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Review Article

Optimization of process parameters by Taguchi based Grey Relational Analysis: A Review

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Abstract

The aim of this paper to review the Taguchi based Grey relational analysis is used to find the best process parameters and improved quality results. The GRA was proposed to optimize the multi response problem by making use of the grey relational coefficient and grey relational grade. GRA is attempted to integrate multiple responses, it is feasible to combine GRA with Taguchi method to provide an optimal constitute of processing parameters for the cases with multiple quality characteristics. The present work focused by using L27 Orthogonal Array (OA) on the processing steps to get the optimal values, and Analysis of variance (ANOVA) was employed to investigate the characteristics and experimental results are provided the effectiveness of this approach. This technology has met the current needs of industry owing to its shorter design cycles and improved the design of quality.

Keywords: *Taguchi based Grey Relational Analysis, Orthogonal array, optimize, Analysis of variance (ANOVA).*

1. Introduction

The Taguchi method is commonly used in the optimization of single quality characteristic. However, the optimization of process parameters could be difficult as more than one quality characteristic are used to represent the overall quality. Therefore, The Grey relational analysis based on the Grey system theory proposed by (Deng, 1989) can be used to solve complicated inter-relationships among the multiple performance characteristics effectively. The grey relational coefficient can express the relationship between the desired and actual experimental results and the grey relational grade is simultaneously computed corresponding to each quality characteristic. A system for which necessary information is completely known is a 'white' system on the other hand if necessary information is completely unknown is called 'black' system and any system between these information is called 'grey' system (Huang J T & Liao Y S, 2003).

2. Plan of experiment

The experimental design is widely used in various fields including industry, medicine and psychology. The fractional factorial designs are effective when the numbers of factors considered are large and when it is difficult to experiment all the combinations. A lot of factors are often taken up at the same time at the early stage of the problem solving. The number of experiment increases in the factorial method as the number of factors considered increases. Then, the obtaining necessary information can be done by an experiment frequency. Taguchi method uses a special design of orthogonal arrays to study the entire parameter space with a small number of experiments(BalaMurganGopalsamy, BiswanathMondal, SukumalGhosh, 2009; D. Philip Selvaraj, P. Chandramohan , 2010; Ross, P. J., 1998.). The methodology of Taguchi for three factors at three levels is used for the implementation of the plan of experiments .Orthogonal arrays (OA) represent a simplified method of putting together an experiment. The original development of the concept was by Sir R.A. Fischer (Mahajan, 2008) of England in 1930s. Taguchi constructed a special set of orthogonal arrays. The 27 in the designation L_{27} represents the number of rows, which is also the number of treatment conditions. Each row thus, represents a trial condition with factor levels indicated by the numbers in the row. The top of the orthogonal array represents the maximum number of factors that can be used; which in this case is thirteen. The levels are assigned by 1, 2 and 3. If more levels occur in the array then 4, 5 and so forth are used. The orthogonal property of the OA is not compromised by changing the rows or the columns. The array forces all experimenters to design almost identical experiments. It assures consistency of design by different experiments. The stages to go through are:

1. Selection of the factors to be evaluated.
2. Selection of the number of levels.
3. Selection of the appropriate orthogonal arrays.
4. Assignment of factors to the columns.
5. Conduct the experiment.
6. Analyze the result.
7. Carry out the experiment.

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In this experiment it consists of three levels and three parameters as shown below.

Table 1: Process control parameters and their levels according to TGRA

Parameter	Units	Symbol	Level 1	Level 2	Level 3
Speed	(rpm)	A	A_1	A_2	A_3
Feed	(mm/tooth)	B	B_1	B_2	B_3
Depth of cut	(mm)	C	C_1	C_2	C_3

L27 orthogonal array

In the factorial experiment, it is necessary to experiment by combining the levels of the factor taken up all. Orthogonal array (OA) is the method of conducting only a part of all possible combinations of factors and their levels. When the levels of all the factors taken up are three, the three-level orthogonal array is used. Table 2. (Benardos and Vosniakos, 2002) shows the *L*27 orthogonal array.

