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Research Article

A Study on Elliptic WP-Bailey Transformation for Hypergeometric Series

Sonia Badesara[Å*]

[Å]DPHE Jhajjar, India

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Abstract

Ramanujan studied hypergeometric series extensively before is gone to England. The chapter 12, 13 and 15 in the first notebook deal with these series. There are many left hand pages in the first notebook that contain facts about hypergeometric series and hypergeometric functions. When I try to understand Ramanujan's impact on mathematics, it is useful to see what was known when he started to work, to try to figure out what he knew, and then see what he rediscovered or added to known mathematics. Making use of an elliptic WP-Bailey pair and certain theorems for constructing elliptic WP-Bailey chains, I has established some transformation formulae for theta hypergeometric series.

Keywords: Hypergeometric functions, Bailey pair

1. Basic Hypergeometric series and their elliptic analogues

[1]For $|q|<1$, the q- shifted factorial for all integers n is defined by,

$$[a;q]_\infty = \prod_{k=0}^{\infty}\left(1-aq^k\right)$$

and

$$[a;q]_n = \frac{[a;q]_\infty}{\left[aq^n;q\right]_\infty}.$$

Specially,

$$[a;q]_n = \begin{cases}(1-a)(1-aq)\ldots\left(1-aq^{n-1}\right), & n>0\\ 0, & n=0.\end{cases}$$

With the usual condensed notation

$$\left[a_1,a_2,\ldots,a_n;q\right]_r = \left[a_1;q\right]_r\left[a_2;q\right]_r\ldots\left[a_n;q\right]_r$$

It can define a basic hypergeometric series as,

$${}_{r+1}\Phi_r\left[\begin{matrix}a_1,a_2,\ldots,a_{r+1};q;z\\ b_1,b_2,\ldots,b_r\end{matrix}\right] = \sum_{n=0}^{\infty}\frac{\left[a_1,a_2,\ldots,a_{r+1};q\right]_n z^n}{\left[q,b_1,b_2,\ldots,b_r;q\right]_n}.$$

Here it is assumed that the b_i are such that none of the terms in the denominator of the right hand side vanishes.

A ${}_{r+1}\Phi_r$

is said to be very well-poised if

$$a_1q = a_2b_1 = a_3b_2 = \ldots = a_{r+1}b_r \text{ and } a_2 = -a_3 = q\sqrt{a_1}.$$

A very well-poised ${}_{r+1}\Phi_r$ series is expressed as

$${}_{r+1}W_r\left[a_1;a_4,\ldots,a_{r+1};q\right] = {}_{r+1}\Phi_r\left[\begin{matrix}a_1,q\sqrt{a_1},-q\sqrt{a_1},a_4,\ldots,a_{r+1};q;q\\ \sqrt{a_1},-\sqrt{a_1},\frac{a_1q}{a_4},\ldots,\frac{a_1q}{a_{r+1}}\end{matrix}\right].$$

Here it always assume that parameters obey the relation,

$$\left(a_4a_5\ldots a_{r+1}\right)^2 = a_1^{r-3}q^{r-5}.$$

To introduce the elliptic analogues of basic hypergeometric series it is denoted by $\theta(z;p)$ the modified Jacobi theta function

$$\theta(z;p) = [z;p]_\infty\left[p/z;p\right]_\infty \text{ for } |p|<1. \tag{1}$$

Using (1) one can define an elliptic analogue of the q-shifted factorial by

$$[a;q,p]_n = \theta(a;p)\theta(aq;p)\theta\left(aq^2;p\right)\ldots\theta\left(aq^{n-1};p\right),$$

$$[a;q,p]_0 = 1$$

and

$$[a;q,p]_{-n} = \frac{1}{\left[aq^{-n};q,p\right]_n} = \frac{q^{n(n+1)/2}}{(-a)^n\left[q/a;q,p\right]_n}.$$

It is defined a balanced, very well poised elliptic hypergeometric series by,

$${}_{r+1}V_r\left[a_1;a_6,a_7,\ldots,a_{r+1};q,p;z\right] = \sum_{n=0}^{\infty}\frac{\theta\left(a_1q^{2n};p\right)\left[a_1,a_6,a_7,\ldots,a_{r+1};q,p\right]_n z^nq^n}{\theta\left(a_1;p\right)\left[q,a_1q/a_6,a_1q/a_7,\ldots,a_1q/a_{r+1};q,p\right]_n}, \tag{2}$$

