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Research Article

A Comparative Study of two Prominent Phase Unwrapping Algorithms used In Digital Holography

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Abstract

Phase wrapping is one of the most important concerns in digital holography. Although many algorithms (David R Burton et al, 1994) were developed to implement phase unwrapping, most of them had several drawbacks such as data corruption due to ambiguities and time consumption. In this paper, we compare two different approaches for phase unwrapping. The first is weighted and unweighted phase unwrapping proposed by Ghiglia. The second is branch cut method proposed by Goldstein. The former gave efficient results even in the case of high levels of noise and solved the problem of regional inconsistencies whereas the latter limited the propagation of local errors as global errors.

Keywords: *Phase unwrapping, digital holography, radar interferometry.*

Introduction

Digital holography is a modern perspective of conventional holography in which chemical recording material is replaced by CCD or CMOS sensor. The electronically detected holograms are called digital holograms and stored in a computer. The reconstruction is performed by simulating the optical reconstruction process using a computer. This will enable us to get both amplitude and phase in the computational domain. The phase information carries 3-D information of the object.
As shown in Fig. 1, for recording a Digital hologram we use a beam splitter to split coherent and monochromatic light (laser beam) into two parts, the first is used to illuminate the object, which is reflected to the recording device .The second one is used to illuminate the recording medium directly called a reference wave. Both waves interfere and the resulting interference pattern is recorded on the CCD/CMOS sensor.

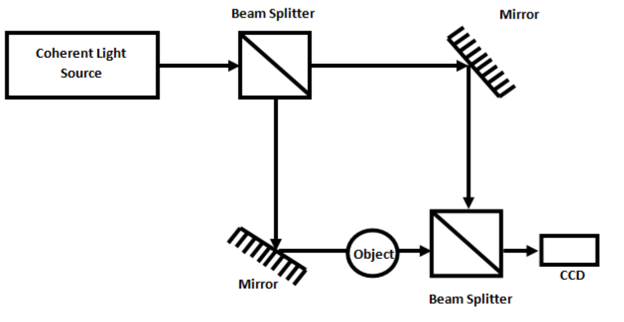

Fig. 1 Mach Zehnder interferometer setup for digital holography

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This interference pattern is a digital hologram which is a two dimensional digital image. By illuminating the hologram with the reference wave again, the original object wave is reconstructed (Thomas M. Kreis *et al*, 1997) In this case this is simulated using a computer. This reconstructed image gives both magnitude and phase information of the object in the form of a complex field.

Thus digital holography is an interesting technological tool for 3-D imaging which can be analyzed by Fourier optics and signal processing. The phase is calculated from the complex wave field *b' (x',y')* using the formula

$$\Phi(x, y) = \tan^{-1}\left\{\frac{Im[b'(x', y')]}{Re[b'(x', y')]}\right\}$$

This is a wrapped phase map with values in the range of [-π, π]. Since many algorithms that compute the phase of a signal gave phases between -π and π, it created problems in variety of applications such as terrain elevation estimation in synthetic aperture radar (SAR) (RM. Goldstein *et al*, 1988) field mapping in magnetic resonance imaging (MRI) and wave front distortion measurement in adaptive optics.

Phase unwrapping is used to reconstruct signal's original phase. Unwrap algorithms add approximate multiples of 2π to each phase input to restore original phase values. Phase unwrapping is not only a concern to digital holography, but it is also present in different fields, it has been the object of research for a long time, and many different algorithms have been developed for its resolution. Different orientations and approaches have been tried, from simple algorithms developed by (A. Oppenheim *et al*, 1975) to complex and effective techniques such as the cellular automata developed by Ghiglia. Unfortunately, speed and accuracy are always a matter of concern, and reliable unwrappers can lead to time-consuming processing.

Another important issue regarding these algorithms is the propagation of errors. This issue was the main reason why unwrappers based on tiles or regions were developed. These unwrappers are based on the partitioning of the image into smaller areas, with these areas being independently unwrapped. Thus errors are limited to relatively small areas and propagation of errors beyond tile boundaries is avoided.

Weighted and Unweighted phase unwrapping

Let the wrapped phase values be ψ and unwrapped phase values be ϕ. Now we define wrapping operator W that wraps all values of its argument into the range [-π, π]. (D. C. Ghiglia *et al*, 1994)

Next step is to compute two sets of differences with respect to i index and those with respect to the j index.

$\Delta^{x}_{i,j} = W\{\psi_{i+1,j} - \psi_{i,j}\}$
$i=0......M\text{-}2,\ j=0.....N\text{-}1.$
$\Delta^{x}_{i,j} = 0;$
$i=0......M\text{-}1,\ j=0.....N\text{-}2.$
$\Delta^{y}_{i,j} = W\{\psi_{i,j+1} - \psi_{i,j}\}$
$\Delta^{y}_{i,j} = 0;$

Using the above differences the least squares solution is found out.

Now

$\rho_{i,j} = (\Delta^{x}_{i,j} + \Delta^{x}_{i-1,j}) + (\Delta^{y}_{i,j} + \Delta^{y}_{i,j-1})$

Using Poisson equation we get

$$\frac{\partial^2}{\partial x^2}\varphi(x,y) + \frac{\partial^2}{\partial y^2}\varphi(x,y) = \rho(x,y)$$

Using cosine transforms we get

$$\varphi = \frac{\rho}{2(\cos\frac{\pi i}{M} + \cos\frac{\pi j}{N} - 2)}$$

The unwrapped phase is now easily obtained by the inverse DCT of above equation.
The main steps in the algorithm are:
1) First we perform 2D forward DCT of the array of values computed by using poisons equation to yield the 2DDCT values.
2) Perform the 2d inverse DCT to obtain the least squares unwrapped phase values.