3. Taguchi Grey Relational Analysis

Experiments are designed using Taguchi method so that effect of all the parameters could be studied with minimum possible number of experiments. Using Taguchi method, appropriate Orthogonal Array has been chosen and experiments have been performed as per the set of experiments designed in the orthogonal array. Signal to Noise ratios are also calculated for analyzing the effect of machining parameters more accurately.There are 2 Signal-to-Noise ratios of common interest for optimization of static problemsused in study as are:

(I) Smaller-the-Better:

$$\eta = -10 \log 1/n\sum_{i=1}^{n} y_i^2 \quad (1)$$

(II) Larger-the-Better:

$$\eta = -10 \log \sum_{i=1}^{n} 1/y_i^2 \quad (2)$$

Where, η- Signal to Noise (S/N) Ratio,
y_i– i^{th}observed value of the response,
n - Number of observations in a trial,
y - Average of observed values (responses)

A. Data Pre Processing

In grey relational analysis, the data pre-processing is the first step performed to normalize the random grey data with different measurement units to transform them to dimensionless parameters. Thus, data pre-processing converts the original sequences to a set of comparable sequences. Different methods are employed to pre-process grey data depending upon the quality characteristics of the original data. Experimental data i.e. measured features of quality characteristics of the product are first normalized ranging from zero to one. This process is known as grey relational generation. (C. C. Tsao, 2009; NihatTosun, 2006).

In grey relational generation, the normalized data corresponding to lower-the-better (LB) criterion can be expressed as:

$x_i(k)$:
$$=\max y_i (k) – y_i (k)/ \max y_i(k) – \min y_i (k) \quad (3)$$

For higher-the-better (HB) criterion, the normalized data can be expressed as:

$x_i(k)$:
$$= y_i (k) – \min y_i (k)/ \max y_i(k) – \min y_i (k) \quad (4)$$

Here $x_i (k)$ is the value after the grey relational generation, min y_i (k) is the smallest value Of y_i (k)for the kth response, and max $y_i(k)$ is the largest value of y_i (k) for the kth response. An ideal sequence x_o (k) is for the responses. The purpose of grey relational grade is to reveal the degrees of relation between the sequences say, [$x_{o\ (k)}$ and $x_i (k)$, i=1,2,3…………,27)

B. Grey Relational Coefficient and Grey Relational Grade

Next, based on normalized experimental data, grey relational coefficient is calculated to represent the correlation between the desired and actual experimental data. Then overall grey relational grade is determined by averaging the grey relational coefficient corresponding to selected responses. The overall performance characteristic of the multiple response process depends on the calculated grey relational grade. This approach converts a multiple-response process optimization problem into a single response optimization situation; the single objective function is the overall grey relational grade. The optimal parametric combination is then evaluated by maximizing the overall grey relational grade.

The grey relational coefficient $\xi_i(k)$:

$$= \Delta_{min\ +} \psi\ \Delta_{max} / \Delta_{0i} (k) + \psi\ \Delta_{max} \quad (5)$$

deviation sequence, Δ_{0i} (k) :

$$= || x_0 (k) – x_i\ (k) || \quad (6)$$

is difference of the absolute value x_o (k) and x_i (k) and ; ψ is the distinguishing coefficient $0 \le \Psi \le 1$;Δ_{min}= the smallest value of Δ_{oi}; and Δ_{max} = largest value ofΔ_{oi} . After averaging the grey relational coefficients, the grey relational gradeγ_i can be computed as

$$\gamma_{i\ =} i/n \sum_{k=1}^{n} \xi i(k) \quad (7)$$

Here n=number of process responses. The higher value of grey relational grade corresponds to intense relational degree between the reference sequence x_0 (k) and the given sequence x_i (k) The reference sequence x_0 (k) represents the best process sequence. Therefore, higher grey relational grade means that the corresponding parameter combination is closer to the optimal. In the aforesaid study, it has been assumed that all quality features are equally important. But in practical case, it may not be so. Depending on the area of application, different response may have different preference and thereby,