(Gasper, G. & Schlosser)

*Corresponding author: **Sonia Badesara**

Where,

$$\frac{\theta\left(aq^{2n};p\right)}{\theta(a;p)}=\frac{\left[q\sqrt{a},-q\sqrt{a},q\sqrt{a/p},-q\sqrt{ap};q,p\right]_n(-q)^{-n}}{\left[\sqrt{a},-\sqrt{a},\sqrt{ap},-\sqrt{a/p};q,p\right]_n}. \tag{3}$$

Warnaar defined the elliptic WP Bailey pair as the pair of sequences

$$\left\{\alpha_n(a,k;q,p),\beta_n(a,k;q,p)\right\}$$

satisfying the equation

$$\beta_n(a,k;q,p)=\frac{\left[k,k/a;q,p\right]_n}{[q,aq;q,p]_n}\times \sum_{r=0}^{n}\frac{\left[kq^n,q^{-n};q,p\right]_r}{\left[aq^{1-n}/k,aq^{1+n};q,p\right]_r}\left(\frac{aq}{k}\right)^r\alpha_r(a,k;q,p). \tag{4}$$

Following are theorems for constructing elliptic WP-Bailey chains.

Theorem 1

If

$$\left\{\alpha_n(a,k;q,p),\beta_n(a,k;q,p)\right\}$$

is an elliptic WP Bailey pair, then so is the pair

$$\left\{\alpha'_n(a,k;q,p),\beta'_n(a,k;q,p)\right\}$$

given by

$$\alpha'_n(a,k;q,p)=\frac{\left[\rho_1,\rho_2;q,p\right]_n}{\left[\frac{aq}{\rho_1},\frac{aq}{\rho_2};q,p\right]_n}(k/m)^n\alpha_n(a,m;q,p), \tag{5}$$

$$\beta'_n(a,k;q,p)=\frac{\left[\frac{mq}{\rho_1},\frac{mq}{\rho_2};q,p\right]_n}{\left[\frac{aq}{\rho_1},\frac{aq}{\rho_2};q,p\right]_n}\sum_{r=0}^{n}\frac{\theta\left(mq^{2r};p\right)\left[\rho_1,\rho_2;q,p\right]_r}{\theta(m;p)\left[\frac{mq}{\rho_1},\frac{mq}{\rho_2};q,p\right]_r}\times \times\frac{[k/m;q,p]_{n-r}[k;q,p]_{n+r}}{[q;q,p]_{n-r}[mq;q,p]_{n+r}}(k/m)^r\beta_r(a,m;q,p), \tag{6}$$

where

$$m=\rho_1\rho_2k/aq.$$

(Warnaar; theorem (1)

Theorem 2

If

$$\left\{\alpha_n(a,k;q,p),\beta_n(a,k;q,p)\right\}$$

is an elliptic WP Bailey pair, then so the pair

$$\left\{\alpha'_n(a,k;q,p),\beta'_n(a,k;q,p)\right\}$$

given by

$$\alpha'_n\left(a^2,k;q^2,p^2\right)=\alpha_n(a,m;q,p) \tag{7}$$

$$\beta'_n\left(a^2,k;q^2,p^2\right)=\frac{[-mq;q,p]_{2n}}{[-aq;q,p]_{2n}}\sum_{r=0}^{\infty}\frac{\theta\left(mq^{2r};p\right)\left[k/m^2;q^2,p^2\right]_{n-r}}{\theta(m;p)\left[q^2;q^2,p^2\right]_{n-r}}\times \times\frac{\left[k;q^2,p^2\right]_{n+r}}{\left[m^2q^2;q^2,p^2\right]_{n+r}}\left(\frac{m}{a}\right)^{n-r}\beta_r(a,m;q,p), \tag{8}$$

where

$$m=k/aq.$$

(Warnnar ; theorem (2)