Branch cut model

Generally there are two types of errors are possible in the image. Local errors in which few points are corrupted and global errors in which local errors may be propagated down the entire sequence. In the case of two dimensional matrixes we have a series of adjacent sequences of phase values. (R. M. Goldstein *et al*, 1988)

0.0 0.1 0.2 0.3
0.0 0.0 0.3 0.4
0.9 0.8 0.6 0.5
0.8 0.8 0.7 0.6

Considering the above phase measurements we would reconstruct the whole phase from these measurements. We know that no two adjacent points differ by more than one half cycle. So we add one cycle where inconsistency is there (between 0.0 & 0.9 and 0.0 & 0.8).

The resulting distribution is

0.0 0.1 0.2 0.3
0.0 0.0 0.3 0.4
-0.1 -0.2 -0.4 -0.5
-0.2 -0.2 -0.3 -0.4

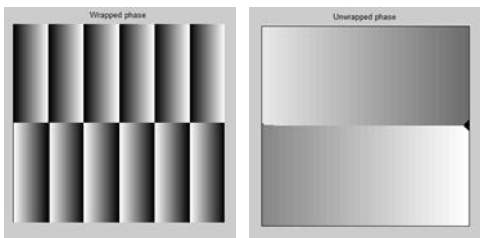
Even now there is inconsistency (between 0.3 and -0.4 and 04 and-0.5).to avoid this we follow branch cut method.

In this method we calculate sum of phase differences clock wise around each set of four adjacent points. It is either zero or plus one cycle or minus one cycle. We refer to the net sum as residues associated with four points.

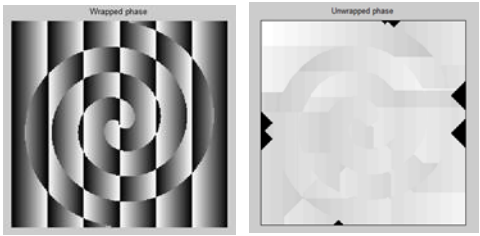
0.0 0.1 0.2 0.3
0 0 0
0.0 0.0 0.3 0.4
0 +1 0
0.9 0.8 0.6 0.5
0 0 0
0.8 0.8 0.7 0.6

Any integration path that encloses residue produces an inconsistency in the unwrapped phase. However if a path has equal number of plus and minus residues there is no inconsistency. So the residues are identified and suitable branch cuts are made between residues to prevent any integration from crossing these cuts. Now we need to connect nearby plus and minus residues with cuts which interdict the integration path. A box of size 3 is placed around the residue and searched for another residue, if found, a cut is placed between them. If the residue is of opposite sign we continue the scan and the signs are designated as uncharged. If the sign of the residue is same as original, the box is moved to the new residue and scan continues and the size of the box is increased by two. In the end all the residues lie on the cuts which are uncharged allowing no global errors. Areas where residues are sparse are connected by cuts and areas where they are dense are isolated.

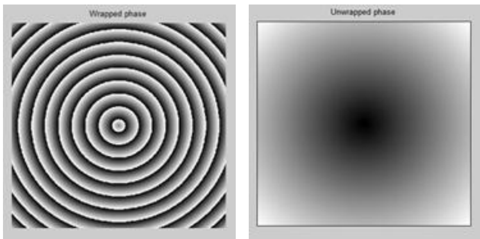
Results and Discussions



(a) (b)



(c) (d)



(e) (f)

Fig.1 (a),(c),(e) represents wrapped phase distribution and (b),(d),(f) represents their unwrapped phase distributions respectively (b) and (d) are generated using Ghiglia method whereas (f) is generated using Goldstein's method.

In the above figure,(a) and (c) are unwrapped using Ghiglia algorithm. In (b) and (d) we can clearly see original phase content. (e) is unwrapped using Goldstein algorithm.(f) is generated without presence of global errors.

Conclusion

In the present paper we have carried out comparative study on two important algorithms used for phase unwrapping. The robustness of the method is analyzed from the results obtained using MATLAB® software. The advantage of unweighted and weighted phase unwrapping is that they permit exact unwrapping of phase data with shears, de-emphasis of suspect phase values, elegant elimination of totally corrupted regions, arbitrarily shaped region unwrapping, and simultaneous unwrapping of multiple isolated arbitrary values. Another advantage is that it is iterative in nature and requires less additional memory compared to previous algorithms. Whereas branch cut method is successful in eliminating global errors.

References

D. C. Ghiglia and L. A. Romero (1994), Robust two-dimensional weighted and unweighted phase unwrapping that uses fast transforms and iterative methods, *J. Opt. Soc. Am*, 11(1), pp 107–117.

Thomas M. Kreis, Werner P.O. Jüptner (1997), Suppression of the dc term in digital holography, *Opt. Eng*, 36(8), pp 2357-2360.

Miguel Arevalillo Herráez, David R. Burton, Michael J. Lalor and David B. Clegg (1996), Robust, simple and fast algorithm for phase unwrapping, *Appl. Op*, 35(29), pp 5847-5852.

Thomas M. Kreis, Mike Adams, Werner P.O. Juptner (1997), Methods of Digital Holography: A Comparison, *Proc. SPIE*, 49(3), pp 224–233

R. M. Goldstein, H. A. Zebker and C.L.Werner (1988), Satellite radar interferometry: two-dimensional phase unwrapping, *Radio Science*, 23(4), pp 713–720.

Anith Nelleri, Joby Joseph and Kehar Singh (2006), Recognition and classification of 3D phase objects by digital Fresnel holography, *Appl. Op*, 45(17), pp 4046-4053.

A. Oppenheim and R. Schafer (1975), Digital Signal Processing *Prentice-Hall*, *Englewood Cliffs*, *N.J*, 10(1), pp 507–511.