Table 2: *L*27 Orthogonal array

RUN NO.	A(1)	B (2)	AB (3)	A^2B (4)	C (5)	AC (6)	A^2C (7)	BC (8)	ABC (9)	A^2BC (10)	B^2C (11)	AB^2C (12)	A^2B^2C (13)
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2	2	3	3	3
5	1	2	2	2	2	2	2	3	3	3	1	1	1
6	1	2	2	2	3	3	3	1	1	1	2	2	2
7	1	3	3	3	1	1	1	3	3	3	2	2	2
8	1	3	3	3	2	2	2	1	1	1	3	3	3
9	1	3	3	3	3	3	3	2	2	2	1	1	1
10	2	1	2	3	1	2	3	1	2	3	1	2	3
11	2	1	2	3	2	3	1	2	3	1	2	3	1
12	2	1	2	3	3	1	2	3	1	2	3	1	2
13	2	2	3	1	1	2	3	2	3	1	3	1	2
14	2	2	3	1	2	3	1	3	1	2	1	2	3
15	2	2	3	1	3	1	2	1	2	3	2	3	1
16	2	3	1	2	1	2	3	3	1	2	2	3	1
17	2	3	1	2	2	3	1	1	2	3	3	1	2
18	2	3	1	2	3	1	2	2	3	1	1	2	3
19	3	1	3	2	1	3	2	1	3	2	1	3	2
20	3	1	3	2	2	1	3	2	1	3	2	1	3
21	3	1	3	2	3	2	1	3	2	1	3	2	1
22	3	2	1	3	1	3	2	2	1	3	3	2	1
23	3	2	1	3	2	1	3	3	2	1	1	3	2
24	3	2	1	3	3	2	1	1	3	2	2	1	3
25	3	3	2	1	1	3	2	3	2	1	2	1	3
26	3	3	2	1	2	1	3	1	3	2	3	2	1
27	3	3	2	1	3	2	1	2	1	3	1	3	2

different tolerance limit. For example, the surface roughness and the MRR; both may be or may not be of equal importance. It depends on the decision maker. Therefore, different weightages have to be assigned to different responses. If different priority weightages have been assigned to different responses, the equation for calculating overall grey relational grade becomes:

$$\gamma_i = \sum_{k=1}^{n} w_k \xi_i(k) \Big/ \sum_{k=1}^{n} w_k \qquad (8)$$

Here γ_i , is the overall grey relational grade for i^{th}experiment, $\xi_i(k)$ is the grey relational coefficient of k^{th} response in i^{th} experiment and w_k is the weightage assigned to the i^{th} response. So basically in this study, a TGRA has been used to establish a correlation between the independent variables and the performance characteristics; therefore, the experiments were performed according to a Taguchi design of experiments.

C. The analysis of variance (ANOVA)

Study of the ANOVA table for a given analysis determines, whether a factor requires control or not. Major part of this chapter has been taken from (Montgomery, 2005 &Mahajan, 2008). Once the optimum condition is determined, it is usually a good practice to run a confirmation experiment. The analysis of variance (ANOVA) test establishes the relative significance of the individual factors and their interaction effects. The steps are as follows:

Step1. Total of all results:

T = sum of all results.

Step2. Correction factor:

$C.F = T^2/n$

Where n = total no. of experiments

Step3. Total sum of squares:

It is a measure of the deviation of the experimental data from the mean value of the data. Summing each squared deviation emphasizes the total deviation. Thus

$$S_T = \sum_{i=1}^{n} y_i^2 - C.F$$

Where S_T is the total sum of square and C.F is the correction factor.

Step4. Factor sum of squares:

$S_A = A_1^2/N_{A1} + A_2^2/N_{A2} + A_3^2/N_{A3} - C.F$

$S_B = B_1^2/N_{B1} + B_2^2/N_{B2} + B_3^2/N_{B3} - C.F$

$S_C = C_1^2/N_{C1} + C_2^2/N_{C2} + C_3^2/N_{C3} - C.F$

$S_{A\times B} = (A\times B)_1^2/N_{(A\times B)1} + (A\times B)_2^2/N_{(A\times B)2} + (A\times B)_3^2/N_{(A\times B)3} - C.F$

$S_{B\times C} = (B\times C)_1^2/N_{(B\times C)1} + (B\times C)_2^2/N_{(B\times C)2} + (B\times C)_3^2/N_{(B\times C)3} - C.F$

$S_{A\times C} = (A\times C)_1^2/N_{(A\times C)1} + (A\times C)_2^2/N_{(A\times C)2} + (A\times C)_3^2/N_{(A\times C)3} - C.F$

$S_e = S_T - (S_A + S_B + S_C + S_{A\times B} + S_{B\times C} + S_{A\times C})$

Where A_1= sum of results y_i where factor A_1 is present.
A_2= sum of results y_i where factor A_2 is present.