Theorem 3

If

$$\left\{\alpha_n(a,k;q,p),\beta_n(a,k;q,p)\right\}$$

is an elliptic WP Bailey pair, then so is the pair

$$\left\{\alpha'_n(a,k;q,p),\beta'_n(a,k;q,p)\right\}$$

given by

$$\alpha'_{2n}[a,k;q,p]=\alpha_n\left[a,m;q^2,p\right],\quad \alpha'_{2n+1}[a,k;q,p]=0 \tag{9}$$

$$\beta'_n(a,k;q,p)=\frac{\left[mq;q^2,p\right]_n}{\left[aq,q^2,p\right]_n}\sum_{r=0}^{[n/2]}\frac{\theta\left(mq^{4r};p\right)[k/m;q,p]_{n-2r}}{\theta(m;p)[q;q,p]_{n-2r}}\times \times\frac{[k;q,p]_{n+2r}}{[mq;q,p]_{n+2r}}(-k/a)^{n-2r}\beta_r\left(a,m;q^2,p\right), \tag{10}$$

where

$$m=k^2/a.$$

[Warnnar ; theorem (3.3)

2. Elliptic WP Bailey pairs

In this section we shall obtain elliptic WP Bailey pairs. Frankel and Turaev established the following summation formula

$${}_{10}V_9\left[a;b,c,d,e,q^{-n};q,p\right]=\frac{[aq,aq/bc,aq/bd,aq/cd;q,p]_n}{[q,aq/b,aq/c,aq/d,aq/bcd;q,p]_n}, \tag{11}$$

Where

$a^2q = bcdeq^{-n}$. (Gasper, G. & Rahman ; p.307)

Putting

kq^n

for e in (11) we get,

$$_{10}V_9\left[a;b,c,\frac{a^2q}{bck},kq^n,q^{-n};q,p\right]$$
$$=\sum_{k=0}^{n}\frac{\theta\left(aq^{2k};p\right)\left[a,b,c,\frac{a^2q}{bck},kq^n,q^{-n};q,p\right]_k q^k}{\theta(a;p)\left[q,aq/b,aq/c,bck/a,aq^{1-n}/k,aq^{1+n};q,p\right]_k}$$
$$=\frac{\left[aq,aq/bc,bk/a,ck/a;q,p\right]_n}{\left[aq/b,aq/c,bck/a,k/a;q,p\right]_n}. \tag{12}$$

Now, choosing

$$\alpha_r(a,k;q,p)=\frac{\theta\left(aq^{2r};p\right)}{\theta(a;p)}\frac{\left[a,b,c,a^2q/bck;q,p\right]_r(k/a)^r}{\left[q,aq/b,aq/c,bck/a;q,p\right]_r}$$

in (4) we get

$$\beta_n(a,k;q,p)=\frac{[k,k/a;q,p]_n\left[aq,aq/bc,bk/a,ck/a;q,p\right]_n}{[q,aq;q,p]_n\left[aq/b,aq/c,bck/a,k/a;q,p\right]_n}$$
$$=\frac{\left[k,aq/bc,bk/a,ck/a;q,p\right]_n}{\left[q,aq/b,aq/c,bck/a;q,p\right]_n}. \tag{13}$$

Thus we find that

$$\alpha_n(a,k;q,p)=\frac{\theta\left(aq^{2r};p\right)}{\theta(a;p)}\frac{\left[a,b,c,a^2q/bck;q,p\right]_n(k/a)^n}{\left[q,aq/b,aq/c,bck/a;q,p\right]_n}$$

and

$$\beta_n(a,k;q,p)=\frac{\left[k,aq/bc,bk/a,ck/a;q,p\right]_n}{\left[q,aq/b,aq/c,bck/a;q,p\right]_n} \tag{14}$$

form an elliptic WP Bailey pair.

Now, we shall make use of the elliptic WP Bailey pair given in (14) and theorems 1 to 3 given in previous section to obtain another elliptic WP Bailey pairs.

