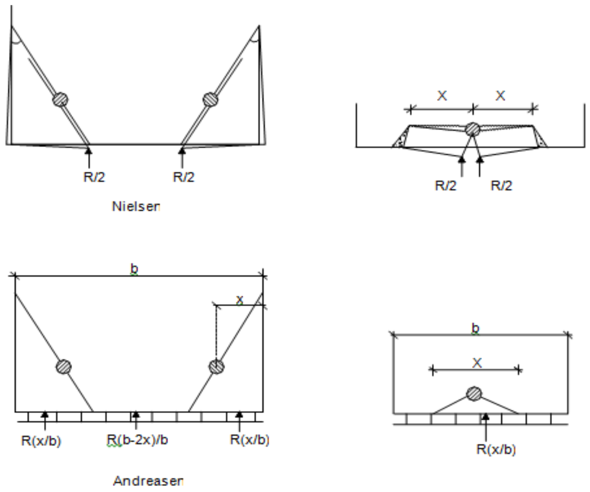


Fig. 27 Distributions of reactions across widths of supports

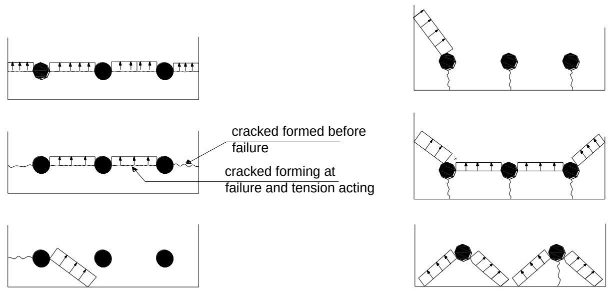


Fig. 28 Splitting pattern types by Tepfers

The approach by Morita and Fujii(Morita and Fujii,1982)considers just three failure patterns and assumes that the bond resistance is proportional to the length of the failure surface. The critical failure pattern is thus readily identified as that giving the minimum length of failure surface. Transverse reinforcement crossing a failure surface is assumed to make an additive contribution to bond resistance which is dependent on the area of transverse steel crossing the failure surface and its inclination to the direction of the movement at the surface. The approach can be extended to treat the influence of transverse pressure by the method developed by Nagatomo and Kaku(Nagatomo and Kaku.2005). The approach of Morita and Fujii has an attraction, as it offers some rationale in the treatment of cover and bar spacings while being simple enough for design use. It is however very close to pure empiricism.

Conclusions

The main interim conclusions drawn from researches on straight anchorages of main bars at simple supports are as follows:

a) There are many empirical equations for bond strength and as their reliability can only be judged in relation to test results.

b) Plastic theory as developed by Nielsen, Andreasen and Hess appears to be the most complete analytical treatment of bond and the description referred to above is generally based upon it. The theory is however complicated and difficult to apply. For this reason it may be seen as a very useful point of reference.

c) Two important characteristics of conditions at simple supports are that anchorage lengths are short and that the support reaction produces compressions perpendicular to the plane of the bars. Because of the first of these the treatment of bond strength must take account of the ratio of the anchorage length to the bar diameter. In relation to the second, there is ample evidence showing that transverse pressure can increase bond strength.

d) The positions and directions of bars during concreting affect bond resistance at least if the concrete exhibits settlement. However as the bars anchored at simple supports are bottom bars, only such bars were tested in order to limit the size of the programme required and top -cast bars are not considered further here.

e) The geometry of the deformations of the bars can affect bond resistance even if codified minima for relative rib areas are respected. However the geometry is not specified in most papers and considerable testing would be required if the effect of various geometries were to be investigated over ranges of other variables. It was therefore decided not to pursue this subject and to accept that there could be some scattering of results from different sources due to differences in bar deformations, but that any detailed information on deformations should be noted.

f) It is clear that the type of test specimen and the way in which loads and reactions are applied to it can materially affect the results of bond tests.

g) Some results of tests with transverse reinforcement at supports are available in the literature. In most cases the amount of transverse reinforcement at a support is small and no very great effect is to be expected from it.

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