A_3= sum of results y_i where factor A_3 is present.
N_{A1}= total no. of experiments in which factor A_1 is present.
N_{A2}= total no. of experiments in which factor A_2 is present.
N_{A3}= total no. of experiments in which factor A_3 is present.
S_e = sum of square for error term.

Step5. Total and Factor Degrees of freedom (DOF):

It is a measure of the amount of information that can be uniquely determined from a given set of data. DOF for data concerning a factor equals one less than the number of levels. An experiment with n trial and r repetitions of each trial has n × r trial runs. The total DOF becomes:
DOF total = total number of experiments – 1

$f_T = n \times r - 1$
DOF of each factor = number of levels of factor – 1
f_A= (number of levels of factor A – 1)
f_B= (number of levels of factor B – 1)
f_C= (number of levels of factor C – 1)
$f_{A\times B} = f_A \times f_B$
$f_{B\times C} = f_B \times f_C$
$f_{A\times C} = f_A \times f_C$

The DOF of the error term is given by:

DOF of error (f_e) = f_T– ($f_A + f_B + f_C + f_{A\times B} + f_{B\times C} + f_{A\times C}$)
Where A, B, C are the three factors and f_A ,f_B , f_C are their DOF

Step6. Mean square (variance):

It measures the distribution of the data about the mean of the data.

Variance = Sum of squares / Degree of freedom

$V_A = S_A/ f_A$
$V_B = S_B/ f_B$
$V_C = S_C/f_C$
$V_{A\times B} = S_{A\times B}/ f_{A\times B}$
$V_{B\times C} = S_{B\times C} / f_{B\times C}$
$V_{A\times C} = S_{A\times C} / f_{A\times C}$
$V_e = S_e/ f_e$

Step7. Percentage contribution:

It is obtained by dividing the sum of square of the factor by total sum of square andmultiplied by 100. It is denoted by P and can be calculated using the following equation.

$P_A = S_A/S_T \times 100$
$P_B= S_B/S_T \times 100$
$P_C= S_C/S_T \times 100$
$P_{A\times B} = (S_{A\times B}/ S_T)\times 100$
$P_{B\times C} = (S_{B\times C}/ S_T)\times 100$
$P_{A\times C} = (S_{A\times C}/ S_T)\times 100$
$P_e = S_e / S_T \times 100$

Step8. Variance ratio (F-ratio):

It is the ratio of variance due to the effect of a factor and variance due to the error term. This ratio is used to measure the significance of the factor under investigation with respect to the variance of all the factors included in the error term. The F value obtained in the analysis is compared with a value from standard F-tables for a given statistical level of significance.

F- Calculated
F ratio = Variance due to factor/ Variance due to error term
$F_A= V_A/V_e$
$F_B= V_B/V_e$
$F_C= V_C/V_e$
$F_{A\times B} = V_{A\times B}/ V_e$
$F_{B\times C}= V_{B\times C}/ V_e$
$F_{A\times C}= V_{A\times C}/ V_e$
F- Tabulated

From the F- table find the F value at confidence level between 90 and 99% at

n_1 = DOF at factor , n_2 = DOF at error term
If F- Calculated > F- Tabulated then that factor is significant otherwise insignificant

D. Predicted optimum conditions

The predicted values of GRG at the optimal levels are calculated by using the relation:

$$\check{n}= nm + \sum_{i=1}^{o}(nim - nm) \quad (9)$$

Where ň= Predicted value after optimization
nm = Total mean value of quality characteristic
nim = Mean value of quality characteristic at optimum level of each parameter
o = Number of main machining parameters that effects the response parameters

4. Confirmation Test

The confirmation tests were performed by selecting the set of parameters and the confidence level is taken as numeric value 90 to 95%, achieving best value of the response. It is an expected result at optimum condition. From the analysis, error associated with all the experimental results were its value also may obtain. Error showed due to the more parameters also be considered while machining or testing of the material property.

Conclusion

The Taguchi Grey Relational Analysis is especially suitable for industrial use, but it can also be used for scientific research purposes and it emphasizes a mean performance characteristic value close to the target value rather than a value within certain specified limits, thus improving the product quality. In present study, design of experiment was performed by L27 orthogonal array was chosen by considering three factors and three levels were employed to analyze the influence of process parameters by using grey relation analysis and analysis of variance to get the optimal conditions and performances that means the best parameters within the experimental results.

Finally, the optimum combinations of parameters are achieved by confirmation tests were conducted.

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