(1) If we use elliptic WP Bailey pair given in (14) in theorem1, equation (5) and (6) we find another elliptic WP Bailey pair given by

$$\alpha'_n(a,k;q,p)=\frac{\left[\rho_1,\rho_2;q,p\right]_n}{\left[\frac{aq}{\rho_1},\frac{aq}{\rho_2};q,p\right]_n}(k/m)^n\frac{\theta\left(aq^{2n};p\right)}{\theta(a;p)}\times$$
$$\times\frac{\left[a,b,c,a^2q/bcm;q,p\right]_n(m/a)^n}{\left[q,aq/b,aq/c,bcm/a;q,p\right]_n} \tag{15}$$

and

$$\beta'_n(a,k;q,p)=\frac{\left[\frac{mq}{\rho_1},\frac{mq}{\rho_2};q,p\right]_n[k,k/m;q,p]_n}{\left[\frac{aq}{\rho_1},\frac{aq}{\rho_2};q,p\right]_n[q,mq;q,p]_n}\times$$
$$_{12}V_{11}\left[m;\rho_1,\rho_2,aq/bc,bm/a,cm/a,kq^n,q^{-n};q,p\right]. \tag{16}$$

(ii) Replacing elliptic WP Bailey pair in equation (4) by new elliptic WP Bailey pair given in (15) and (16) we get a transformation formula

$$_{12}V_{11}\left[m;\rho_1,\rho_2,aq/bc,bm/a,cm/a,kq^n,q^{-n};q,p\right]$$
$$=\frac{\left[\frac{aq}{\rho_1},\frac{aq}{\rho_2},mq,k/a;q,p\right]_n}{\left[\frac{mq}{\rho_1},\frac{mq}{\rho_2},k/m,aq;q,p\right]_n}\times$$
$$_{12}V_{11}\left[a;b,c,\frac{a^2q}{bcm},\rho_1,\rho_2,kq^n,q^{-n};q,p\right], \tag{17}$$

Where

$m=\frac{\rho_1\rho_2 k}{a}$.

(iii) Putting the elliptic WP Bailey pair given in (14) in theorem 2, equation (7) and (8) we get another elliptic WP Bailey pair given by (17)

$$\beta'_n\left(a^2,k;q^2,p^2\right)=\frac{[-mq;q,p]_{2n}}{[-aq;q,p]_{2n}}\frac{\left[\frac{k}{m^2},k;q^2,p^2\right]_n}{\left[q^2,m^2q^2;q^2,p^2\right]_n}\left(\frac{m}{a}\right)^n\times$$
$$\times\sum_{r=0}^{n}\frac{\theta\left(mq^{2r};p\right)\left[kq^{2n},q^{-2n};q^2,p^2\right]_r}{\theta(m;p)\left[\frac{m^2}{k}q^{2-2n},m^2q^{2+2n};q^2,p^2\right]_r}\left(\frac{maq^2}{k}\right)^r\times$$
$$\times\frac{\left[m,aq/bc,mb/a,mc/a;q,p\right]_r}{\left[q,aq/b,aq/c,bcm/a;q,p\right]_r},$$
$$=\frac{\left[-mq,-mq^2;q^2,p\right]_n\left[k,k/m^2;q^2,p^2\right]_n\left(\frac{m}{a}\right)^n}{\left[-aq,-aq^2;q^2,p\right]_n\left[q^2,m^2q^2;q^2,p^2\right]_n}\times$$
$$\times{}_{12}V_{11}\left[m;\frac{aq}{bc},\frac{mb}{a},\frac{mc}{a},\sqrt{k}q^n,-\sqrt{k}q^n,q^{-n},-q^{-n};q,p;\frac{maq}{k}\right], \tag{18}$$

where m = k / aq.

(iv) Replacing elliptic WP Bailey pair of equation (4) by elliptic WP Bailey pair given in (17) and (18) we obtain the transformation

$$_{12}V_{11}\left[m;\frac{aq}{bc},\frac{mb}{a},\frac{mc}{a},\sqrt{k}q^n,-\sqrt{k}q^n,q^{-n},-q^{-n};q,p;\frac{maq}{k}\right]$$
$$=\frac{\left[k/a^2,m^2q^2;q^2,p^2\right]_n\left[-aq,-aq^2;q^2,p\right]_n}{\left[a^2q^2,k/m^2;q^2,p^2\right]_n\left[-mq,-mq^2;q^2,p\right]_n}\times$$

$$\times {}_{12}V_{11}\left[a;b,c,\frac{a^2q}{bcm},\sqrt{k}q^n,-\sqrt{k}q^n,q^{-n},-q^{-n};q,p;\frac{maq}{k}\right], \quad (19)$$

where m = k / aq.

(v) Putting the elliptic WP Bailey pair given in (14) in theorem 3, equation (18) and (19) we get another elliptic WP Bailey pair given by,

$$\alpha'_{2n}(a,k;q,p)=\frac{\theta\left(aq^{4n};p\right)\left[a;b,c,\frac{a^2q^2}{bcm};q^2,p\right]_n\left(\frac{m}{a}\right)^n}{\theta(a;p)\left[q^2,aq^2/b,aq^2/c,bcm/a;q^2,p\right]_n} \quad (20)$$

$$\alpha'_{2n+1}(a,k;q,p)=0$$

$$\beta'_n(a,k;q,p)=\frac{\left[mq;q^2,p\right]_n\left[\frac{k}{m};q,p\right]_n[k;q,p]_n\left(-\frac{k}{a}\right)^n}{\left[aq,q^2,p\right]_n[q;q,p]_n[mq;q,p]_n}\times$$

$$\times\sum_{r=0}^{[n/2]}\frac{\theta\left(mq^{4r};p\right)\left[q^{-n};q,p\right]_{2r}}{\theta(m;p)\left[\frac{m}{k}q^{1-n};q,p\right]_{2r}}\left(\frac{mq}{a}\right)^{2r}\frac{\left[kq^n;q,p\right]_{2r}}{\left[mq^{1+n};q,p\right]_{2r}}\left(\frac{a}{k}\right)^{2r}\times$$

$$\times\frac{\left[m,aq^2/bc,mb/a,mc/a;q^2,p\right]_r}{\left[q^2,aq^2/b,aq^2/c,bcm/a;q^2,p\right]_r}, \quad (21)$$

Where

$m=\frac{k^2}{a}$.

(vi) Replacing elliptic WP Bailey pair in the equation (4) by elliptic WP Bailey pair given in (11) & (12) we get the transformation

$${}_{12}V_{11}\left[m;\frac{aq^2}{bc},\frac{mb}{a},\frac{mc}{a},kq^n,kq^{n+1},q^{-n},q^{-n+1};q^2,p;\frac{m^2a^2}{k^4}\right]$$

$$=\frac{[k/a,mq;q,p]_n\left[aq;q^2,p\right]_n}{[aq,k/m;q,p]_n\left[mq;q^2,p\right]_n}\left(-\frac{a}{k}\right)^n\times$$

$$\times {}_{12}V_{11}\left[a;b,c,\frac{a^2q^2}{bcm},kq^n,kq^{n+1},q^{-n},q^{-n+1};q^2,p;\frac{ma}{k^2}\right], \quad (22)$$

Where

$m=\frac{k^2}{a}$.

3. Special Cases

Taking p = 0 in (17), (10) and (13) we get the following transformations respectively,

(i) $${}_{10}W_9\left[m;\rho_1,\rho_2,aq/bc,mb/a,mc/a,kq^n,q^{-n};q;q\right]$$

$$=\frac{\left[aq/\rho_1,aq/\rho_2,mq,k/a;q\right]_n}{\left[mq/\rho_1,mq/\rho_2,k/m,aq;q\right]_n}\times$$

$$\times {}_{10}W_9\left[a;b,c,\frac{a^2q}{bcm},\rho_1,\rho_2,kq^n,q^{-n};q;q\right] \quad (23)$$

Where

$m=\frac{\rho_1\rho_2k}{a}$.

(ii) $${}_{10}W_9\left[m;\frac{aq}{bc},\frac{mb}{a},\frac{mc}{a},q^n\sqrt{k},-q^n\sqrt{k},q^{-n},-q^{-n};q;\frac{maq^2}{k}\right]$$

$$=\frac{\left[k/a^2,m^2q^2;q^2\right]_n\left[-aq,-aq^2;q^2\right]_n}{\left[a^2q^2,k/m^2,-mq,-mq^2;q^2\right]_n}\times$$

$${}_{10}W_9\left[a;b,c,\frac{a^2q}{bcm},q^n\sqrt{k},-q^n\sqrt{k},q^{-n},-q^{-n};q;\frac{maq^2}{k}\right], \quad (24)$$

where m = k / aq.

(iii) $${}_{10}W_9\left[m;\frac{aq^2}{bc},\frac{mb}{a},\frac{mc}{a},kq^n,kq^{n+1},q^{-n},q^{-n+1};q^2;\frac{m^2a^2q^2}{k^4}\right]$$

$$=\frac{[k/a,mq;q]_n\left[aq;q^2\right]_n(-a/k)^n}{[aq,k/m;q]_n\left[mq;q^2\right]_n}\times$$

$$\times {}_{10}W_9\left[a;b,c,\frac{a^2q^2}{bcm},kq^n,kq^{n+1},q^{-n},q^{-n+1};q^2;\frac{maq^2}{k^2}\right], \quad (25)$$

Where

$m=\frac{k^2}{a}$.

(24) can be written as,

$${}_8\Phi_7\left[\begin{matrix}\frac{\rho_1\rho_2k}{a},q\sqrt{\frac{\rho_1\rho_2k}{a}},-q\sqrt{\frac{\rho_1\rho_2k}{a}},\frac{aq}{bc},\frac{\rho_1\rho_2kb}{a^2},\frac{\rho_1\rho_2kc}{a^2},kq^n,q^{-n};q;q\\ \sqrt{\frac{\rho_1\rho_2k}{a}},-\sqrt{\frac{\rho_1\rho_2k}{a}},\frac{\rho_1\rho_2kbc}{a^2},\frac{aq}{b},\frac{aq}{c},\frac{\rho_1\rho_2q^{1-n}}{a},\frac{\rho_1\rho_2kq^{1+n}}{a}\end{matrix}\right]$$

$$=\frac{\left[\frac{aq}{\rho_1},\frac{aq}{\rho_2},\frac{\rho_1\rho_2kq}{a},\frac{k}{a};q\right]_n}{\left[\frac{\rho_1kq}{a},\frac{\rho_2kq}{a},\frac{a}{\rho_1\rho_2},aq;q\right]_n}\times$$

$$\times {}_8\Phi_7\left[\begin{matrix}a,q\sqrt{a},-q\sqrt{a},b,c,\frac{a^3q}{\rho_1\rho_2kbc},\rho_1,\rho_2,kq^n,q^{-n};q;q\\ \sqrt{a},-\sqrt{a},\frac{aq}{b},\frac{aq}{c},\frac{\rho_1\rho_2kbc}{a^2},\frac{aq}{\rho_1},\frac{aq}{\rho_2},\frac{aq^{1-n}}{k},aq^{1+n}\end{matrix}\right]. \quad (26)$$

Taking $\rho_1=\sqrt{a},\rho_2=-\sqrt{a}$ in (26) we get

$${}_8\Phi_7\left[\begin{matrix}-k,q\sqrt{-k},-q\sqrt{-k},\frac{aq}{bc},\frac{-kb}{a},\frac{-kc}{a},kq^n,q^{-n};q;q\\ \sqrt{-k},-\sqrt{-k},\frac{-kbc}{a},\frac{aq}{b},\frac{aq}{c},-q^{1-n},-kq^{1+n}\end{matrix}\right]$$

$$=\frac{\left[q\sqrt{a},-q\sqrt{a},-kq,\frac{k}{a};q\right]_n}{\left[\frac{kq}{\sqrt{a}},-\frac{kq}{\sqrt{a}},-1,aq;q\right]_n}\times$$

$$\times{}_6\Phi_5\left[\begin{matrix}a,b,c,-\frac{a^2q}{kbc},kq^n,q^{-n};q;q\\ \frac{aq}{b},\frac{aq}{c},\frac{-kbc}{a},\frac{aq^{1-n}}{k},aq^{1+n}\end{matrix}\right]. \qquad (27)$$

Taking b = 1 in (27) we have

$${}_6\Phi_5\left[\begin{matrix}-k,iq\sqrt{k},-iq\sqrt{k},-\frac{k}{a},kq^n,q^{-n};q;q\\ i\sqrt{k},-i\sqrt{k},aq,-q^{1-n},-kq^{1+n}\end{matrix}\right].$$

$$=\frac{\left(1-aq^{2n}\right)\left[-kq,k/a;q\right]_n}{\left(1-\frac{k^2}{a}q^{2n}\right)\left[-1,aq;q\right]_n}. \qquad (28)$$

A number of similar results can also be scored.

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